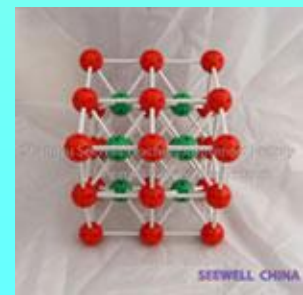


CHAPTER 3: CRYSTAL STRUCTURES & PROPERTIES

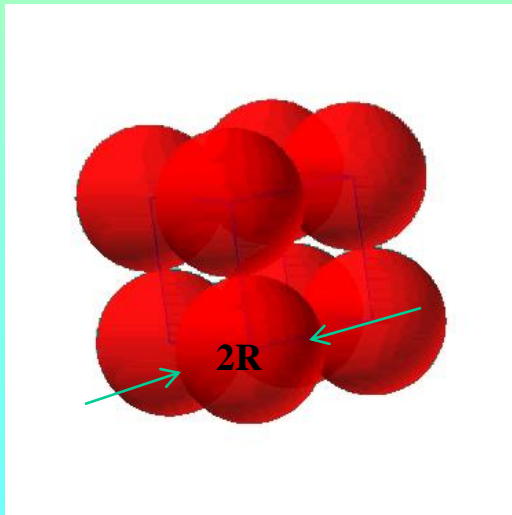
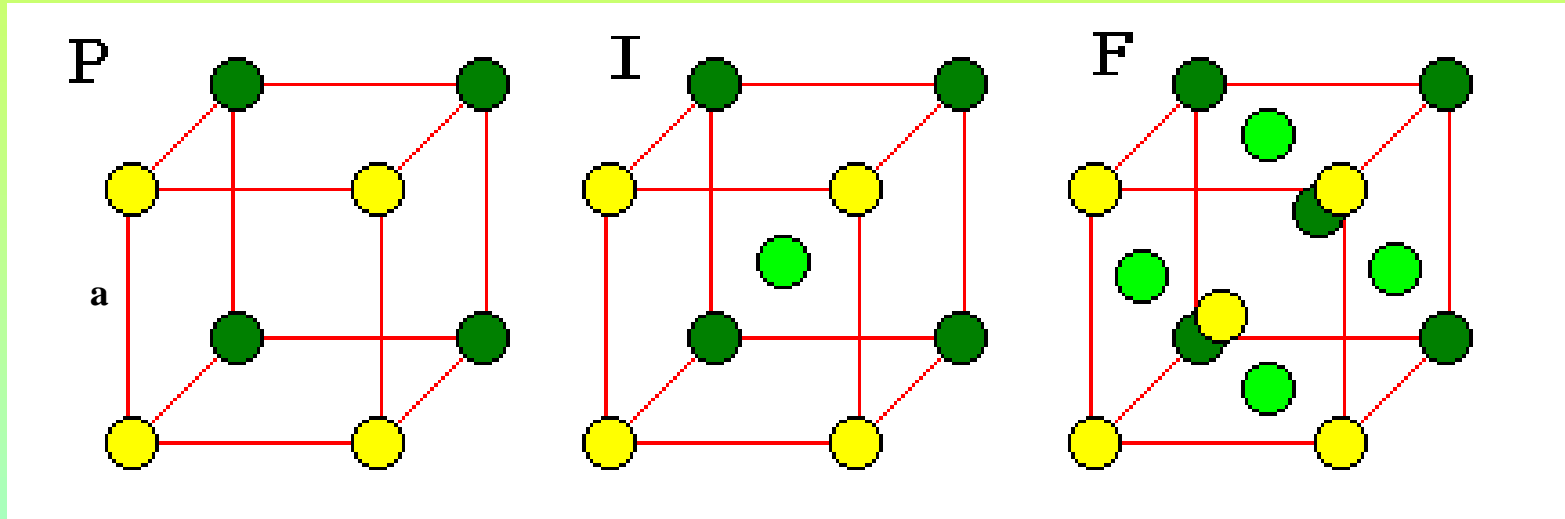


