# CHAPTER 8:
# MECHANICAL FAILURE

# ISSUES TO ADDRESS

- How do **flaws in a material** initiate failure?
- How is fracture resistance quantified; how do different **material classes compare**?
- How do we estimate the ***stress to fracture***?
- How do **loading rate, loading history, and temperature** affect the failure stress?

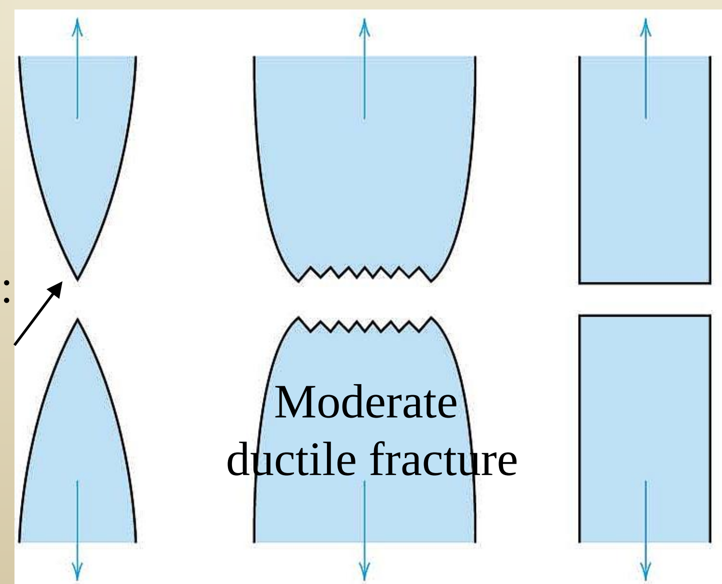
# DUCTILE VS BRITTLE FAILURE

***Simple fracture*** is the separation of a body into two or more pieces in response to an imposed static stress at temperatures that are lower than material's melting point.

***Two*** limiting fracture ***modes*** are possible for engineering materials: **ductile** and **brittle**

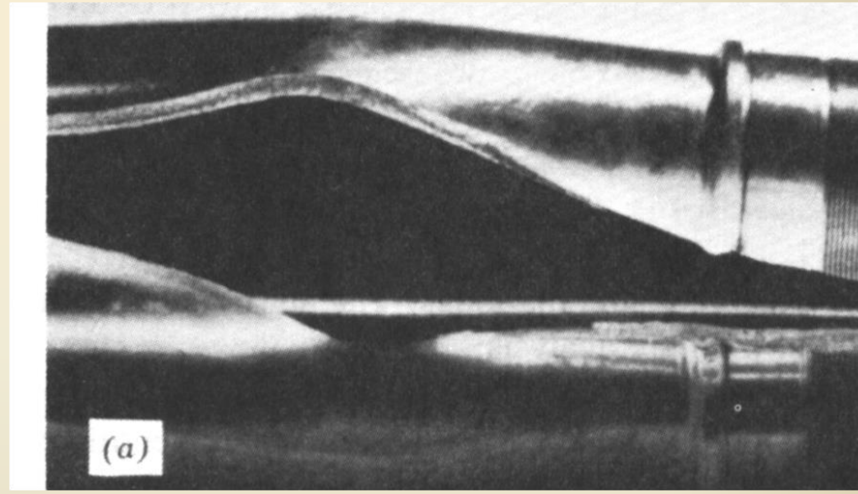
Absolute brittle fracture: without any plastic deformation

