A black and white micrograph showing a complex, layered metal microstructure, possibly a lamellar structure or a heavily deformed material. The image features a prominent curled corner effect in the bottom right, suggesting it's a page from a book or document. The text is overlaid on the central part of the image.

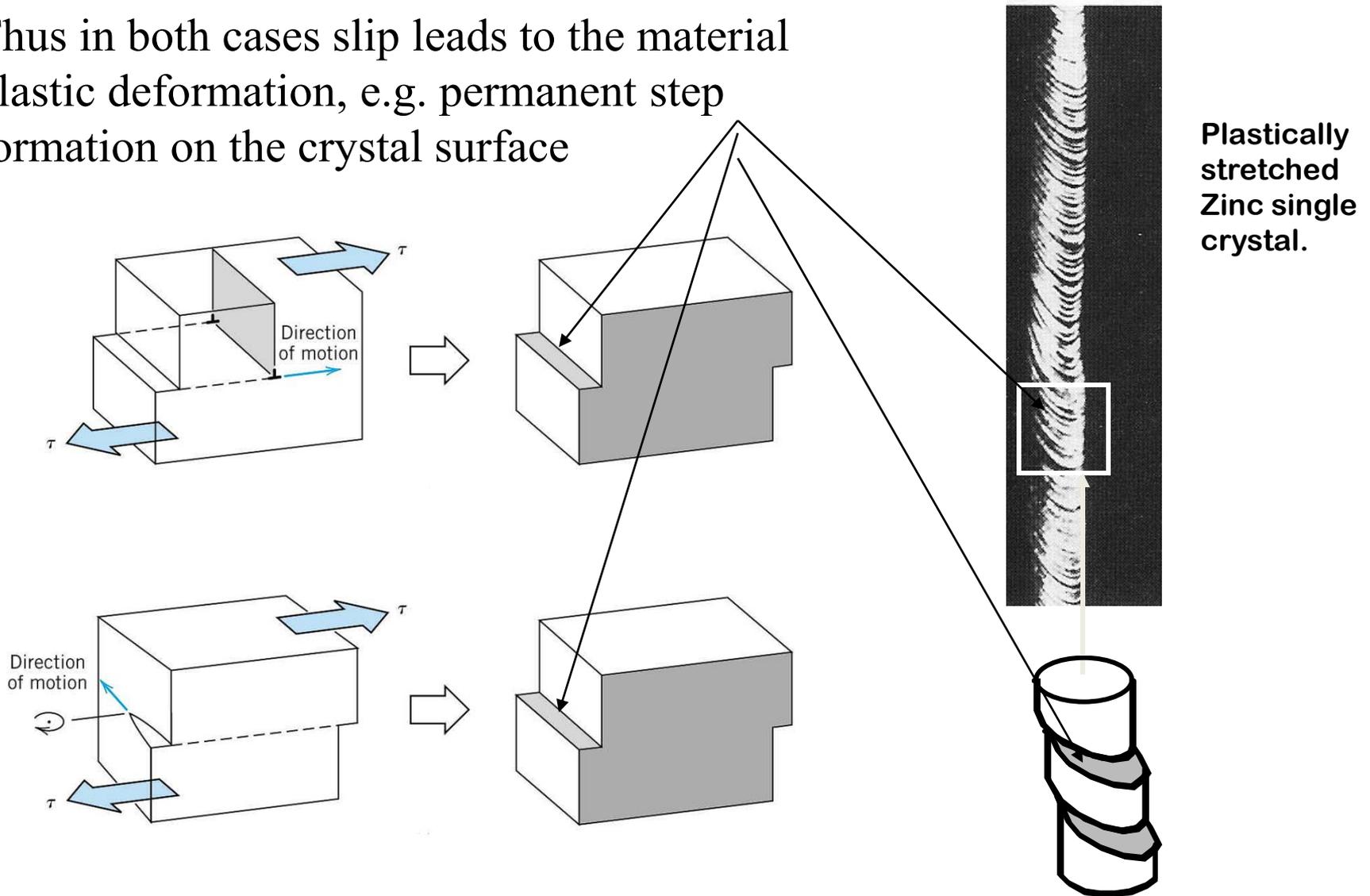
**CHAPTER 7**  
**DISLOCATIONS AND STRENGTHENING**  
**MECHANISMS**

# Slip and Deformation: Conclusion

- Dislocations are the elementary carriers of plastic flow thus they define material mechanical properties
- Dislocations allow deformation at much lower stress than in a perfect crystal because slip does not require all bonds across the slip line to break simultaneously, but only small fraction of the bonds are broken at any given time.

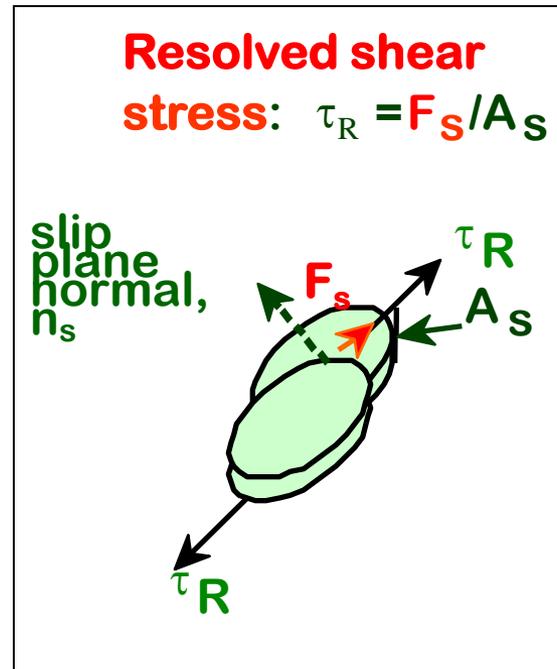
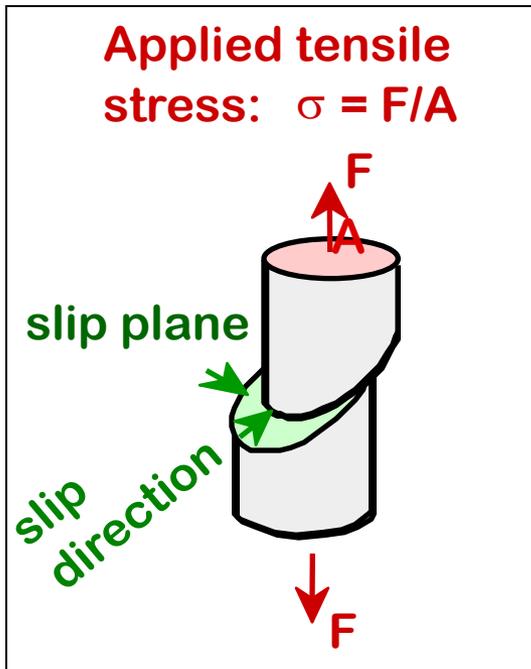
# Slip and Deformation

Thus in both cases slip leads to the material plastic deformation, e.g. permanent step formation on the crystal surface



# SLIP IN SINGLE CRYSTAL: Resolved Shear Stress

- Crystals slip in particular direction on particular plane due to a **resolved shear stress,  $\tau_R$** , i.e. shear stress applied along these plane and direction



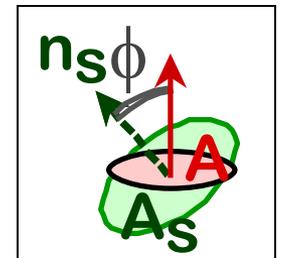
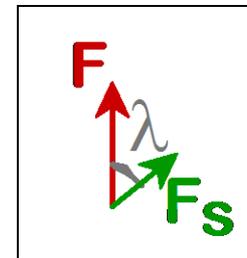
## Relation between