Metallic Crystals

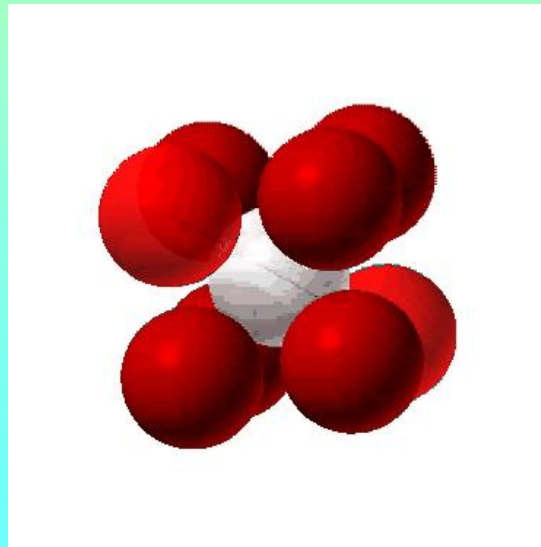
- **tend to be densely packed.**
- **have several reasons for dense packing:**
 - Typically, only one element is present, so all atomic radii are the same.
 - Metallic bonding is not directional.
 - Nearest neighbor distances tend to be small in order to lower bond energy.
- **have the simplest crystal structures.**

We will look at three such structures...

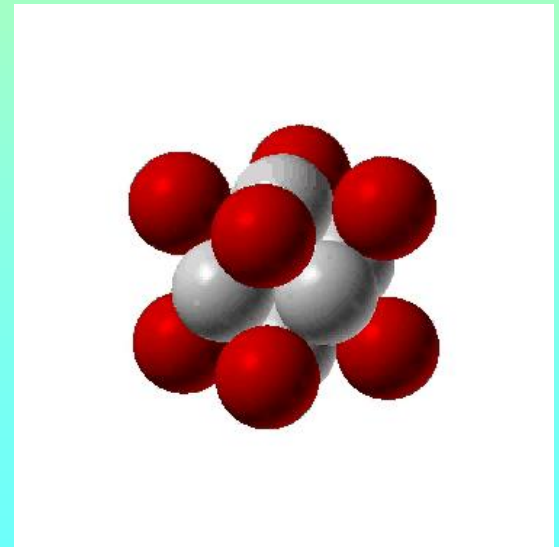
Cubic Unit Cells



**SIMPLE CUBIC
STRUCTURE (SC)**



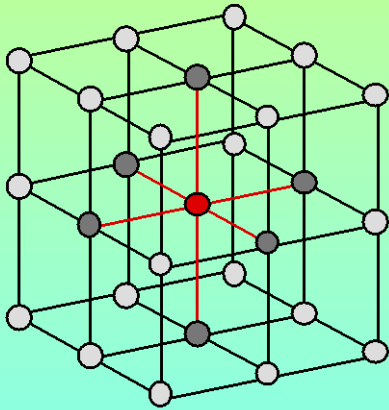
**BODY CENTERED
CUBIC STRUCTURE (BCC)**



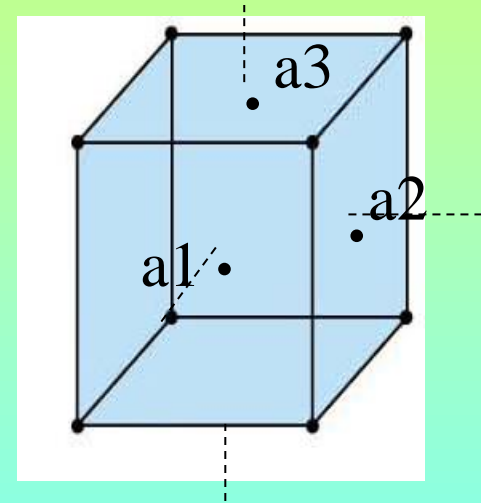
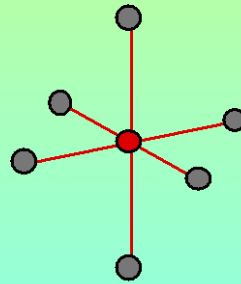
**FACE CENTERED
CUBIC STRUCTURE (FCC)**

Simple Cubic (SC) Structure

- **Coordination number** is the number of nearest neighbors
- **Linear density (LD)** is the number of atoms per unit length along a specific crystallographic direction