Highly ductile fracture: neck down to a point

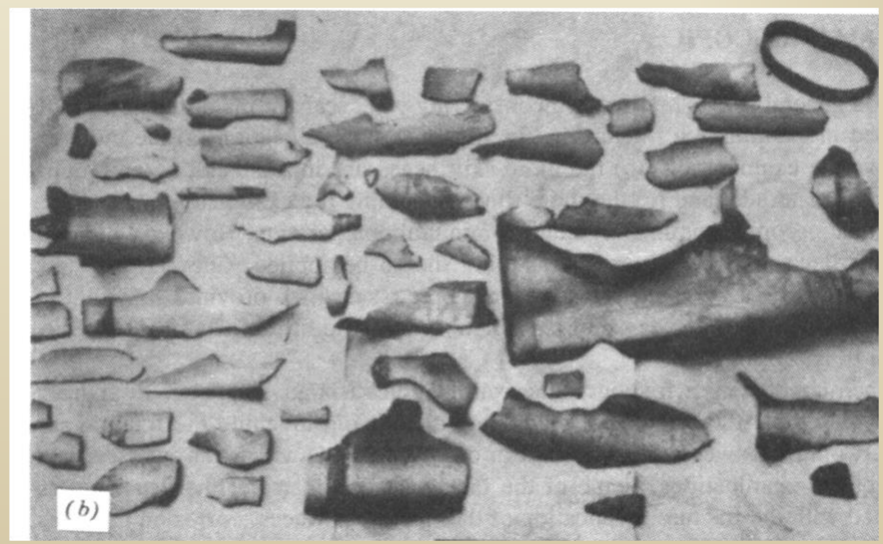


# Example: FAILURE OF A PIPE

- **Ductile failure**:
  --one piece
  --large deformation

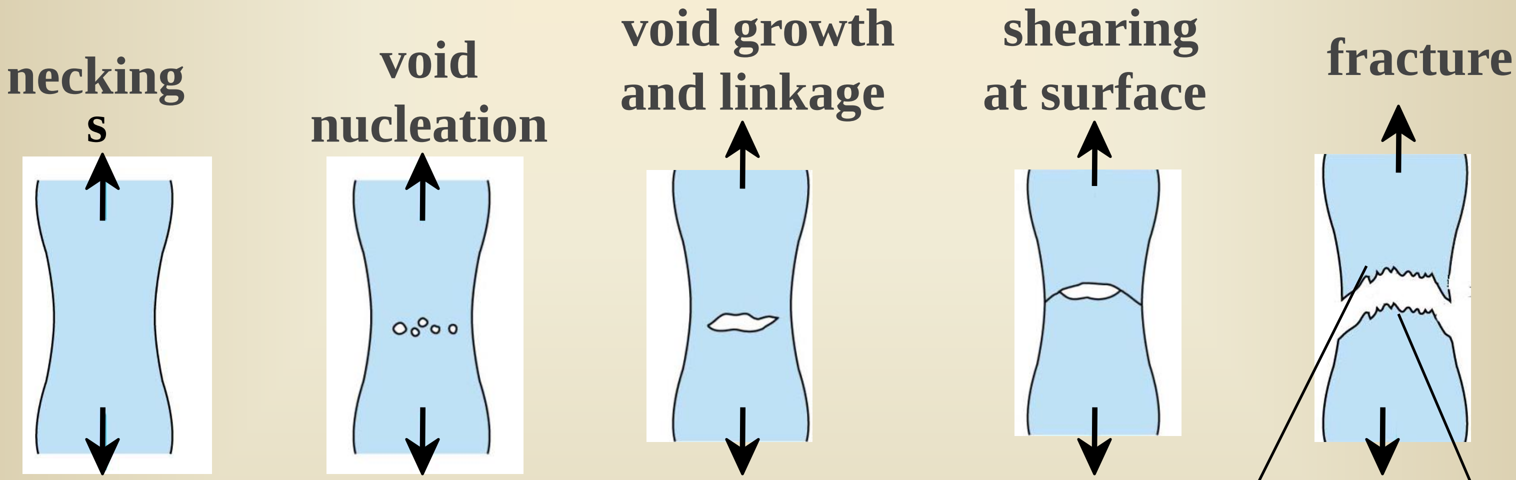


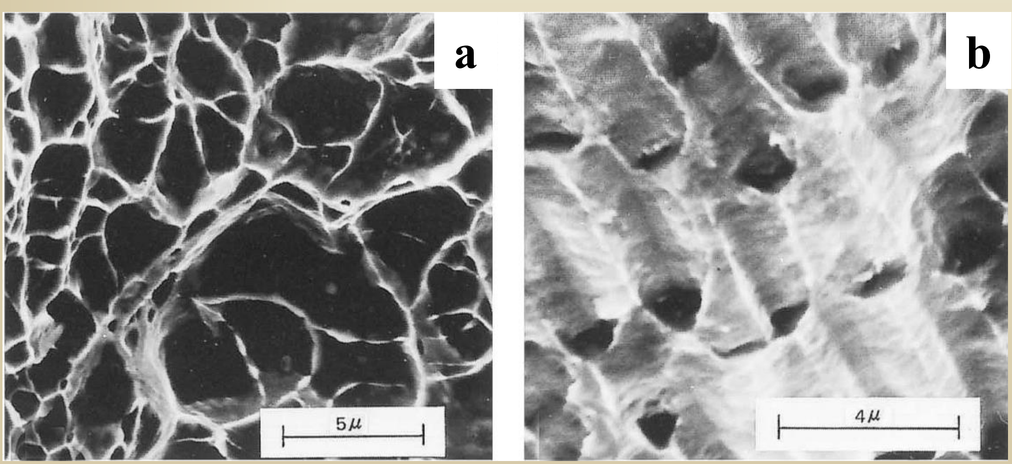

- **Brittle failure:**
  --many pieces
  --small deformation



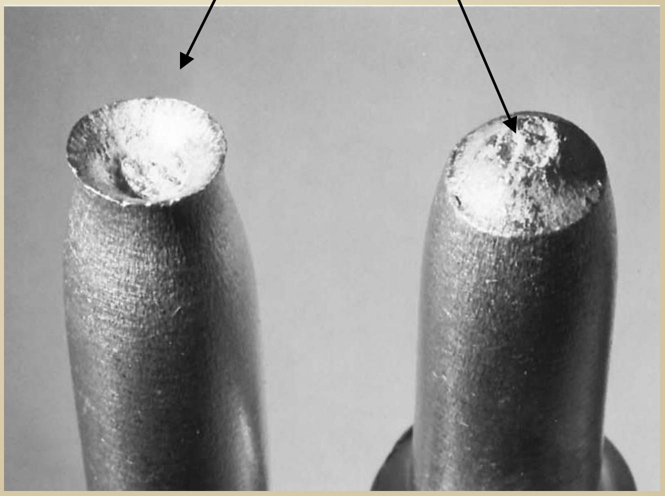
# MODERATELY DUCTILE FAILURE

• Evolution of **cup-and-cone** fracture to failure:



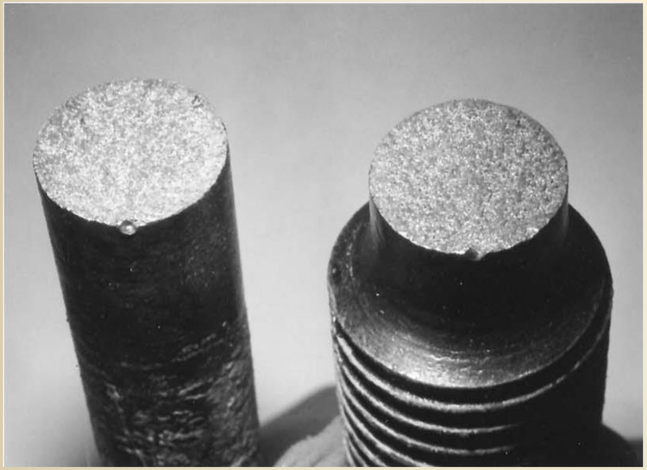




Spherical (a) and parabolic (b) “dimples” characteristic for ductile fracture from uniaxial tensile and shear loadings

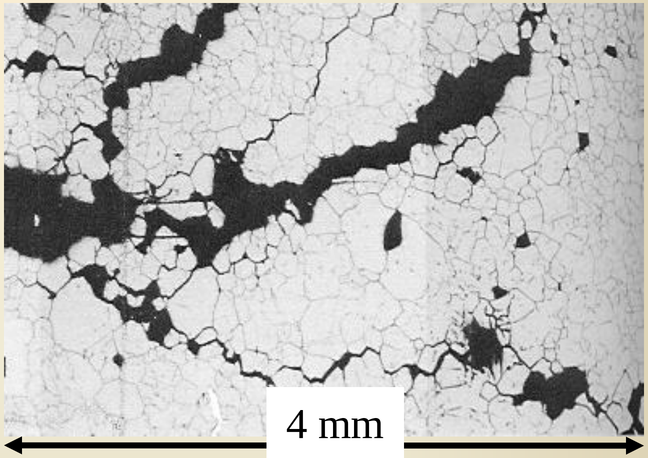


**Cup-and cone fracture of Al**

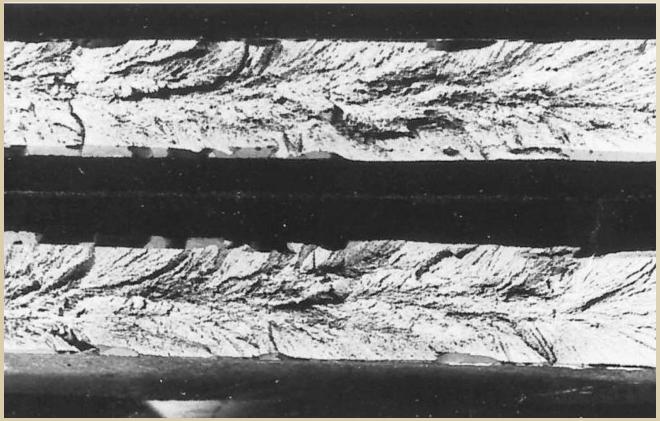
# BRITTLE FAILURE



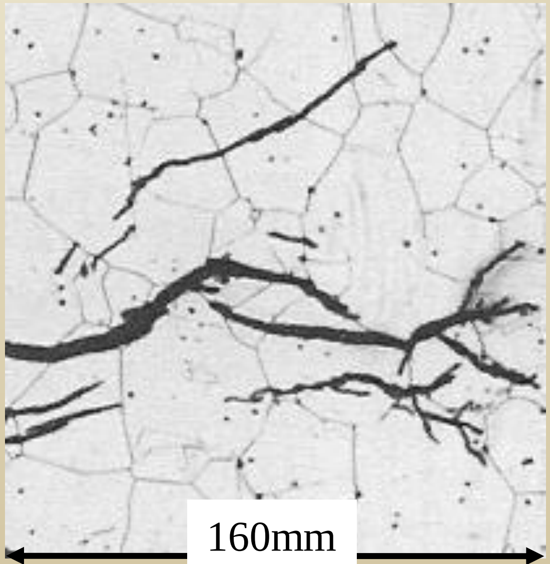
Brittle fracture in a mild steel




**Intergranular** fracture (**between grains**)



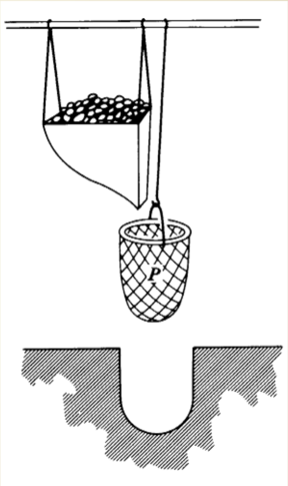
V-shaped "chevron" markings
Characteristic of brittle fracture