$\sigma$  and  $\tau_R$

$$\tau_R = F_s / A_s$$

$F \cos \lambda$

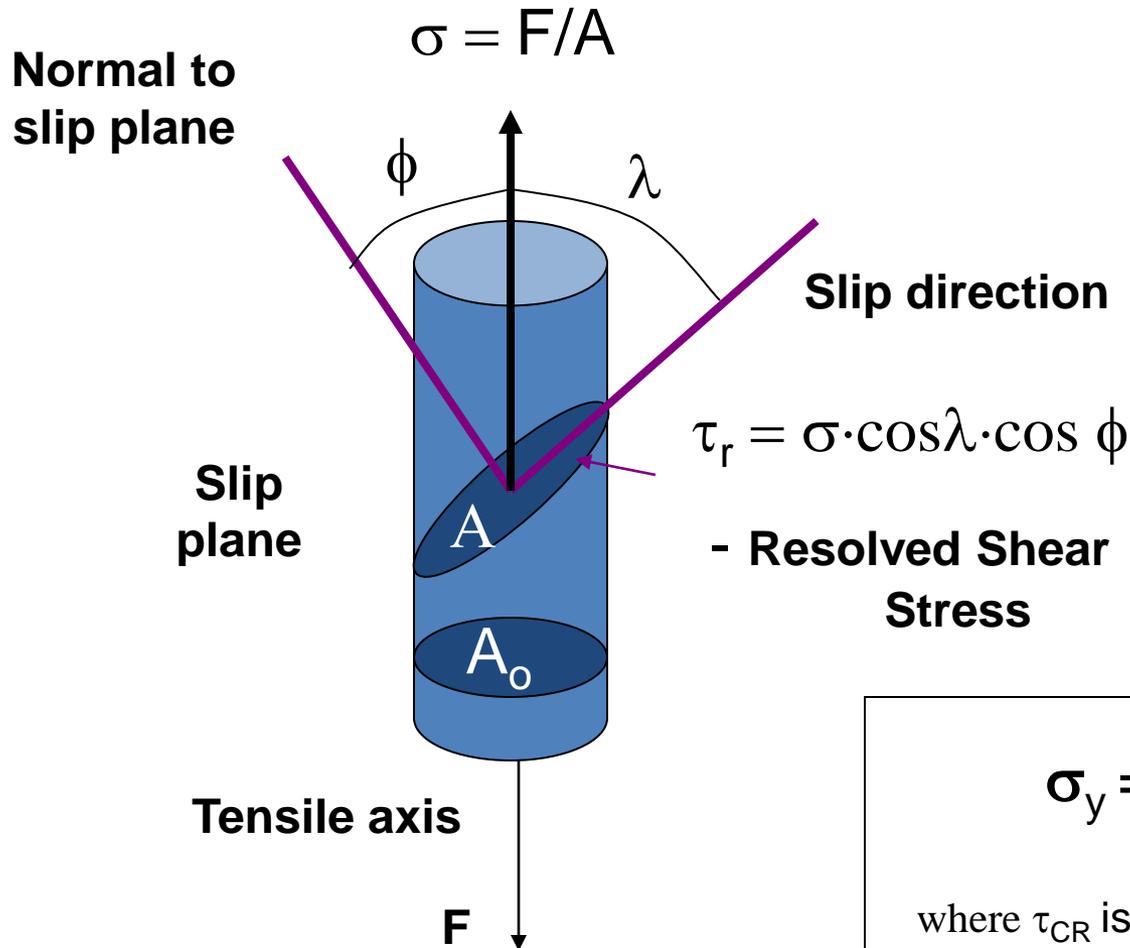
$A / \cos \phi$



$$\tau_R = \sigma (\cos \lambda \cos \phi)$$

Schmidt's factor

# Conclusion: Schmidt's Law



$$\sigma_y = \tau_{CR} / \cos \lambda \cdot \cos \phi$$

where  $\tau_{CR}$  is **critical resolved shear stress**,  
at which plastic deformation occurs  
 $\sigma_C$  is the corresponding **critical normal stress**

In this space we operate with crystallographic directions. Typically, we need to calculate the angle between crystallographic directions,  $[u \ v \ w]$  and  $[u^* \ v^* \ w^*]$ , within a certain crystal. By using full expressions of these vectors, namely,  $\mathbf{r} = u\mathbf{a}_1 + v\mathbf{a}_2 + w\mathbf{a}_3$  and  $\mathbf{r}^* = u^*\mathbf{a}_1 + v^*\mathbf{a}_2 + w^*\mathbf{a}_3$ , we find:

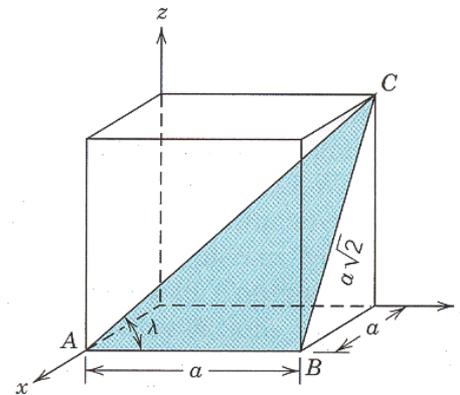
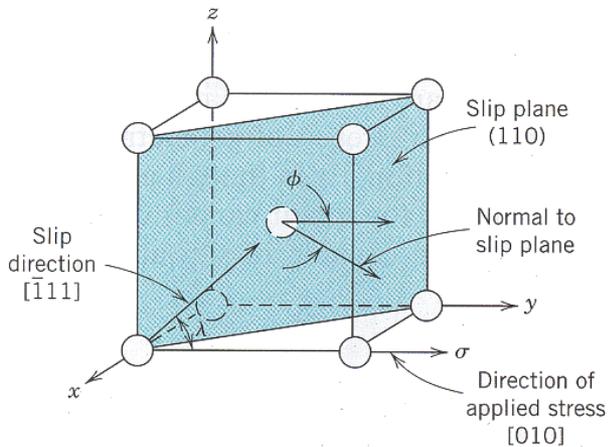
$$\begin{aligned} \cos \varphi &= \frac{\mathbf{r} \cdot \mathbf{r}^*}{|\mathbf{r}| \cdot |\mathbf{r}^*|} = \\ &= \frac{(u\mathbf{a}_1 + v\mathbf{a}_2 + w\mathbf{a}_3) \cdot (u^*\mathbf{a}_1 + v^*\mathbf{a}_2 + w^*\mathbf{a}_3)}{\sqrt{(u\mathbf{a}_1 + v\mathbf{a}_2 + w\mathbf{a}_3) \cdot (u\mathbf{a}_1 + v\mathbf{a}_2 + w\mathbf{a}_3)} \sqrt{(u^*\mathbf{a}_1 + v^*\mathbf{a}_2 + w^*\mathbf{a}_3) \cdot (u^*\mathbf{a}_1 + v^*\mathbf{a}_2 + w^*\mathbf{a}_3)}} \end{aligned}$$

e.g. between  $[010]$  and  $[-111]$  in BCC:  $\cos \lambda = \frac{1}{\sqrt{3}}$

Or between  $[110]$  and  $[010]$  in BCC:  $\cos \phi = \frac{1}{\sqrt{2}}$

# Typical Problem

- A single crystal of BCC iron. Tensile stress (52MPa) is applied along [010] direction. Compute the resolved shear stress along (110) plane and [-111] direction.
- For the same slip system and direction of the applied tensile stress, calculate the magnitude of the applied tensile stress necessary to initiate yielding if critical resolved shear stress known to be equal to 30MPa



$$\tau_R = \sigma (\cos \lambda \cos \phi)$$

$$\tau_R = 52 (1/\sqrt{2}) (1/\sqrt{3}) = 21.2 \text{ MPa}$$

$$\sigma_y = \frac{\tau_R^{\text{critical}}}{(\cos \phi \cdot \cos \lambda)_{\text{max}}}$$

$$\sigma_y = 30 / [(\cos 45^\circ) 1/\sqrt{3}] = 73.4 \text{ MPa}$$

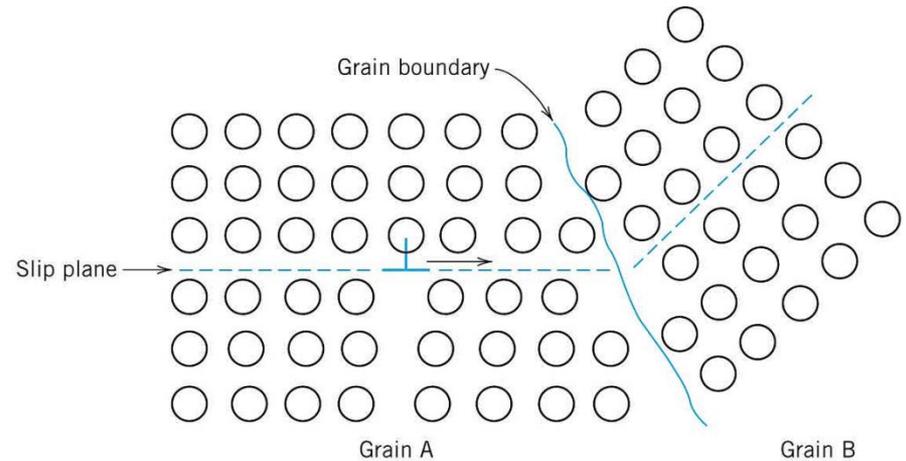
# Mechanical strength is the ability of a material to withstand plastic deformation

- On *a microscopic level* this ability is related to the stress required to initiate the movement of slip systems and the dislocation density.
- The primary principle of strengthening is therefore the **restriction and hindrance of dislocation motion**.
- *Grain Size Reduction* (increase area of grain boundaries):  
**hard to move dislocations across grain boundaries**
- *Solid Solution Strengthening*:  
**pin dislocations**
- *Work-hardening (Strain-hardening)*:  
**increased dislocation density so more dislocation interactions)**

# 1: Reduction of Grain Size

Grain boundaries are **barriers** to slip:

- ❑ Owing to ***misalignment*** of the slip planes in adjacent grains, a dislocation passing the grain boundary have to change its direction and thus lose its energy.
- ❑ A single grain may be favorably oriented for slip, but cannot deform until the adjacent grains (less favorable) are also capable to slip;
- ❑ ***Low angle boundaries*** are less effective in blocking than high angle ones.