Coordination number = 6



$$LD_{110} = 1 \text{ atoms}/2\sqrt{2} R$$
$$LD_{100} = 1 \text{ atoms}/2R$$

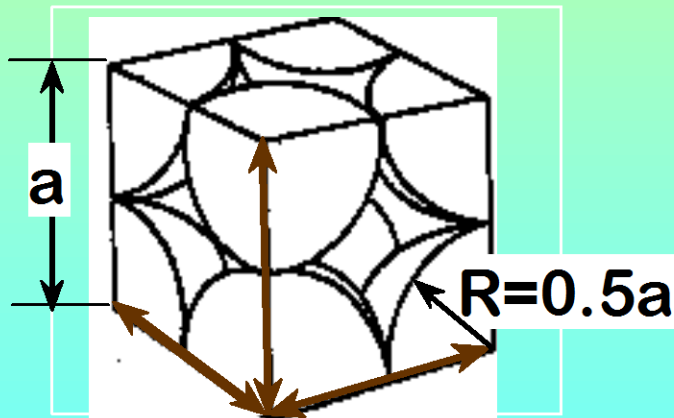
- **Rare due to poor packing (only Po [84] has this structure)**
- **Close-packed directions** are cube edges.

Atomic Packing Factor (APF)

$$\text{APF} = \frac{\text{Volume of atoms in unit cell}^*}{\text{Volume of unit cell}}$$

*assume hard spheres

- APF for a simple cubic structure = 0.52



close-packed directions

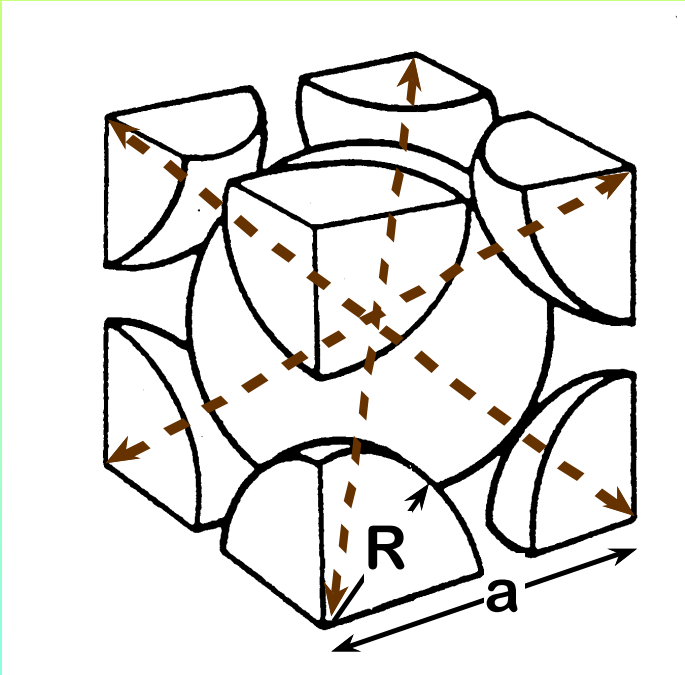
contains $8 \times 1/8 =$

1 atom/unit cell

$$\text{APF} = \frac{\text{atoms unit cell} \times \frac{4}{3} \pi (0.5a)^3}{a^3}$$

Annotations:
- **atoms unit cell**: points to the number 1 in the numerator.
- **volume atom**: points to the term $\frac{4}{3} \pi (0.5a)^3$ in the numerator.
- **volume unit cell**: points to the term a^3 in the denominator.

Body Centered Cubic (BCC) Structure



- Coordination number = 8
- Close packed directions are cube diagonals:

$$LD_{110} = 1 \text{ atom} / (4R\sqrt{2/3}) = 1 / (2R\sqrt{8/3})$$

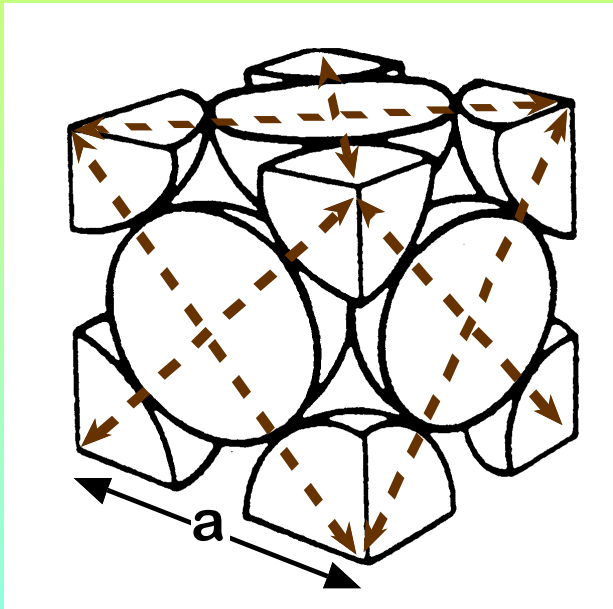
$$LD_{001} = 1 \text{ atom} / (4R/\sqrt{3}) = 1 / (2R\sqrt{4/3})$$