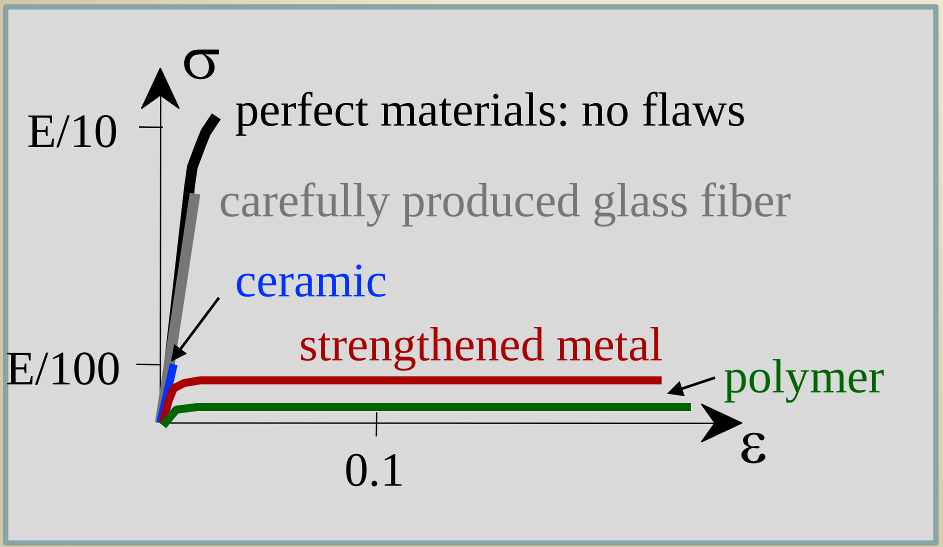



**Intragranular** fracture (**within grains**)

# Again: IDEAL vs. REAL MATERIALS

- **History:** **Leonardo DaVinci** (500 years ago!) observed: the **longer the wire**, the **smaller** the **failure-causing load**.



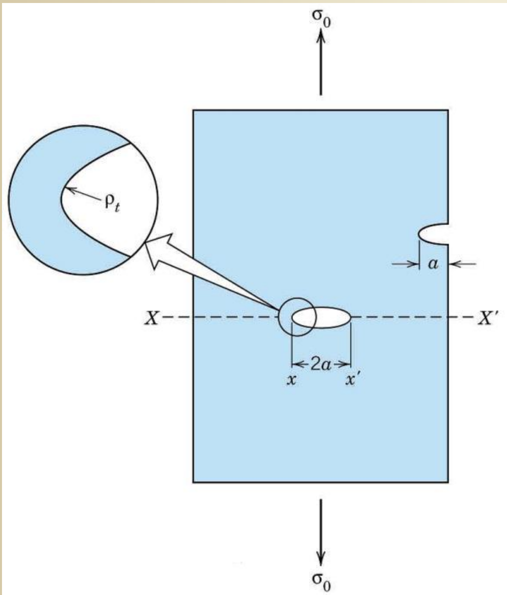



- **Reason:**
  - **flaws** cause premature failure.
  - larger samples are more flawed!

$TS_{\text{Eng. Mater.}} << TS_{\text{Perfect Mater}}$

# FLAWS ARE STRESS CONCENTRATORS!

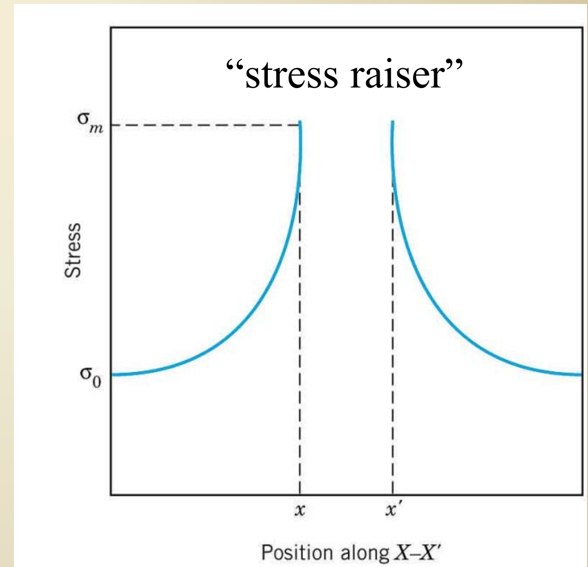
- Schematic of surface and internal cracks




Characterized by:
length (2**a**)
curvature ($\boldsymbol{\rho_t}$)

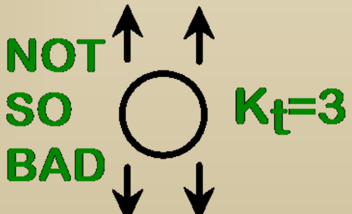
$$\boxed{\sigma_m = 2\sigma_o\left(\frac{a}{\rho_t}\right)^{1/2}}$$

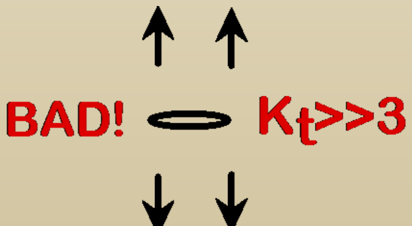
- Stress distribution in front of a crack:




- Stress concentration factor: $K_t = \sigma_{max}/\sigma_o$ - stress amplification!

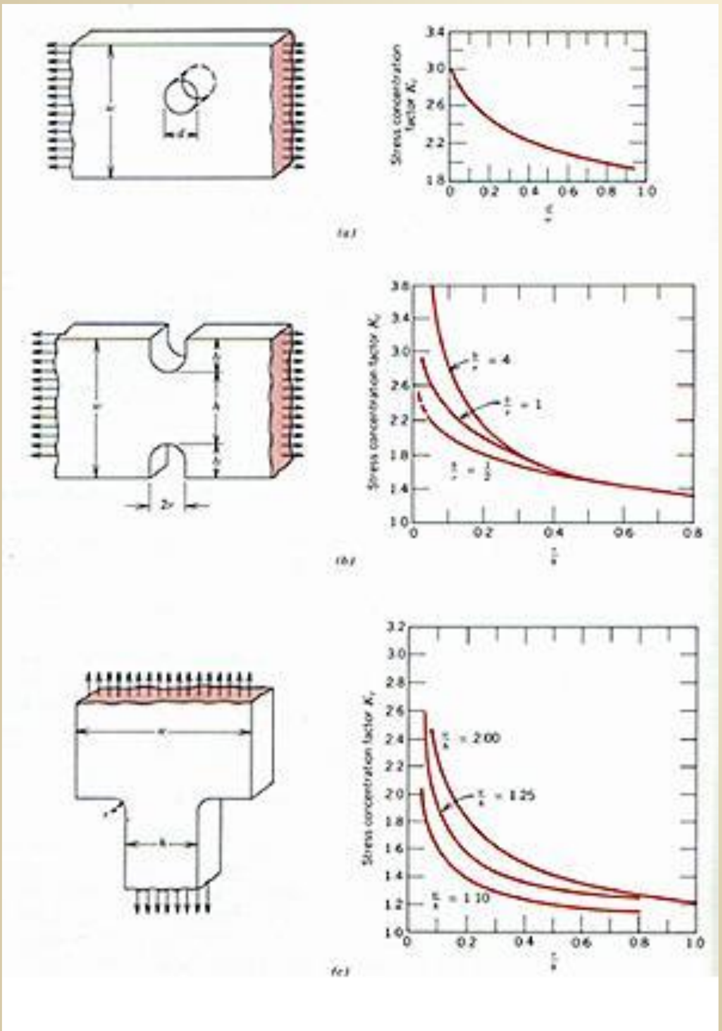
<u>Large $K_t$ promotes failure:</u>




Plastic deformation ($\sigma_m > \sigma_y$) leads to more uniform stress distribution!!