- Small grain size: **higher structural disorder**
- Smaller grain size: **more barriers for slip**
- More barrier for slip: **higher material strength**

# Strengthening by Reduction of Grain Size

The *Hall-Petch equation* describes dependence of yield strength,  $\sigma_y$ , as a function of average grain diameter,  $d$ :

$$\sigma_y = \sigma_0 + k_y d^{-0.5}$$

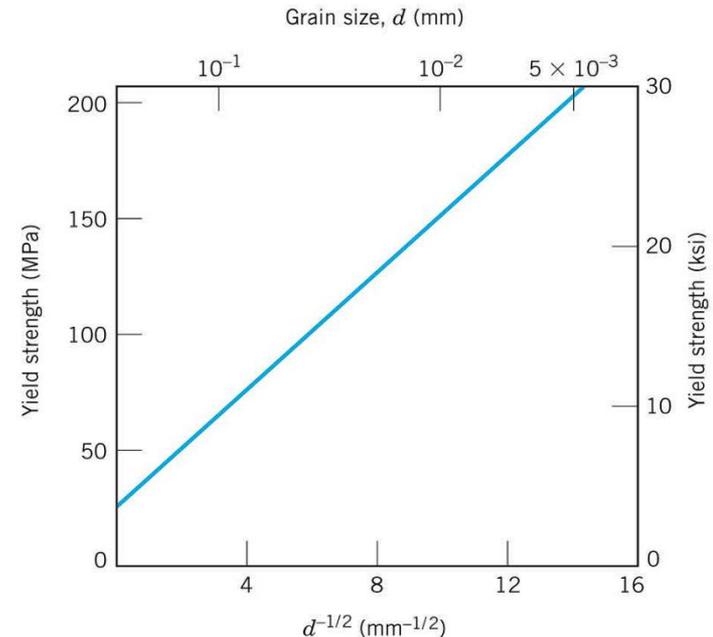
The  $\sigma_0$  is the Peierls (frictional) stress and is the minimum stress needed to induce dislocation glide in a single crystal and  $k_y$  is the Hall-Petch slope.

- Valid for many materials but not for **nano-structures !!**

- Example:  
70wt%Cu-30wt%Zn brass alloy



← 0.75mm →



- Grain size can be changed for example by plastic deformation or heat-treatment

# Typical Problem

*The lower yield point for an iron that has an average grain diameter of  $1 \times 10^{-2}$  mm is 230 MPa (33,000 psi). At a grain diameter of  $6 \times 10^{-3}$  mm, the yield point increases to 275 MPa (40,000 psi). At what grain diameter will the lower yield point be 310 Mpa?*

$$\sigma_y = \sigma_o + k_y d^{-0.5}$$

# CHAPTER 8: MECHANICAL FAILURE



# 1: FRACTURE TOUGHNESS

- Thus it is possible to show that critical stress for crack propagation is:

Stress intensity factor  $\rightarrow$   $\sigma\sqrt{\pi a} = \sqrt{E\gamma_c}$   $\leftarrow$  Material properties only!!!

- **Fracture toughness** is a property that is a measure of a materials' resistance for brittle fracture when cracks are present. This property can be defined by parameter  $K_c$  that relates the critical stress for crack propagation and geometry of the crack:

$$K_c = Y\sigma_c\sqrt{\pi a}$$

$$K_{ic} = [\text{Pa} \cdot \text{m}^{1/2}]!!!$$

where **Y** – a dimensionless parameter that depends on both crack and specimen sizes and geometries (is tabulated for different crack-specimen geometries), as well as the type of load application

# GEOMETRY, LOAD, & MATERIAL

- Condition for crack propagation:

**Stress Intensity Factor:**  $\longrightarrow$

$$K \geq K_c$$

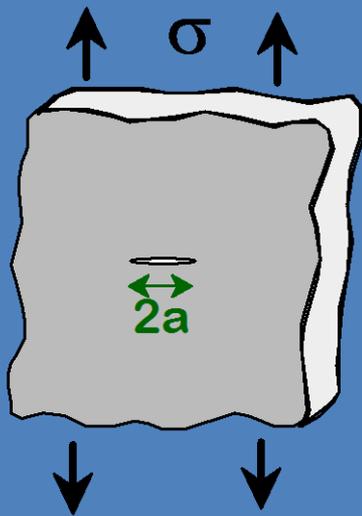


**Fracture Toughness:**

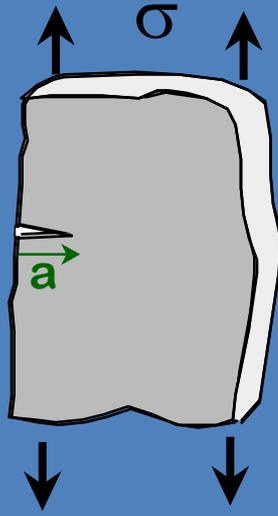
-Depends on load & geometry

-Depends on the material, temperature, environment, & rate of loading.

- Values of  $K$  for some standard loads & geometries:



$$K = \sigma \sqrt{\pi a}$$



$$K = 1.1 \sigma \sqrt{\pi a}$$

When sample thickness  $\gg$  crack dimensions

plane-strain conditions occur,

$$K_c = K_{1c}$$

that is independent of thickness

$$K_{1c} = Y \sigma_c \sqrt{\pi a}$$

Plane strain fracture toughness

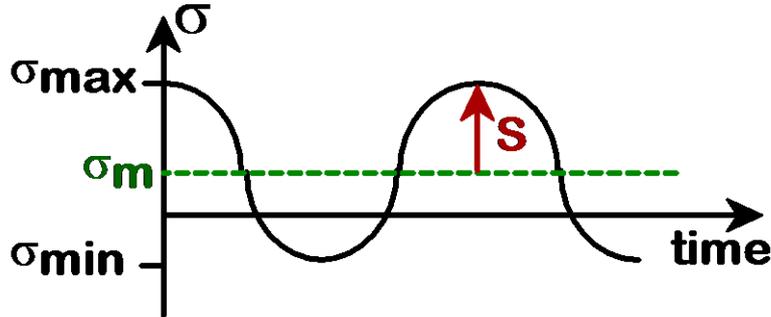
# Typical Problem

*An aircraft component is fabricated from an aluminum alloy that has a plane strain fracture toughness of 40 MPa . It has been determined that fracture results at a stress of 300 MPa when the maximum (or critical) internal crack length is 4.0 mm. For this same component and alloy, will fracture occur at a stress level of 260 MPa when the maximum internal crack length is 6.0 mm? Why or why not?*

$$K_c = Y\sigma_c \sqrt{\pi a}$$

# FATIGUE

• **Fatigue** is a failure **under cyclic stress** (bridges, aircraft etc.).



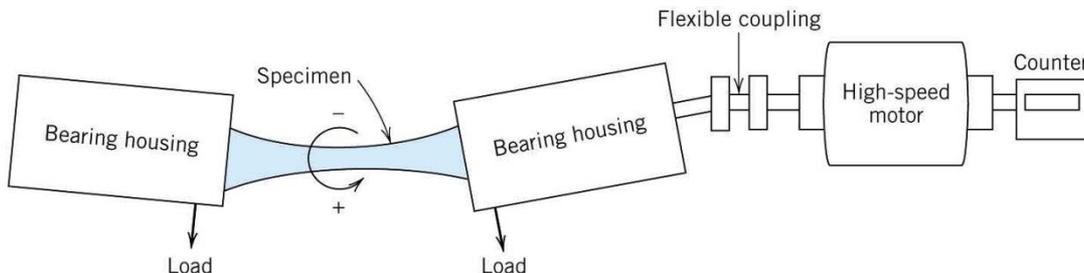
- Stress varies with time:
  - key parameters are  $S$ ,  $\sigma_{\text{mean}}$ ,  $\Delta\sigma$

$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$

$$\Delta\sigma_a = S = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$

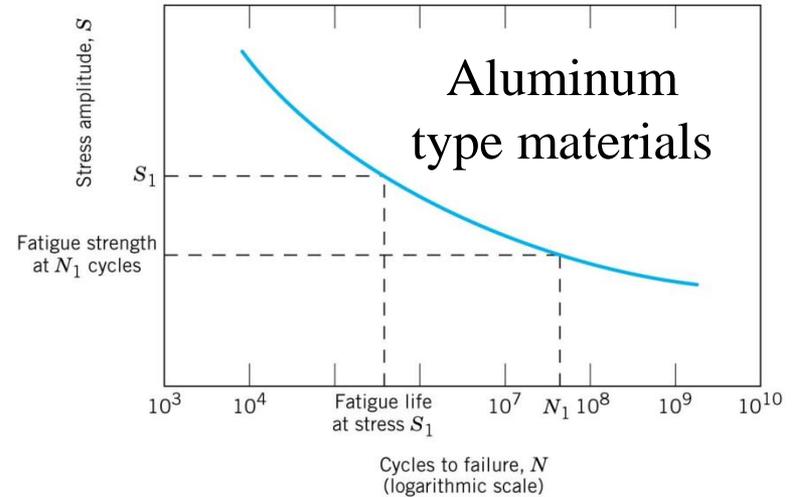
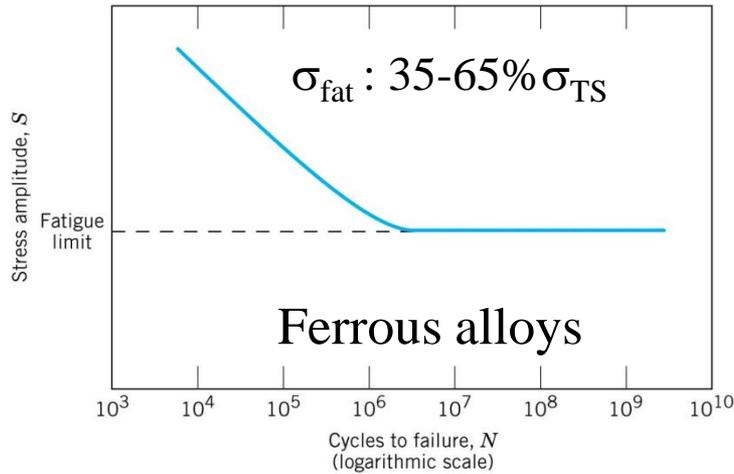
• **Key points:**

- Fatigue can cause part failure, even though  $\sigma_{\max} < \sigma_c$ , i.e. at lower strength than for a static conditions;
- Fatigue causes ~ 90% of mechanical engineering failures.