$$LD_{111} = 2 \text{ atoms} / 4R = 1 / (2R)$$

- Unit cell contains:
 $1 + 8 \times 1/8 = 2$ atoms/unit cell
- APF = 0.68:

$$\text{APF} = \frac{\text{atoms/unit cell} \times \text{volume/atom}}{\text{volume/unit cell}} = \frac{2 \times \frac{4}{3} \pi (\sqrt{3}a/4)^3}{a^3}$$

Face-Centered Cubic (FCC) Structure



- Coordination number = 12
- Close packed directions are face diagonals:

$$LD_{110} = 2 \text{ atom}/(4R) = 1/2R$$

$$LD_{001} = 1 \text{ atom}/(2R\sqrt{2}) = 1/(2R\sqrt{2})$$

$$LD_{111} = 1 \text{ atoms}/4R = 1/(2R\sqrt{6})$$

- Unit cell contains:

$$6 \times 1/2 + 8 \times 1/8 = 4 \text{ atoms/unit cell}$$

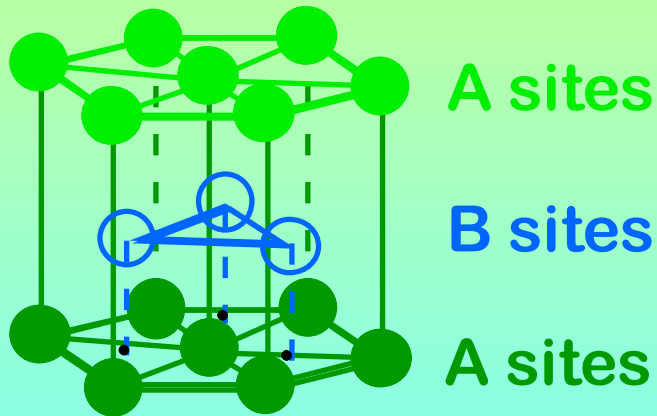
- APF = 0.74

$$\text{APF} = \frac{\frac{\text{atoms}}{\text{unit cell}} \cdot \frac{\text{volume}}{\text{atom}}}{\frac{\text{volume}}{\text{unit cell}}}$$

$$\text{APF} = \frac{4 \cdot \frac{4}{3} \pi (\sqrt{2}a/4)^3}{a^3}$$

Hexagonal Close-Packed (HCP) Structure

- ABAB... Stacking Sequence
- 3D Projection



- 2D Projection



- Coordination number = ?
- APF = ?

Characteristics of Selected Elements at 20C

Element	Symbol	At. Weight (amu)	Density (g/cm ³)	Crystal Structure	Atomic radius (nm)
Aluminum	Al	26.98	2.71	FCC	0.143
Argon	Ar	39.95	-----	-----	-----
Barium	Ba	137.33	3.5	BCC	0.217
Beryllium	Be	9.012	1.85	HCP	0.114
Boron	B	10.81	2.34	Rhomb	-----
Bromine	Br	79.90	-----	-----	-----
Cadmium	Cd	112.41	8.65	HCP	0.149
Calcium	Ca	40.08	1.55	FCC	0.197
Carbon	C	12.011	2.25	Hex	0.071
Cesium	Cs	132.91	1.87	BCC	0.265
Chlorine	Cl	35.45	-----	-----	-----
Chromium	Cr	52.00	7.19	BCC	0.125
Cobalt	Co	58.93	8.9	HCP	0.125
Copper	Cu	63.55	8.94	FCC	0.128
Flourine	F	19.00	-----	-----	-----
Gallium	Ga	69.72	5.90	Ortho.	0.122
Germanium	Ge	72.59	5.32	Dia. cubic	0.122
Gold	Au	196.97	19.32	FCC	0.144
Helium	He	4.003	-----	-----	-----
Hydrogen	H	1.008	-----	-----	-----