# STRESS CONCENTRATORS: Macroscopic Level

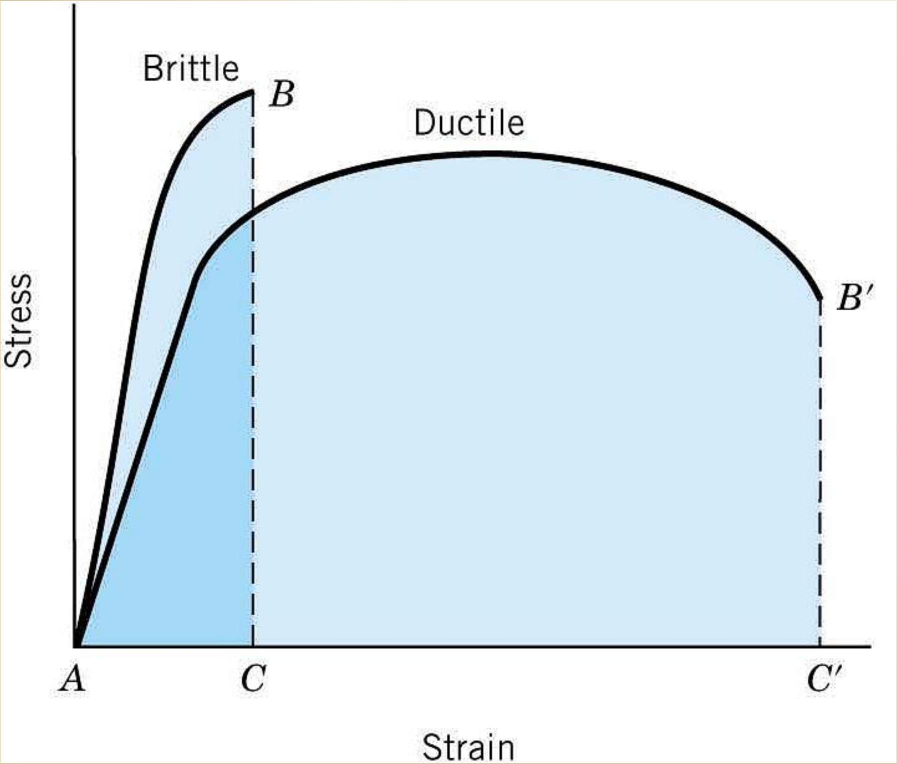


- It is important to remember that stress amplification not only occurs on a ***microscopic level*** (e.g. small flaws or cracks,) but can also can take place on ***the macroscopic level*** in the case of sharp corners, holes, fillets, and notches.

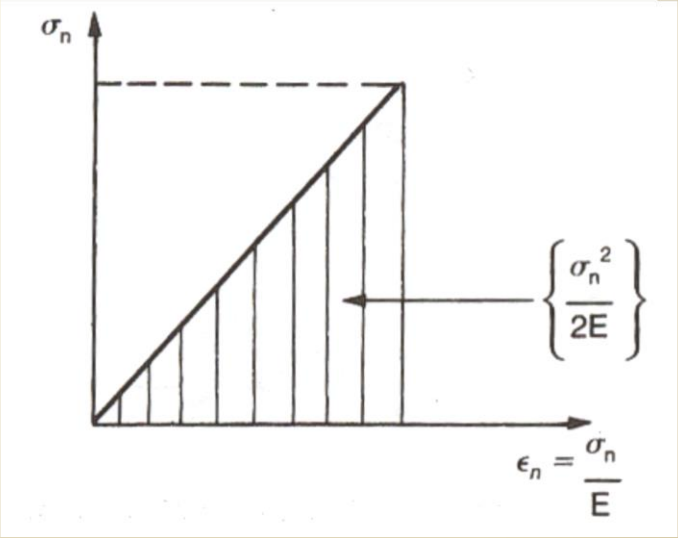
- The figure depicts the theoretical stress concentration factor curves for several simple and common material geometries.

# Reminder: TOUGHNESS

- Energy to break a unit volume of material
- Approximate by the area under the stress-strain curve.
- Units: [J/m$^2$]




**For Elastic Strain**

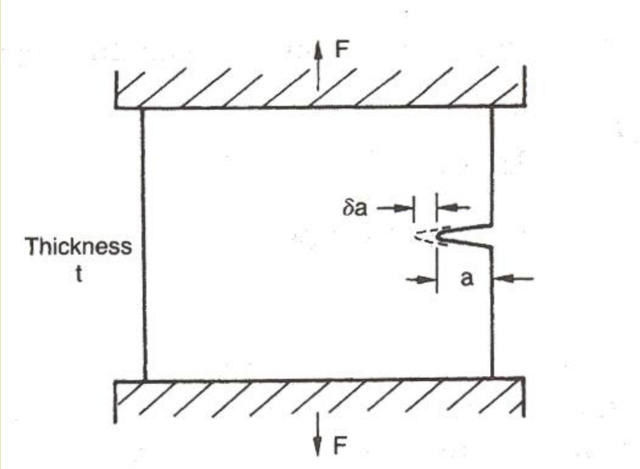


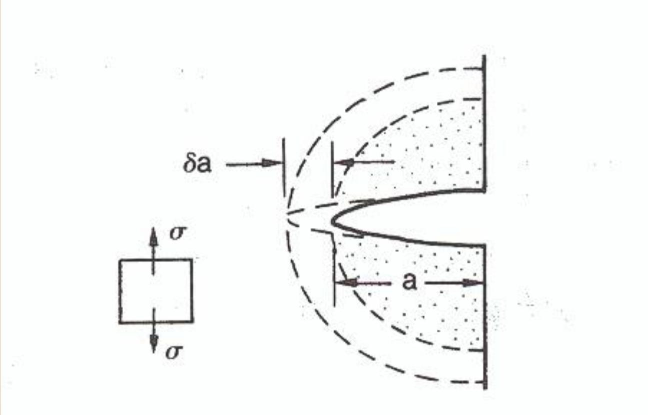

$$\boldsymbol{\sigma}_n = E\boldsymbol{\varepsilon}_n$$

$$U^{el} = \int \boldsymbol{\sigma}_n d\boldsymbol{\varepsilon}_n = \int \boldsymbol{\sigma}_n \frac{d\boldsymbol{\sigma}_n}{E} = \left\{\frac{\boldsymbol{\sigma}_n^2}{2E}\right\}$$

# Material Fast Fracture

**Fixed displacements case**
**(the boundary of the plate are fixed)**




$$\delta W \geq \delta U^{el} + \gamma_c t \delta a$$

$$-\delta U^{el} = \gamma_c t \delta a$$

$$U^{el} = -\frac{\sigma^2}{2E} \frac{\pi a^2 t}{2}$$

$$\delta U^{el} = \frac{\delta U^{el}}{\delta a} \delta a = -\frac{\sigma^2}{2E} \frac{2\pi a t}{2} \delta a$$

$$\frac{\sigma^2 \pi a}{2E} = \gamma_c$$

$$\sigma_c = \left( \frac{2E\gamma_c}{\pi a} \right)^{1/2}$$

where E – module of elasticity; $\sigma_c$ – critical stress; $\gamma_s$ – **toughness** (J $m^{-2,}$ specific surface energy); W-work; $U^{el}$- energy related to elastic deformation

# 1: FRACTURE TOUGHNESS

- Thus it is possible to show that critical stress for crack propagation is:

Stress intensity factor $\rightarrow$ $$\sigma\sqrt{\pi a} = \sqrt{E\gamma_c}$$ $\longleftarrow$ **Material properties only!!!**

• **Fracture toughness** is a property that is a measure of a materials' resistance for brittle fracture when cracks are present. This property can be defined by parameter $K_c$ that relates the critical stress for crack propagation and geometry of the crack:

$$K_c = Y\sigma_c\sqrt{\pi a}$$

$K_{ic}$ = [Pa ·m$^{1/2}$]!!!

where **Y** – a dimensionless parameter that depends on both crack and specimen sizes and geometries (is tabulated for different crack-specimen geometries), as well as the type of load application