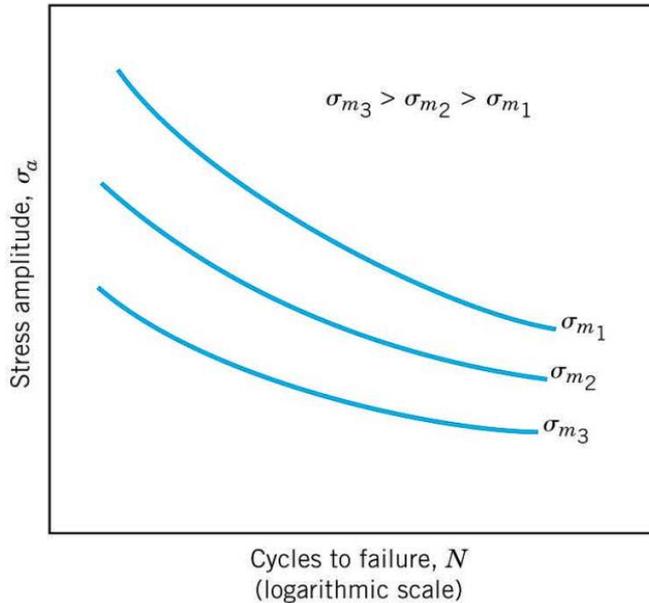


Fatigue-testing apparatus for rotating-bending test: main parameters  $S$  and number of cycles ( $N$ )

# S-N DIAGRAMS: FATIGUE DESIGN PARAMETERS



- **Fatigue limit,  $S_{fat}$** : -no fatigue if  $S < S_{fat}$



- Sometimes, the fatigue limit is **zero!**

## Factors affect fatigue life:

- increasing the **mean stress** leads to a decrease in fatigue life
- **surface effects**: design factor, surface treatment

# Typical Problem

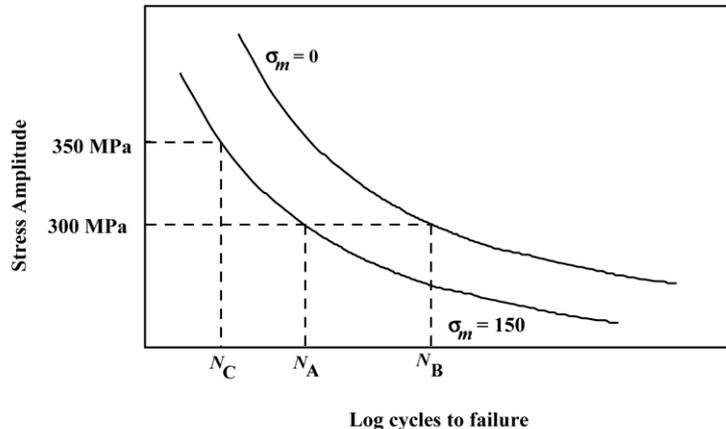
Three identical fatigue specimens (denoted A, B, and C) are fabricated from a nonferrous alloy. Each is subjected to one of the maximum-minimum stress cycles listed in the following table; the frequency is the same for all three tests.

- (a) Rank the fatigue lifetimes of these three specimens from the longest to the shortest.  
 (b) Now justify this ranking using a schematic S–N plot.

Specimen	$\sigma_{\max}$ (MPa)	$\sigma_{\min}$ (MPa)
A	+450	-150
B	+300	-300
C	+500	-200

$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$

$$\Delta\sigma_a = S = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$



$$S_a(A) = \frac{450 \text{ MPa} - (-150 \text{ MPa})}{2} = 300 \text{ MPa}$$

$$S_a(B) = \frac{300 \text{ MPa} - (-300 \text{ MPa})}{2} = 300 \text{ MPa}$$

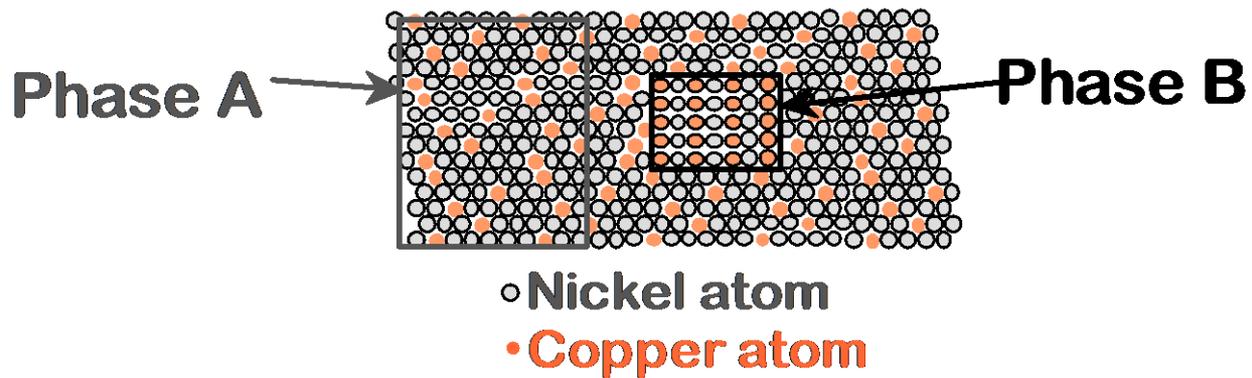
$$S_a(C) = \frac{500 \text{ MPa} - (-200 \text{ MPa})}{2} = 350 \text{ MPa}$$

$$S_m(A) = \frac{450 \text{ MPa} + (-150 \text{ MPa})}{2} = 150 \text{ MPa}$$

$$S_m(B) = \frac{300 \text{ MPa} + (-300 \text{ MPa})}{2} = 0 \text{ MPa}$$

$$S_m(C) = \frac{500 \text{ MPa} + (-200 \text{ MPa})}{2} = 150 \text{ MPa}$$

# CHAPTER 9: PHASE DIAGRAMS



# Pb-Sn EUTECTIC SYSTEM (2)

● For point B,  
i.e. alloy 40%Sn-60wt%Pb at  
150°C, *find...*

-What are the compositions  
of the phases?

- Answer:

$$C_{\alpha} = 11\% \text{ Sn} - 89\text{wt}\% \text{ Pb}$$

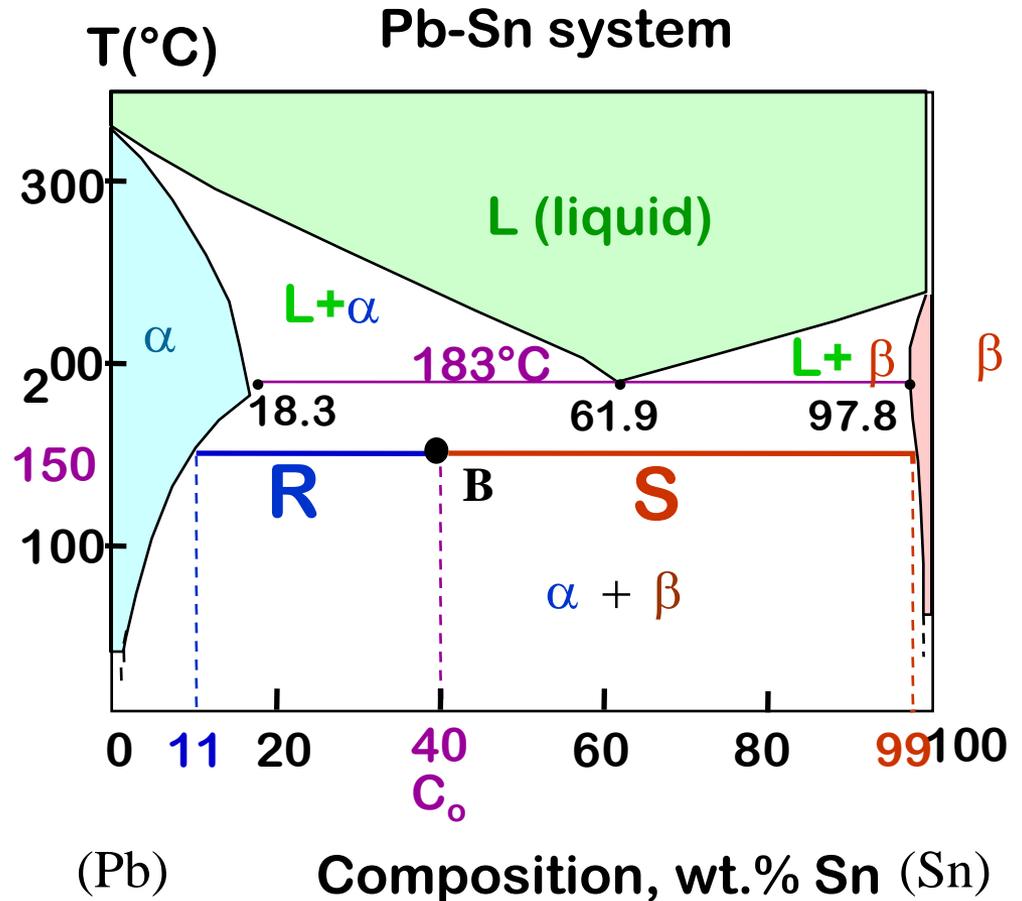
$$C_{\beta} = 99\% \text{ Sn} - 1\text{wt}\% \text{ Pb}$$

-What are the relative amounts  
of each phase?