Theoretical Density, ρ

$$\rho = \frac{n A}{V_c N_A}$$

atoms/unit cell \rightarrow n Atomic weight (g/mol) \rightarrow A
Volume/unit cell (cm³/unit cell) \rightarrow V_c Avogadro's number (6.023 x 10²³ atoms/mol) \rightarrow N_A

Example: Copper

- crystal structure **FCC**
- # atoms/unit cell = **4**
- atomic weight = **63.55 g/mol**
- atomic radius $R = \mathbf{0.128 \text{ nm}}$
- for FCC $\mathbf{a = 2R\sqrt{2}}$; $V_c = a^3$; $V_c = 4.75 \cdot 10^{-23} \text{ cm}^3$

Result: theoretical $\rho_{\text{Cu}} = 8.89 \text{ g/cm}^3$

Compare to actual: $\rho_{\text{Cu}} = 8.94 \text{ g/cm}^3$

Densities of Materials Classes

$$\rho_{\text{metal}} > \rho_{\text{ceramics}} > \rho_{\text{polymers}}$$

Why?

Metals have...

- close-packing (metallic bonding)
- large atomic mass

Ceramics have...

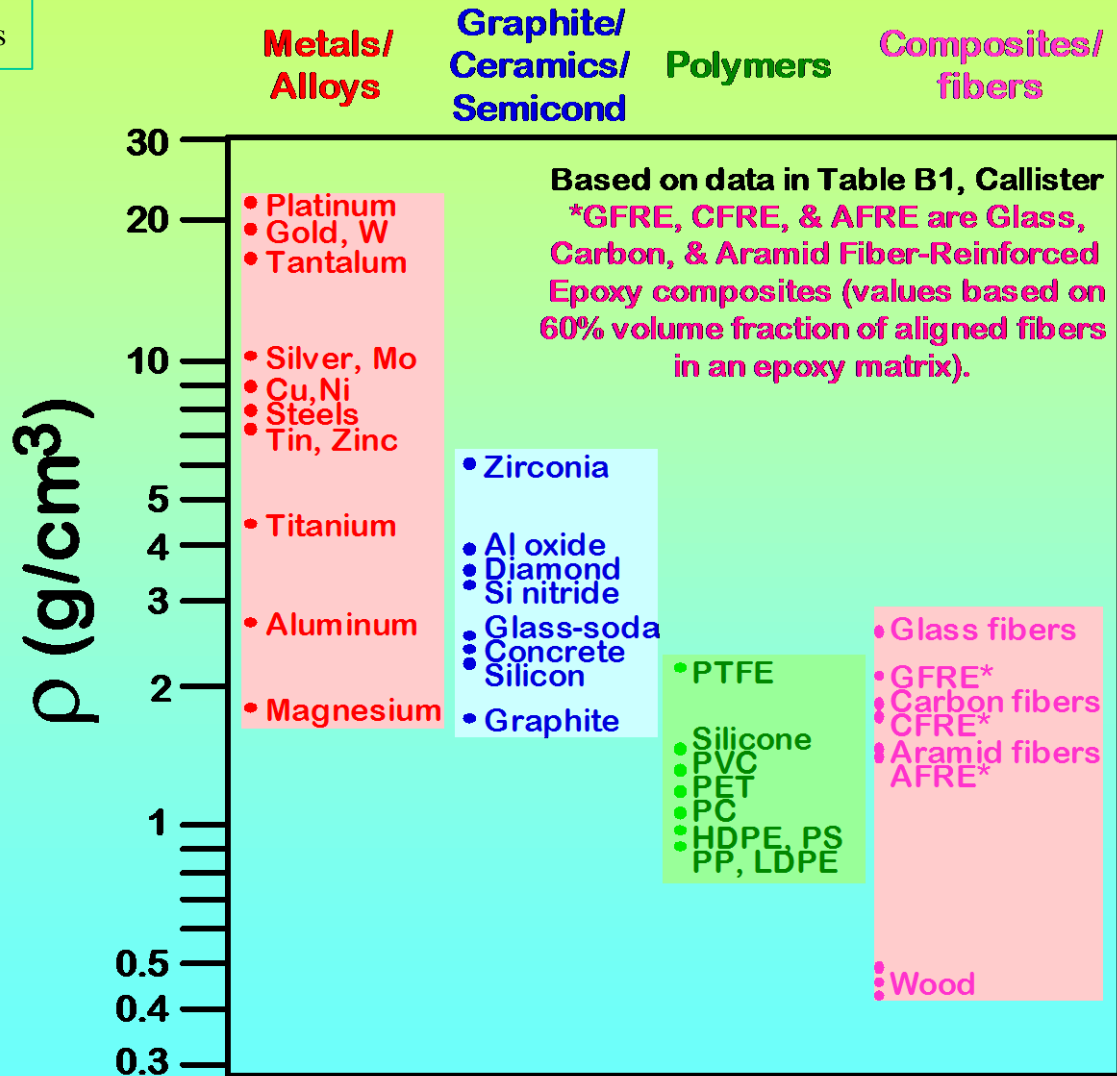
- less dense packing (covalent bonding)
- often lighter elements