# GEOMETRY, LOAD, & MATERIAL

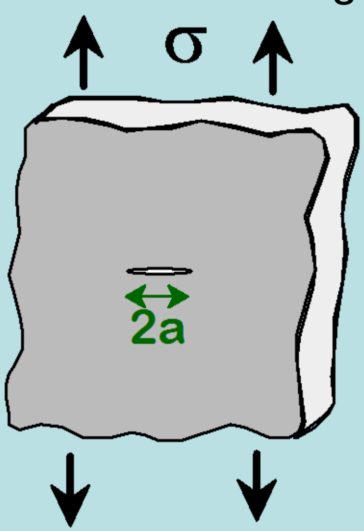
- Condition for crack propagation:

**Stress Intensity Factor**: → $K \geq K_c$ ← **Fracture Toughness:**

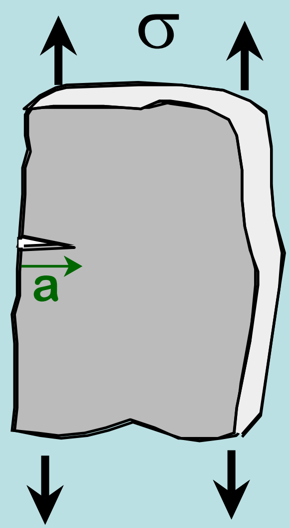
-Depends on load & geometry

-Depends on the material, temperature, environment,& rate of loading.

- Values of K for some standard loads & geometries:




$$K = \sigma\sqrt{\pi a}$$



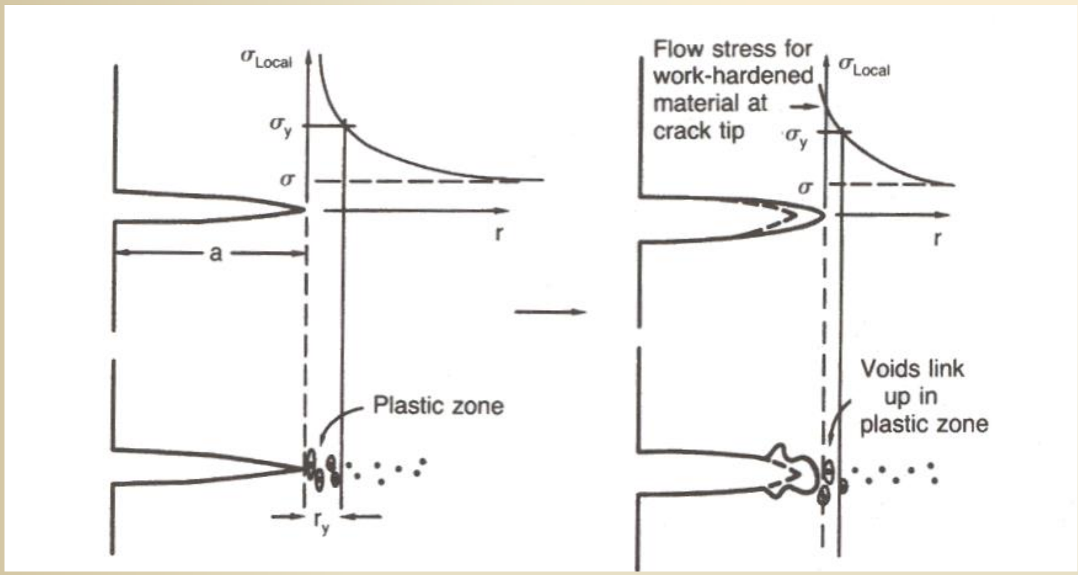

$$K = 1.1\sigma\sqrt{\pi a}$$

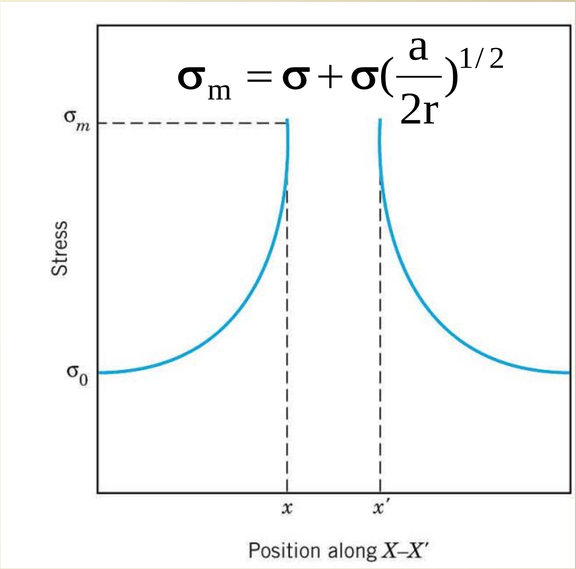
**When sample thickness>>crack dimensions**
**plane-strain** conditions occur,
$K_c = K_{1c}$
that is independent of thickness

$$K_{1c} = Y\boldsymbol{\sigma}_c\sqrt{\pi a}$$

Plane strain fracture toughness

# Mechanisms of Crack Propagation: ductile tearing

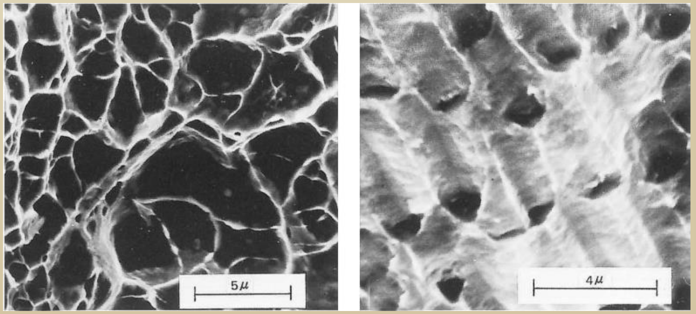


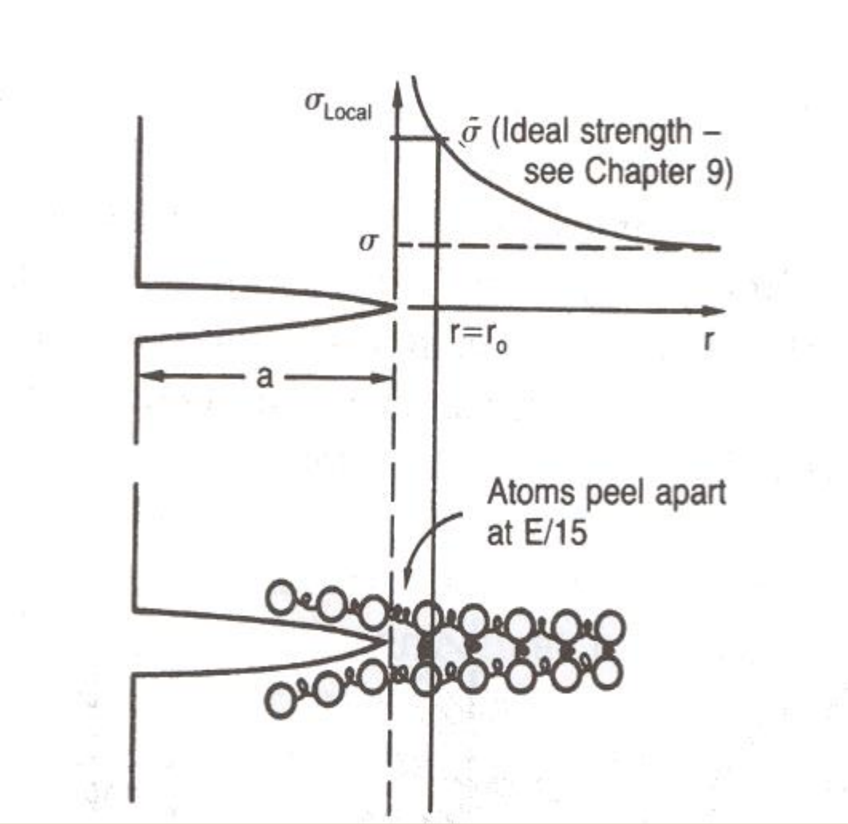

$$r_y = \frac{\sigma^2 a}{2\sigma_y{}^2} = \frac{K^2}{2\pi\sigma_y^2}$$

$$K = K_c$$

The plastic flow at the crack tip naturally turns the initially sharp crack into a ***blunt crack***. Crack blunting decreases $\sigma_m$ so that crack tip itself can keep on plastically deforming. Thus ductile tearing ***consumes a lot of energy by plastic flow.*** This is why ductile materials are so ***tough***.