-Answer:

$$W_{\alpha} = \frac{59}{88} = 67\text{wt}\%$$

$$W_{\beta} = \frac{29}{88} = 33\text{wt}\%$$



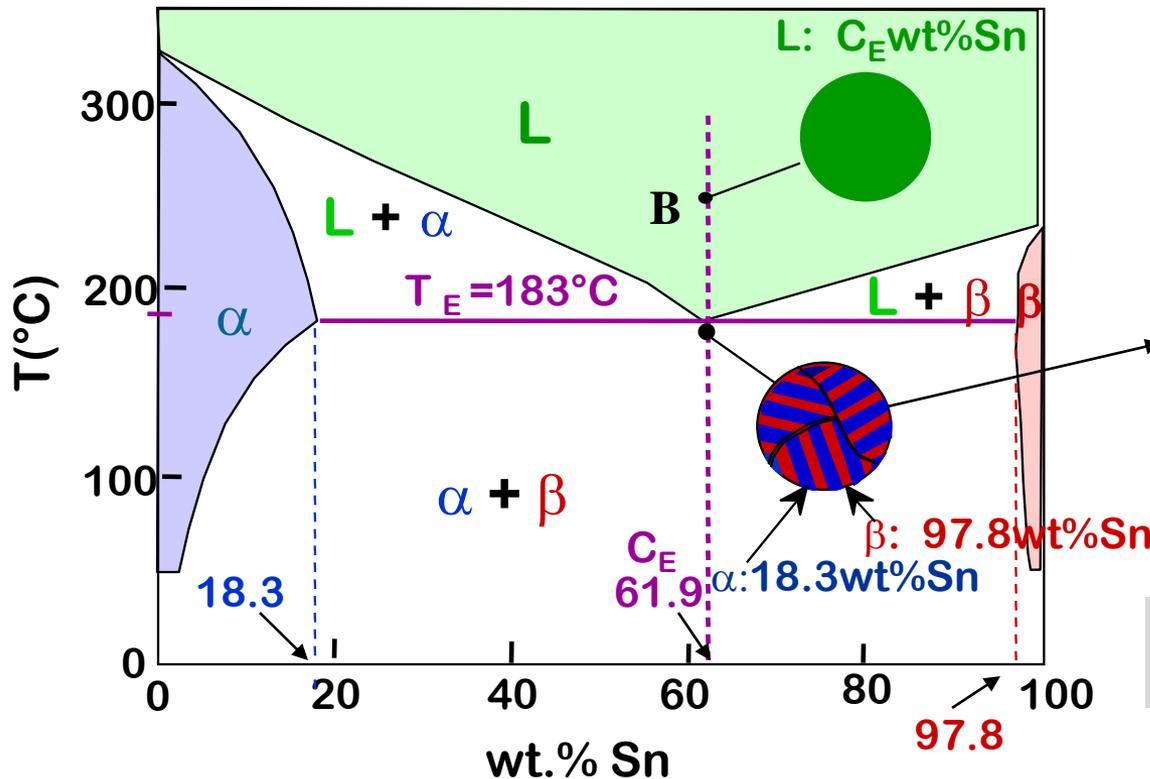
# MICROSTRUCTURES IN EUTECTIC SYSTEMS:

## Equilibrium Cooling (3)

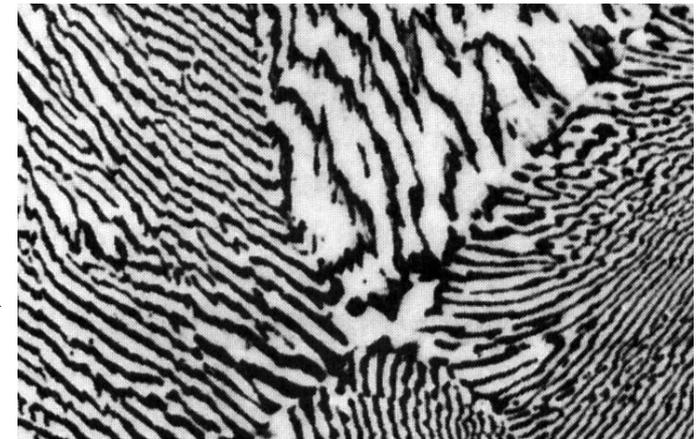
- **Composition range:  $C = C_E$**

- **Result:**

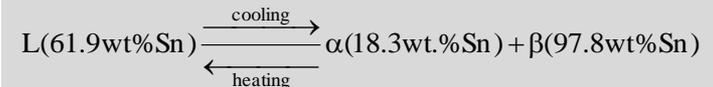
- $T > T_E$ : **liquid** with  $C = C_E = 61.9$  wt.% Sn
- $T < T_E$ : **alternating layers of  $\alpha$  and  $\beta$  crystals.**



**Microstructure of Pb-Sn eutectic lamellae**

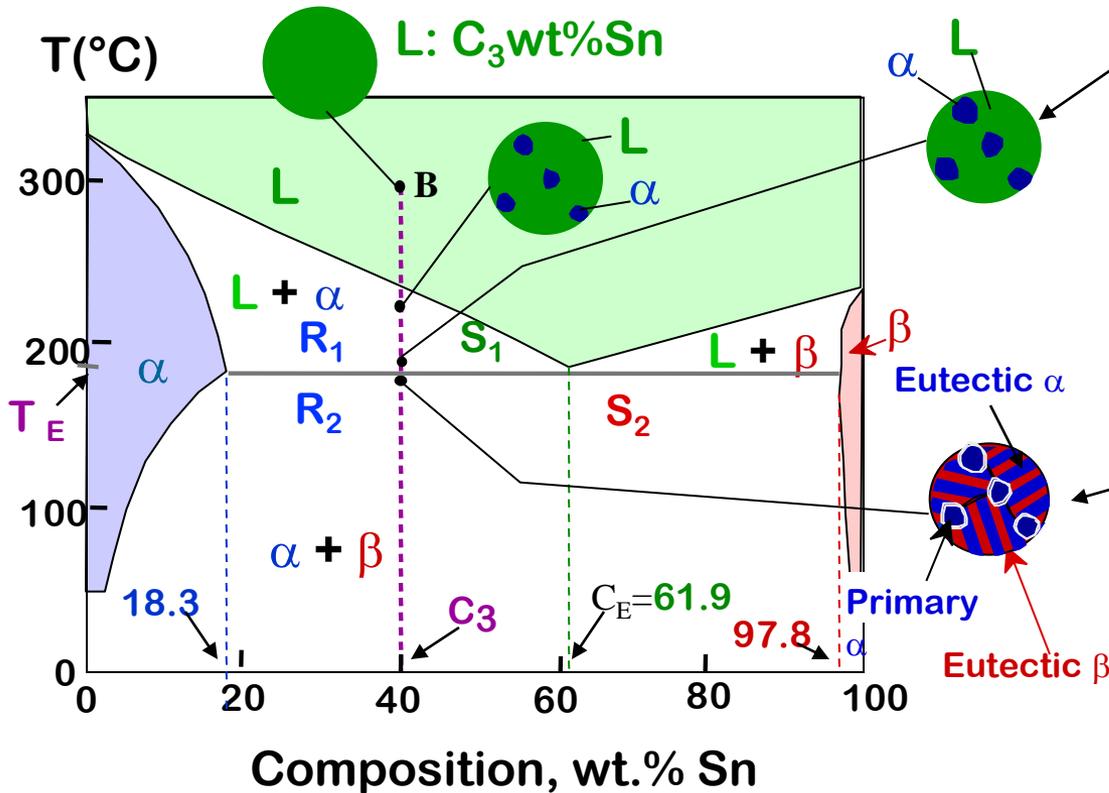


160  $\mu\text{m}$



# MICROSTRUCTURES IN EUTECTIC SYSTEMS: Equilibrium Cooling (4)

• **Composition range:**  
 $18.3\text{wt}\% \text{Sn} < C_3 < 61.9\text{wt}\% \text{Sn}$   
 (e.g. point B with  $C_B = C_3 = 40\text{wt}\% \text{Sn}$ )



Results:

• Just above  $T_E$ :  
*solid primary  $\alpha$  phase in liquid*

$$C_\alpha \approx 18.3\text{wt}\% \text{Sn}$$

$$C_L \approx 61.9\text{wt}\% \text{Sn}$$

$$W_\alpha = \frac{S_1}{R_1 + S_1} \approx 50\text{wt}\%$$

$$W_L = (1 - W_\alpha) \approx 50\text{wt}\%$$

• Just below  $T_E$ :  
*crystals and a eutectic microstructure*

$$C_\alpha \approx 18.3\text{wt}\% \text{Sn}$$

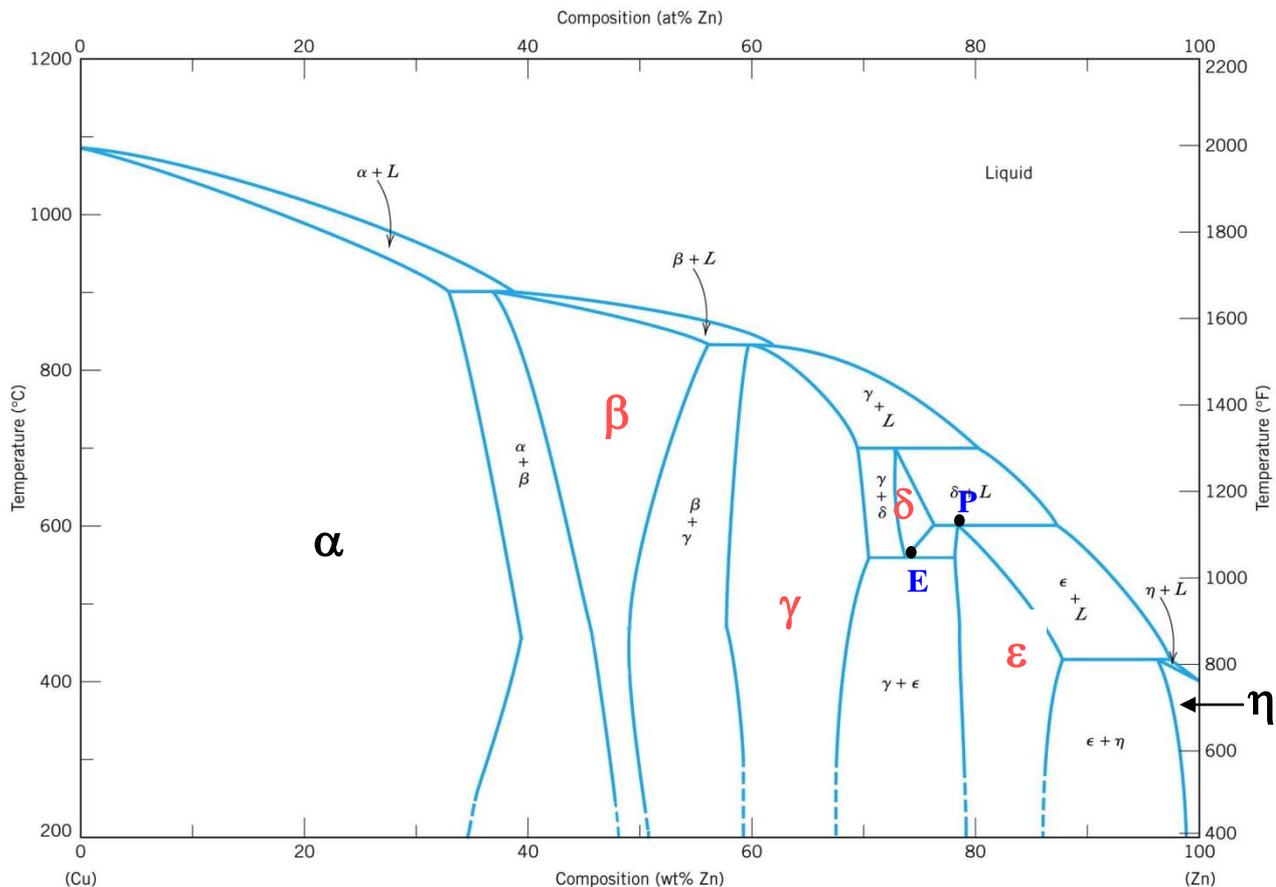
$$C_\beta \approx 97.8\text{wt}\% \text{Sn}$$

$$W_\alpha = \frac{S_2}{R_2 + S_2} \approx 73\text{wt}\%$$

$$W_\beta \approx 27\text{wt}\%$$

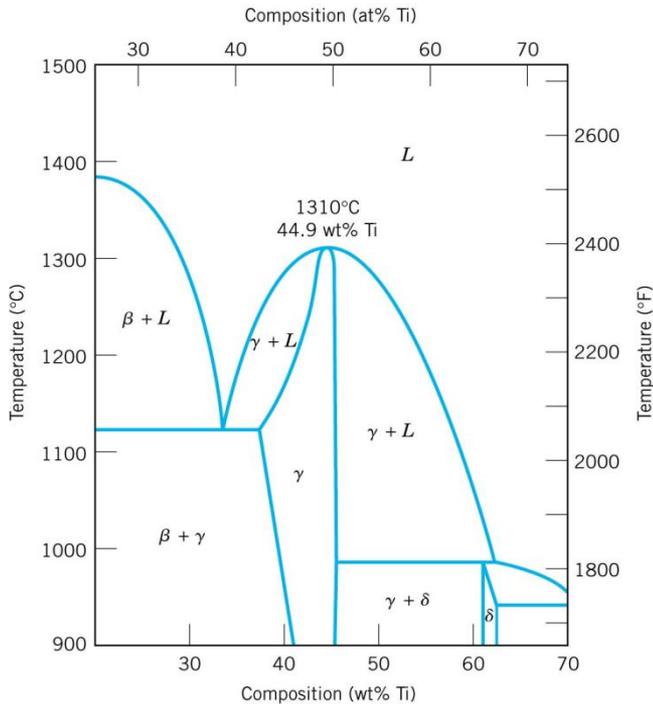
# Equilibrium Diagrams with Intermediate Phases

## Example: The Copper-Zinc System

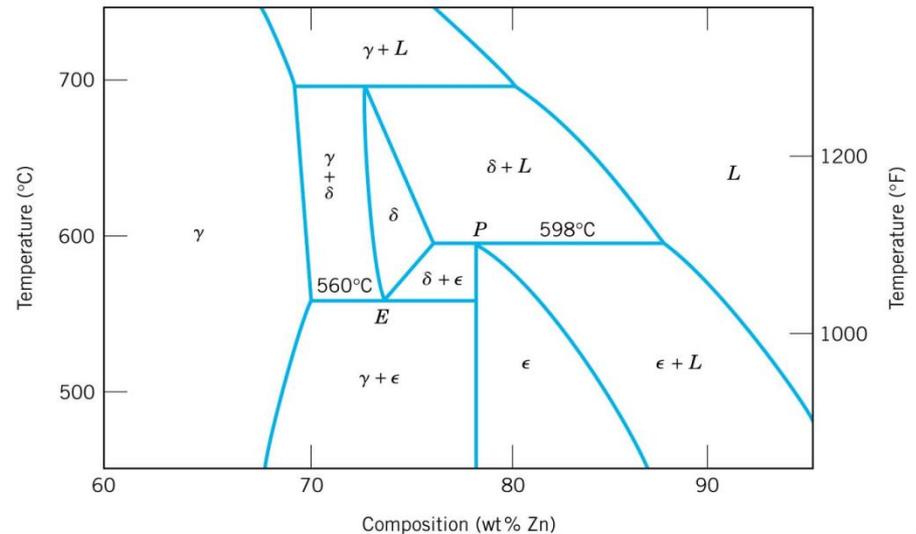


- $\alpha$  and  $\eta$  are **terminal solid solutions**: exist near the concentration *extremities* of the phase diagram
- $\beta, \gamma, \epsilon, \delta$  are **intermediate solid solutions** (or *intermediate phases*)
- new types (not eutectic) of **invariant points** (e.g. **E**, **P**) and corresponding reactions are shown up

# Types of Phase Transformations

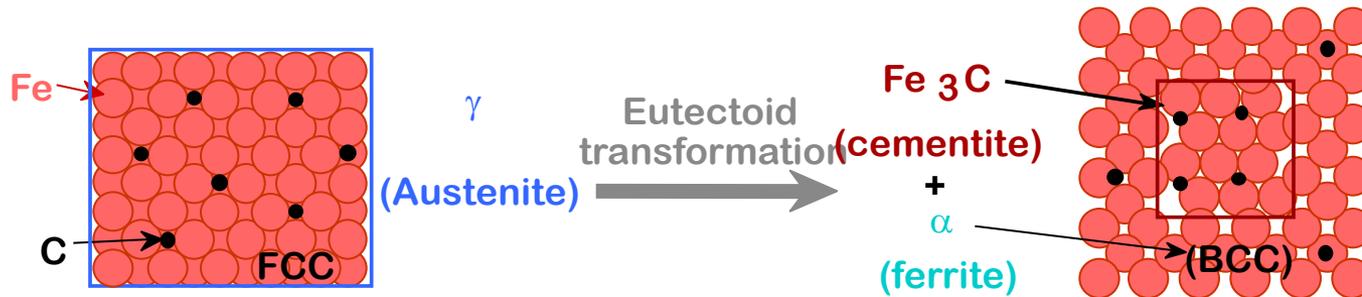


- *γ* **solid solution** at 1310°C and  $C = 44.9$  wt.% Ti melts without changing of the composition – **congruent transformation**
- melting of pure metals, allotropic transformations are **congruent**



- P melting at 598°C:  $\epsilon \Rightarrow \delta + L$  (peritectic reaction) occurs with **changing of phase** composition – **incongruent phase transformation**
- Eutectic, eutectoid and peritectic reactions are examples of incongruent transformations