Polymers have...

- poor packing (often amorphous)
- lighter elements (C,H,O)

Composites have...

- intermediate values



CRYSTALS AS BUILDING BLOCKS

- *Some* engineering applications require single crystals:



diamond single
crystals for abrasives

turbine blades



- Crystal properties reveal features of atomic structure.

Ex: Certain crystal planes in quartz fracture more easily than others.



POLYCRYSTALS

- *Most* engineering materials are polycrystals.

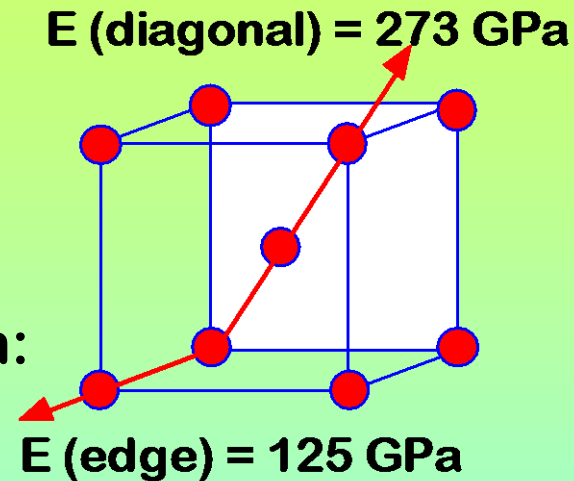


- Nb-Hf-W plate with an electron beam weld.
- Each "grain" is a single crystal.
- If crystals are randomly oriented, overall component properties are not directional.
- Crystal sizes typ. range from 1 nm to 2 cm (i.e., from a few to millions of atomic layers).

SINGLE VS POLYCRYSTALS

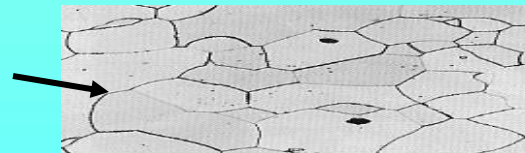
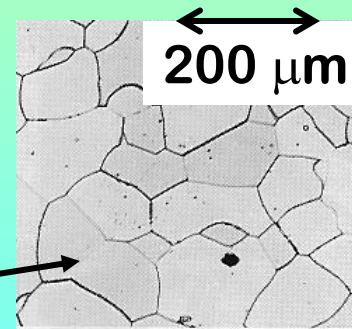
- Single Crystals

- Properties vary with direction: **anisotropic**.
- Example: the modulus of elasticity (E) in BCC iron:



- Polycrystals

- Properties may/may not vary with direction.
- If grains are randomly oriented: **isotropic**.
($E_{\text{poly iron}} = 210$ GPa)
- If grains are **textured**, anisotropic.



SUMMARY

- Atoms may assemble into **crystalline** or **amorphous** structures.
- We can predict the **density** of a material, provided we know the **atomic weight**, **atomic radius**, and **crystal geometry** (e.g., FCC, BCC, HCP).
- Material properties generally vary with single crystal orientation (i.e., they are **anisotropic**), but properties are generally non-directional (i.e., they are **isotropic**) in polycrystals with randomly oriented grains.