# Mechanisms of Crack Propagation: cleavage




Blunting of the sharp crack does not occur/ The local stress at the crack tip is large enough to ***break apart the inter-atomic bonds***!!
The crack spreads between a pair of atomic planes leading to the formation of flat surfaces by ***cleavage***.

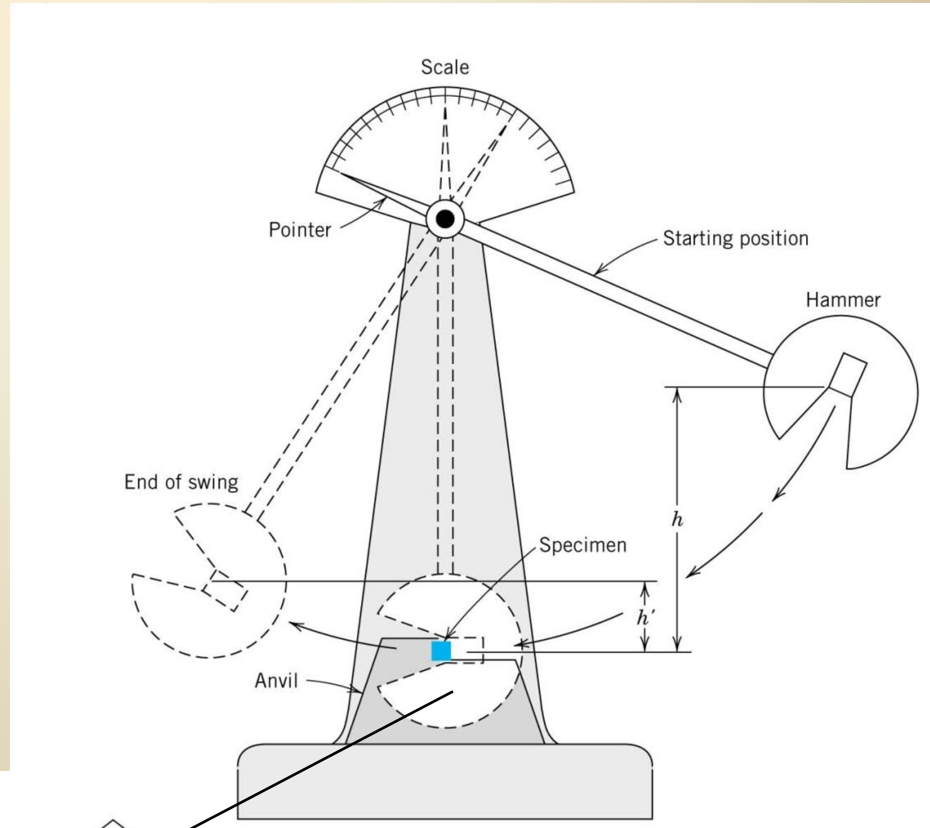
$$r = \frac{\sigma^2 a}{2\sigma_y^{\;2}} = \frac{K^2}{2\pi\sigma_y^2}$$

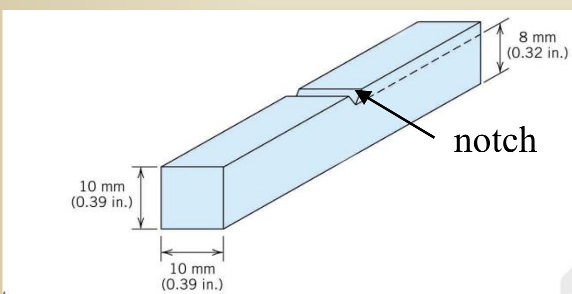
$$K = K_c$$

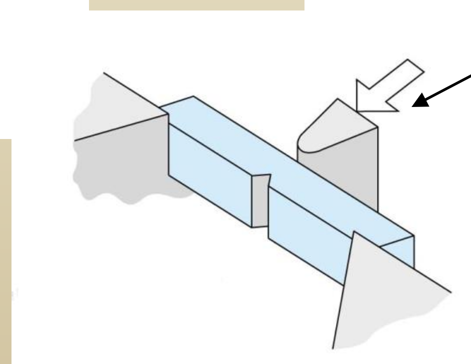
# IMPACT FRACTURE TESTING

**Charpy V-notch (CVN) technique** represents the most severe relative to the potential for fracture conditions:

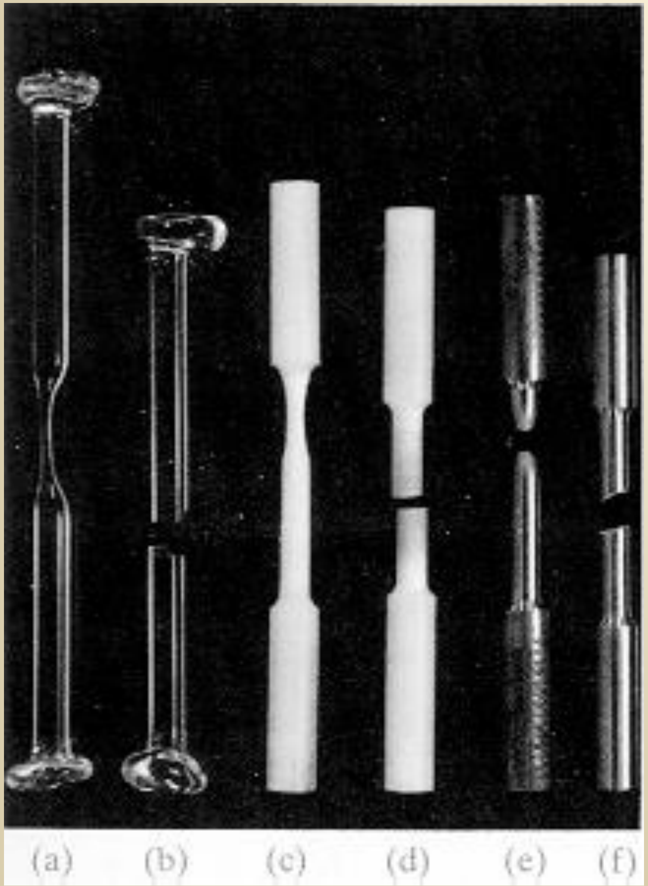
- deformation at low temperature;
- a  high strain rate
- a tri-axial stress state introduced by a notch and is used to measure the **impact energy** or notch toughness