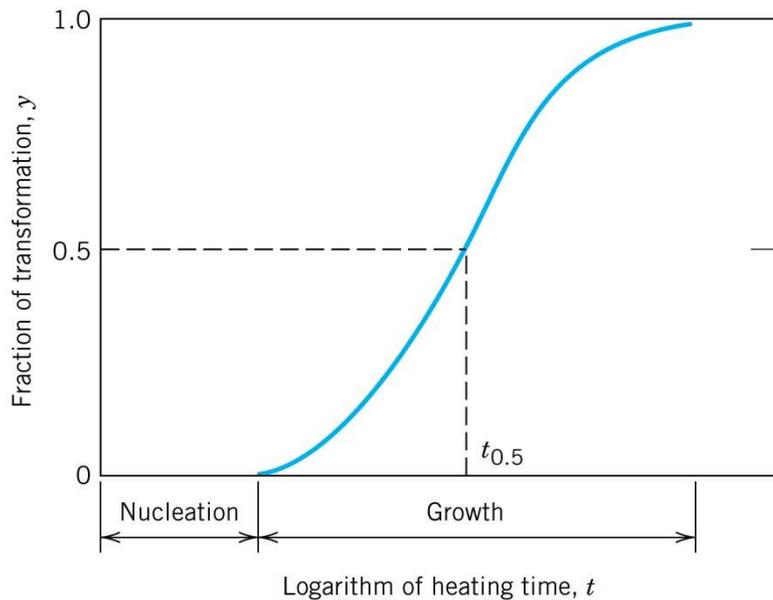
# CHAPTER 10: PHASE TRANSFORMATIONS



# KINETICS of SOLID STATE TRANSFORMATION

- **Kinetics** - a time dependence of transformation rate.

Example: typical kinetics for variety of solid-state transformation at constant temperature



- Can be described by so-called **Avrami Equation**:

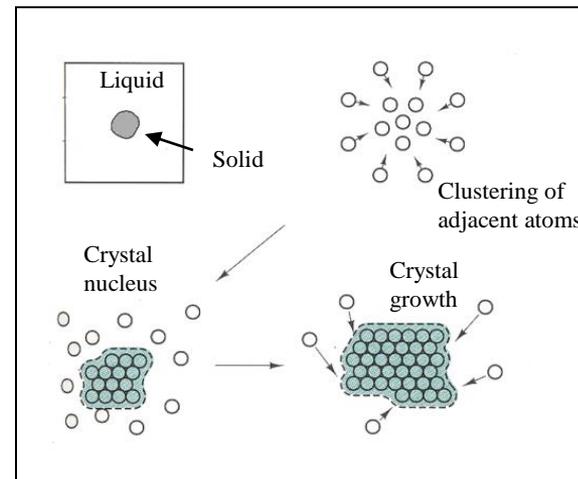
$$y = 1 - e^{-kt^n}$$

fraction transformed  $y$       time  $t$

- By convention, the rate of transformation  $r$  is taken as the reciprocal of time required to reach half completion:

Two stages of transformation process:

- **Nucleation** – formation of extremely fine particles, i.e. **nuclei**, of **new phases**
- **Growth** – increase of nuclei size



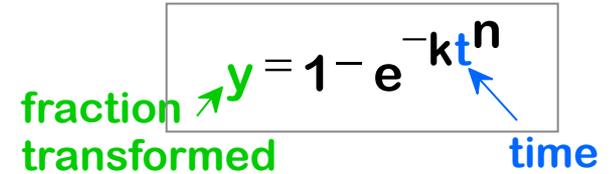
$$r = 1/t_{0.5}$$

# Typical Problem

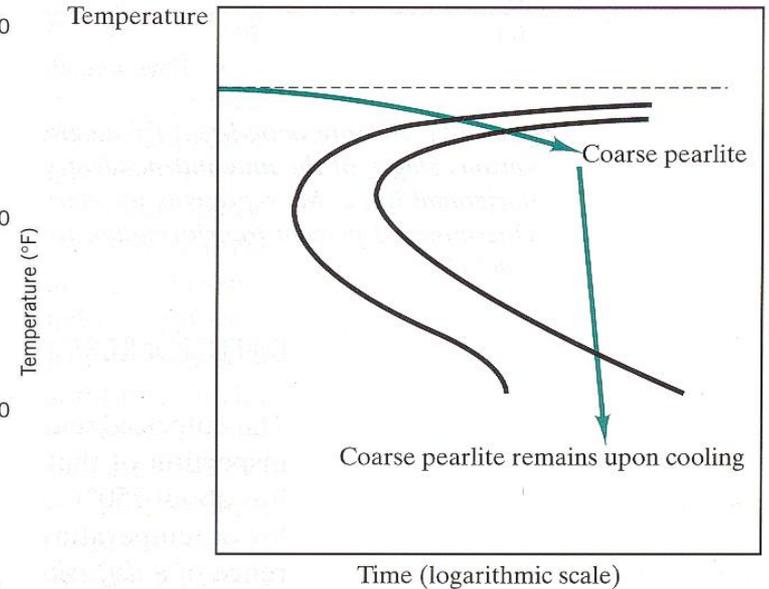
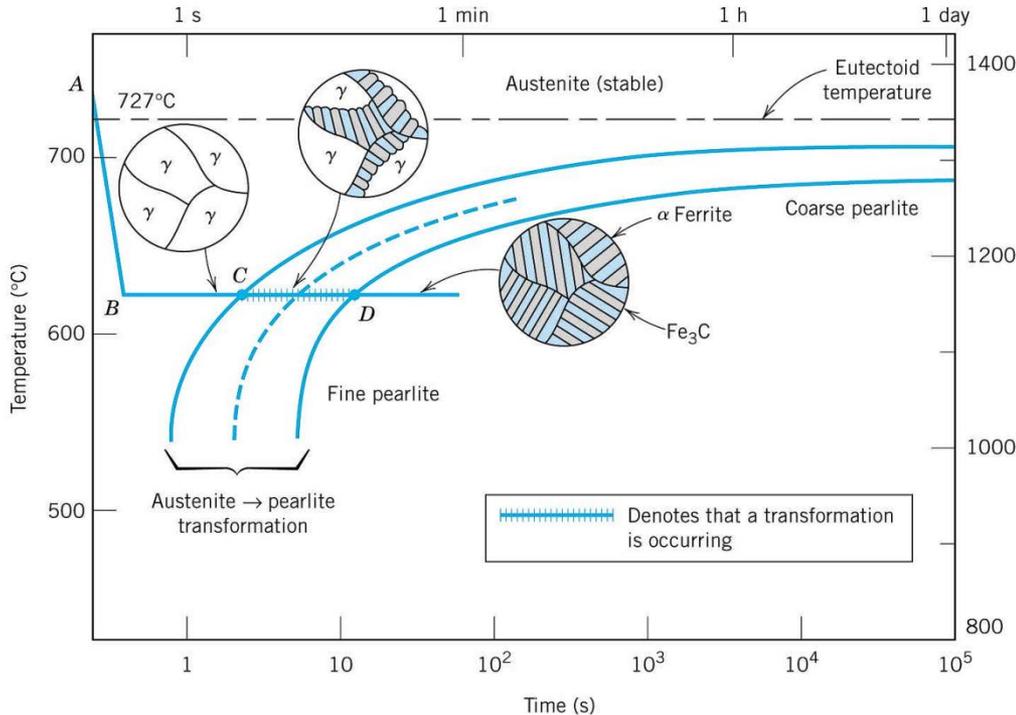
*For some transformation having kinetics that obey the Avrami equation, the parameter  $n$  is known to have a value of 1.5. If the reaction is 25% complete after 125 s, how long (total time) will it take the transformation to go to 90% completion?*

$$y = 1 - e^{-kt^n}$$

fraction transformed time



# TTT Diagram and Steel Structure



At relatively high  $T < T_E$  *diffusion is fast* enough to produce relatively long and thick lamellae.

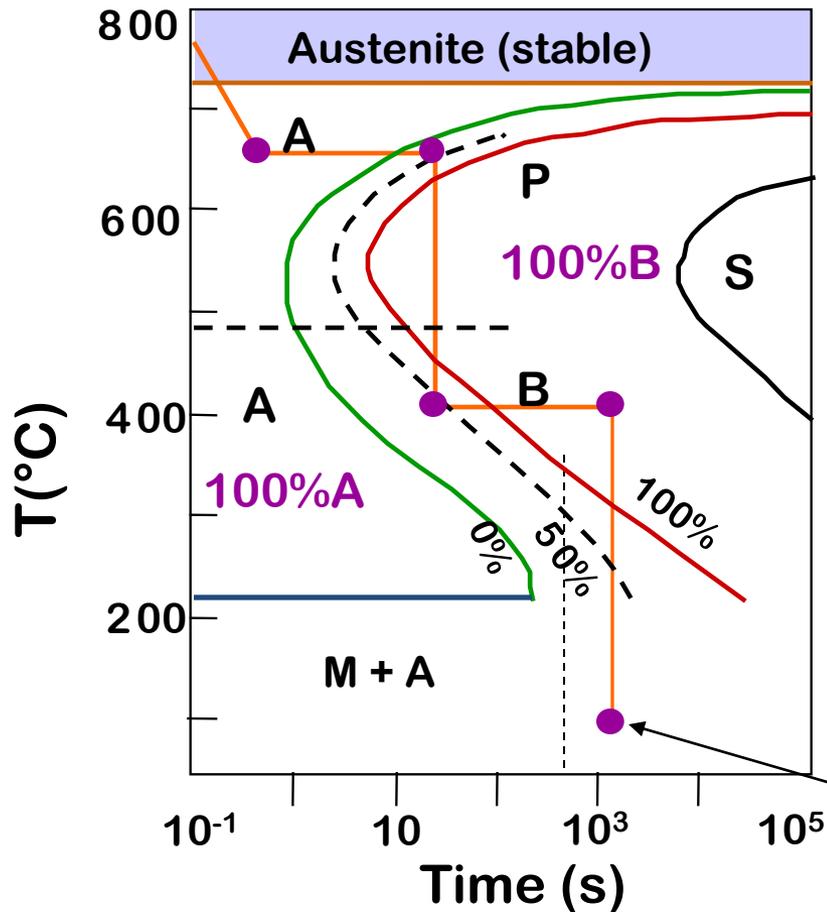
- A slow cooling path leads to coarse pearlite formation, which once formed, remains stable upon further cooling