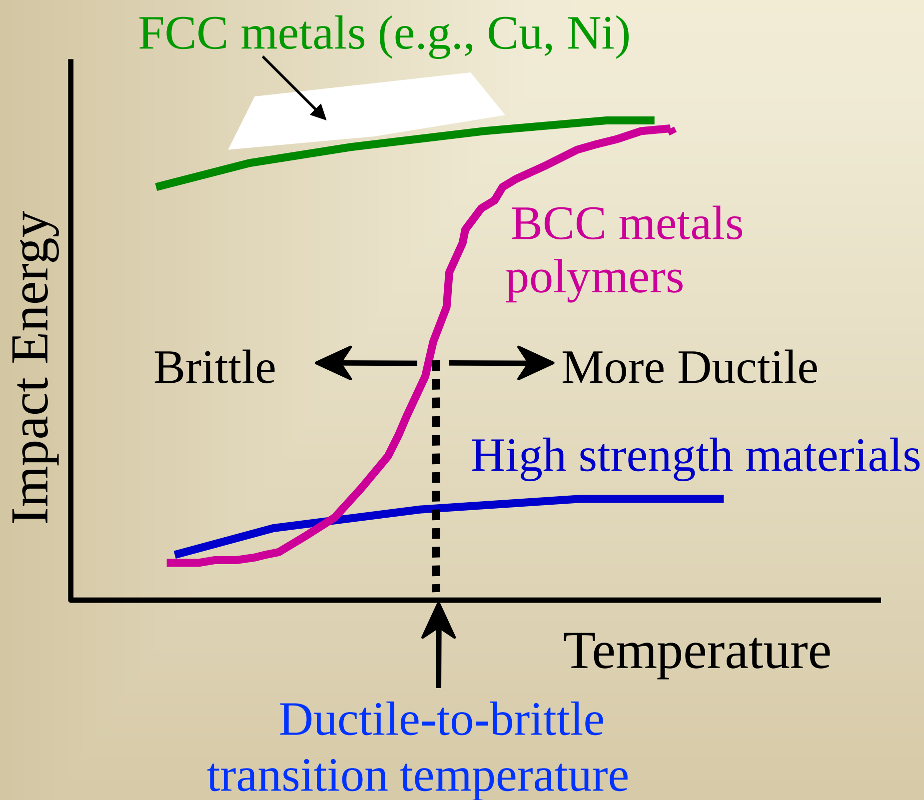
# Temperature and Strain Rate

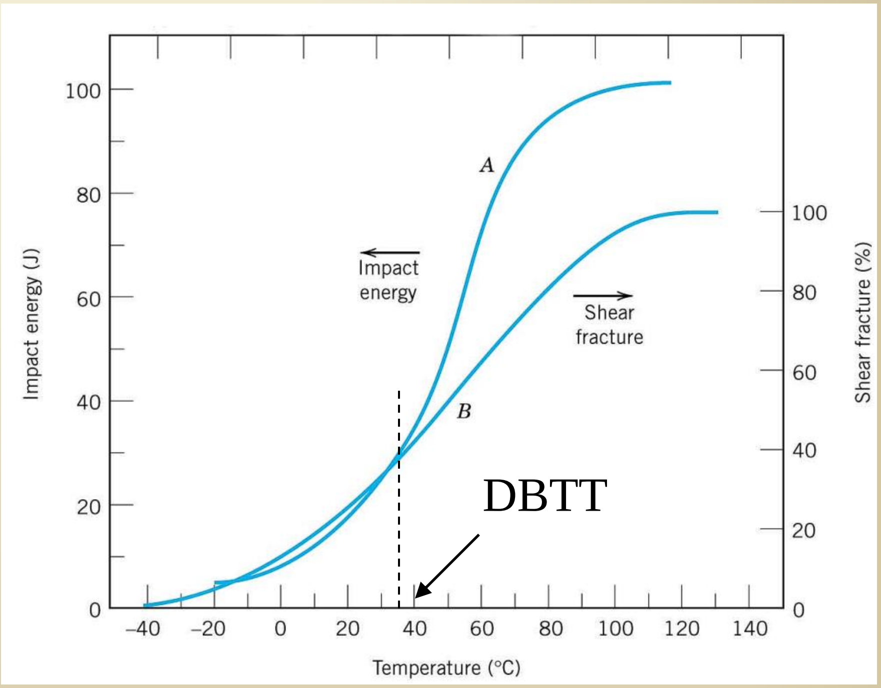



- Samples (a) and (b) are both glass, but (a) was tested **at 800 K** and behaved in **a ductile** (viscoelastic) way whereas (b) was tested **at 273 K** and showed **brittle** behavior.
- Poly-propylene samples (c) and (d) were tested at the same temperature but at **different strain rates.** At low strain rates (c) this polymer is **ductile** but at high strain rates (d) it shows **brittle** fracture.

# TEMPERATURE

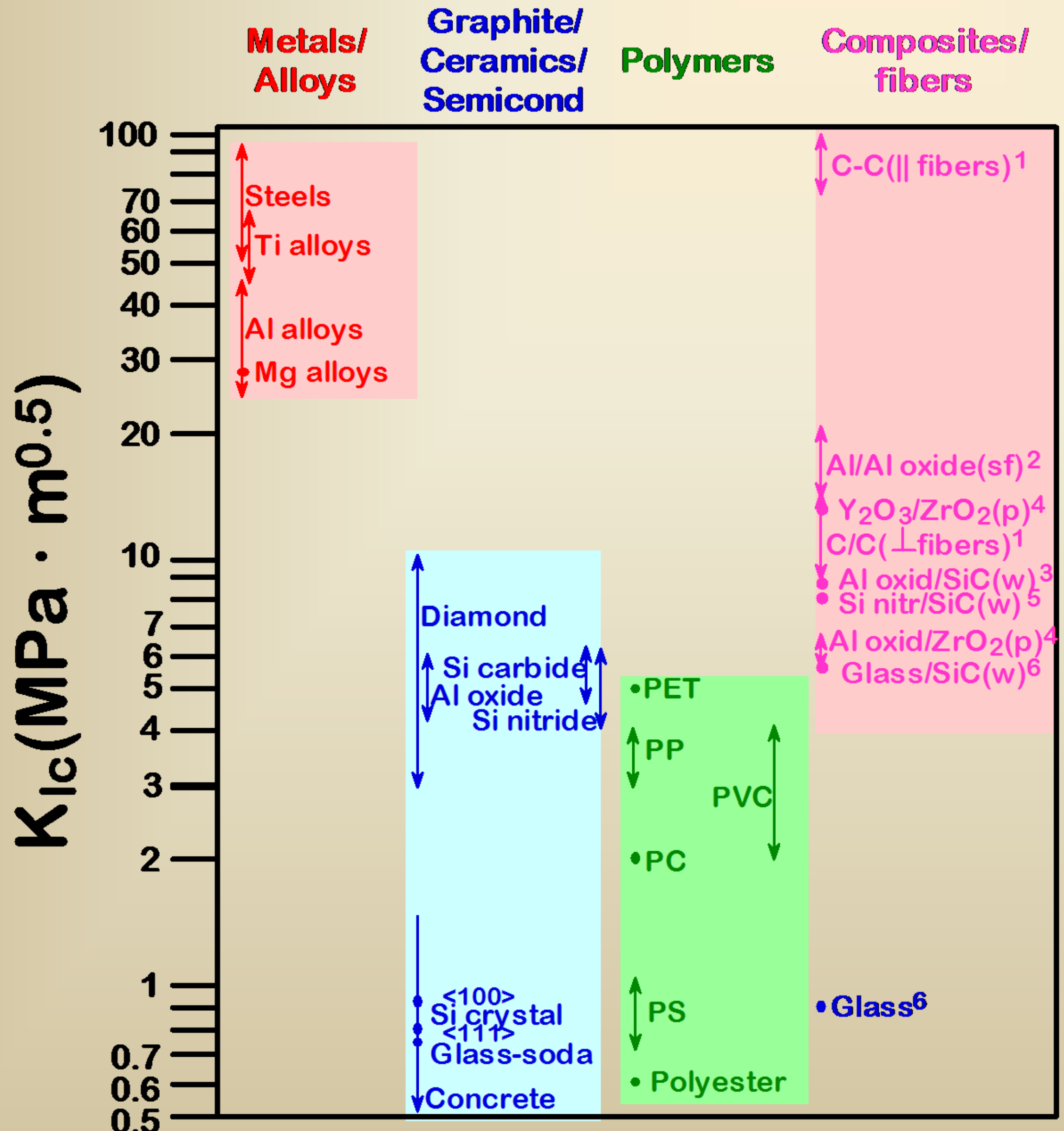
- Increase of temperature leads to the increase of shear fracture and $K_c$
- **Ductile-to-brittle transition temperature (DBTT)**