**Q: What will be at higher under cooling ? Answer: Fine pearlite !!**

# COOLING EX: Fe-C SYSTEM (1)

- $C_o = C$  eutectoid

## Case III



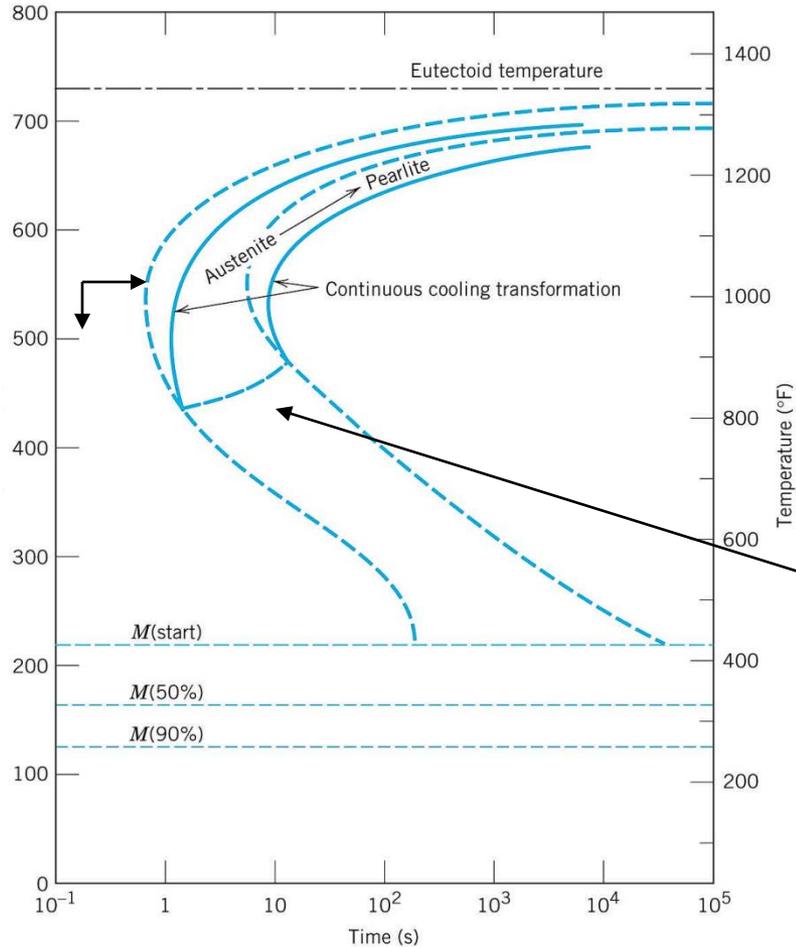
## Temperature-time path # 1:

Rapid	Hold	Rapid	Hold	Rapid
cool to:	for:	cool to:	for:	cool to:
650°C	20s	400°C	10 <sup>3</sup> s	T <sub>room</sub>

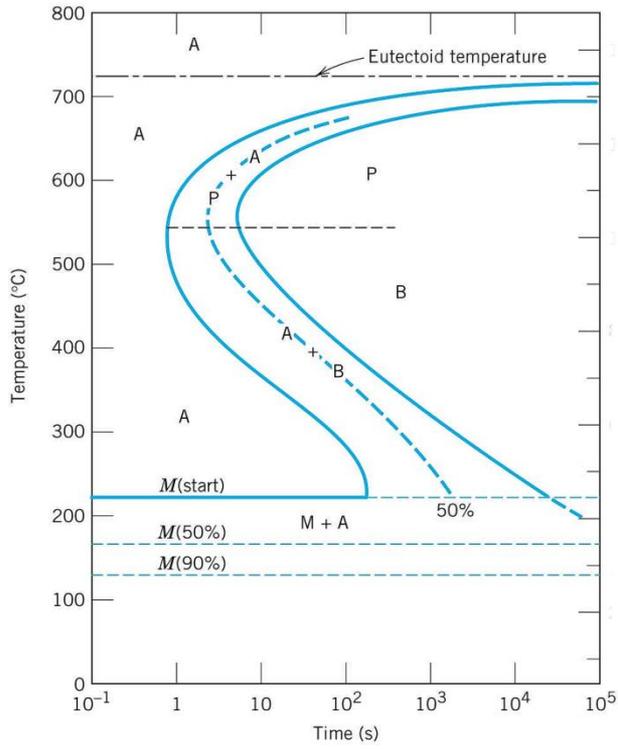
- At 650°C in 7s  $\gamma$ -Fe begins isothermal transformation to pearlite and after 20s ~50% of specimen has pearlite structure;
- Rapid cooling to 400°C does not significantly contribute to the changing of phase composition
- While 10<sup>3</sup> s treatment at this temperature completely converts remaining austenite to bainite.
- Finally, cooling down to room temperature results in no further transformations occur.

**50% Pearlite + 50% Bainite**

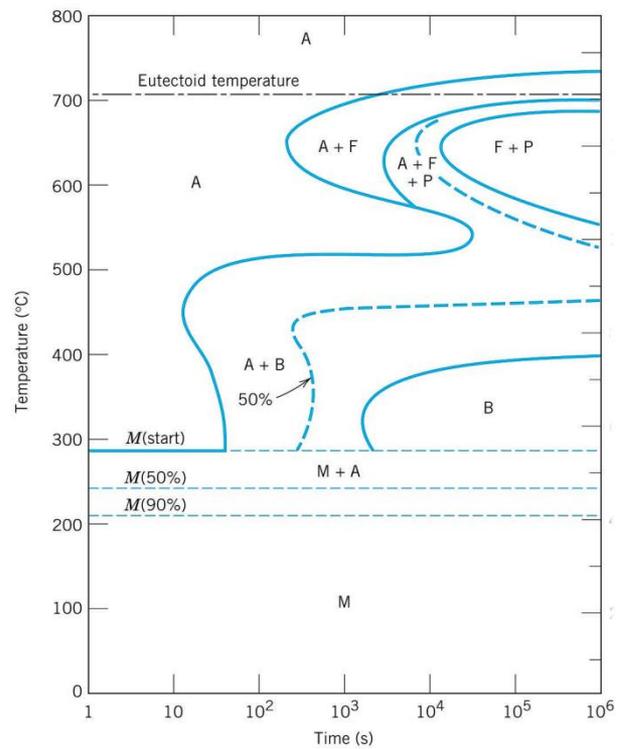
# Continuous Cooling Transformation (CCT) Diagram



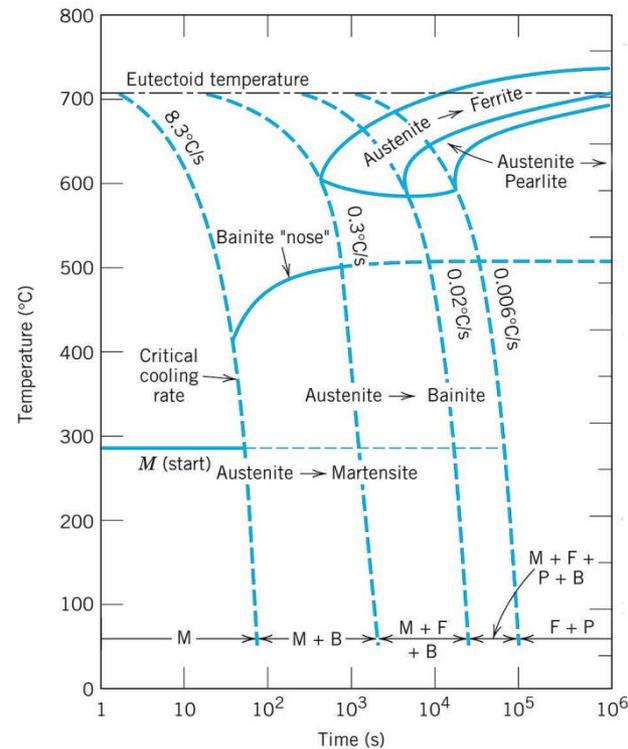
- The positions of transformation curves on CCT diagrams are shifted slightly *downwards* and towards the *right* as compared to TTT diagrams: the transformations start and end are delayed.
- Also, specifically for considered eutectoid composition in plain steel the *bainite* does not form during continuous cooling to room temperature.



**Plain carbon steel :**  
 carbon is prime  
 alloying element



**Alloy steel :**  
 e.g. type 4340 with alloying  
 elements: *Ni, Cr, Mo*



**Critical cooling rates**  
 for alloy steel: compare  
 with plane carbon steel