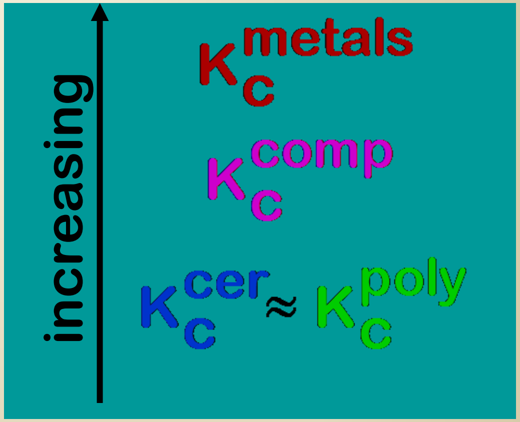



Example: A283 Steel

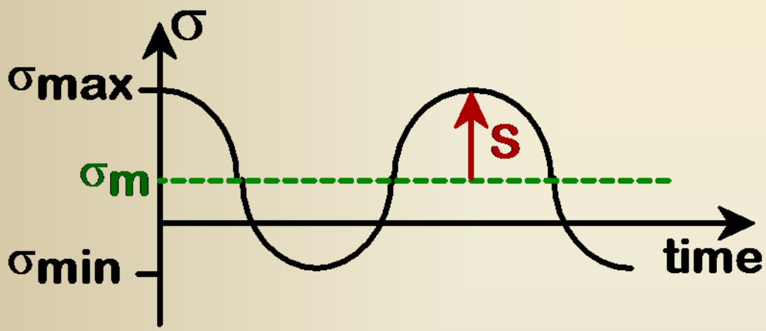
# FRACTURE TOUGHNESS



# FATIGUE

• **Fatigue** is a failure **under cyclic stress** (bridges, aircraft etc.).

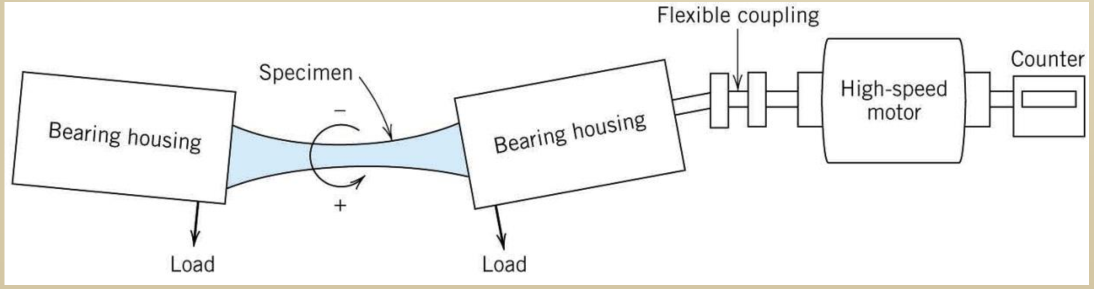



• Stress varies with time:
- key parameters are S, $\sigma_{mean}$, $\Delta\sigma$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

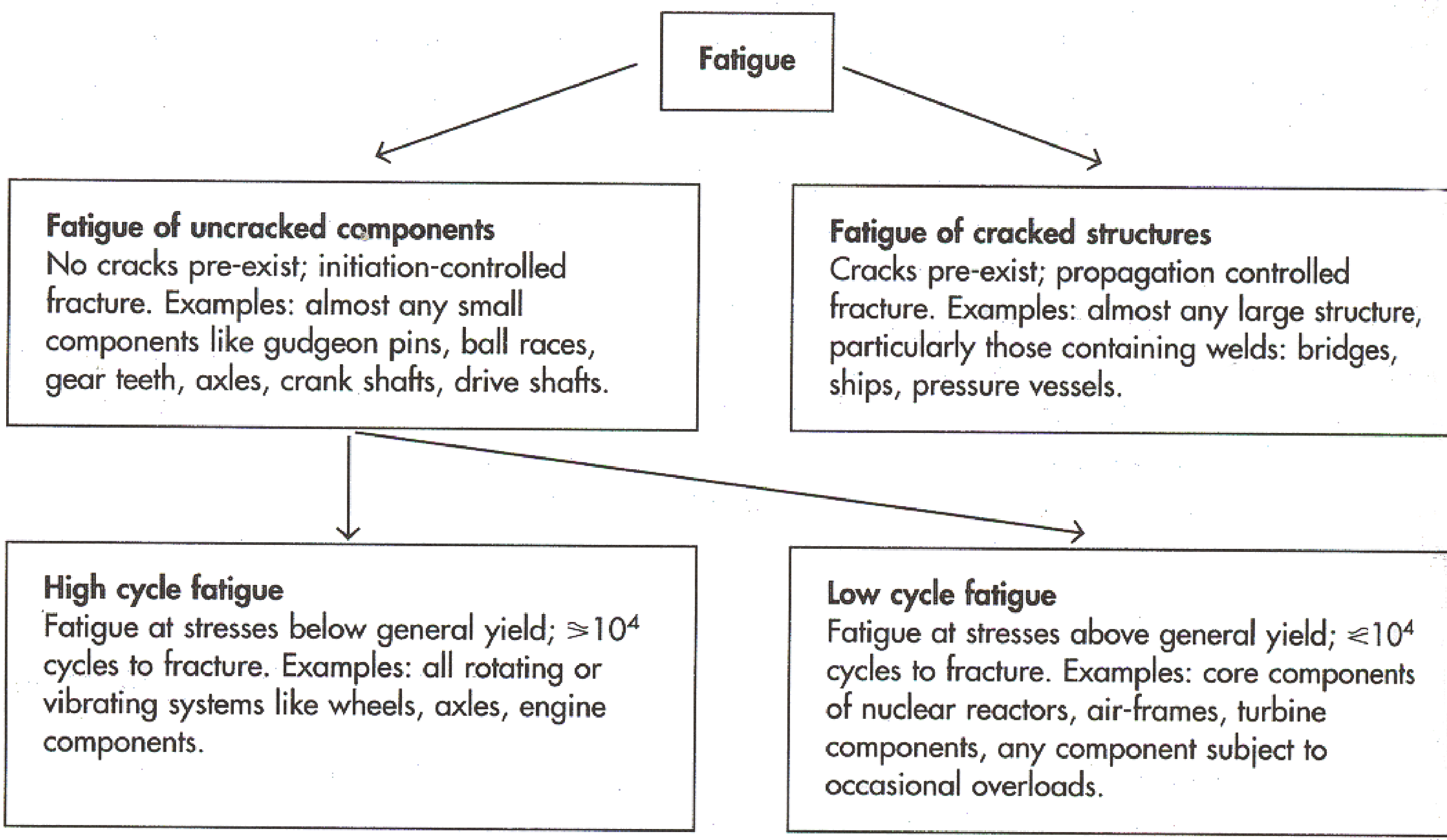
$$\Delta\sigma_a = S = \frac{\sigma_{max} - \sigma_{min}}{2}$$

• **Key points**:
- Fatigue can cause part failure, even though $\sigma_{max} < \sigma_c$, i.e. at lower strength than for a static conditions;
- Fatigue causes ~ 90% of mechanical engineering failures.

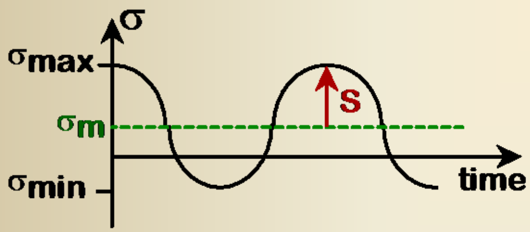



Fatigue-testing apparatus for rotating-bending test: main parameters **S** and number of cycles (N)

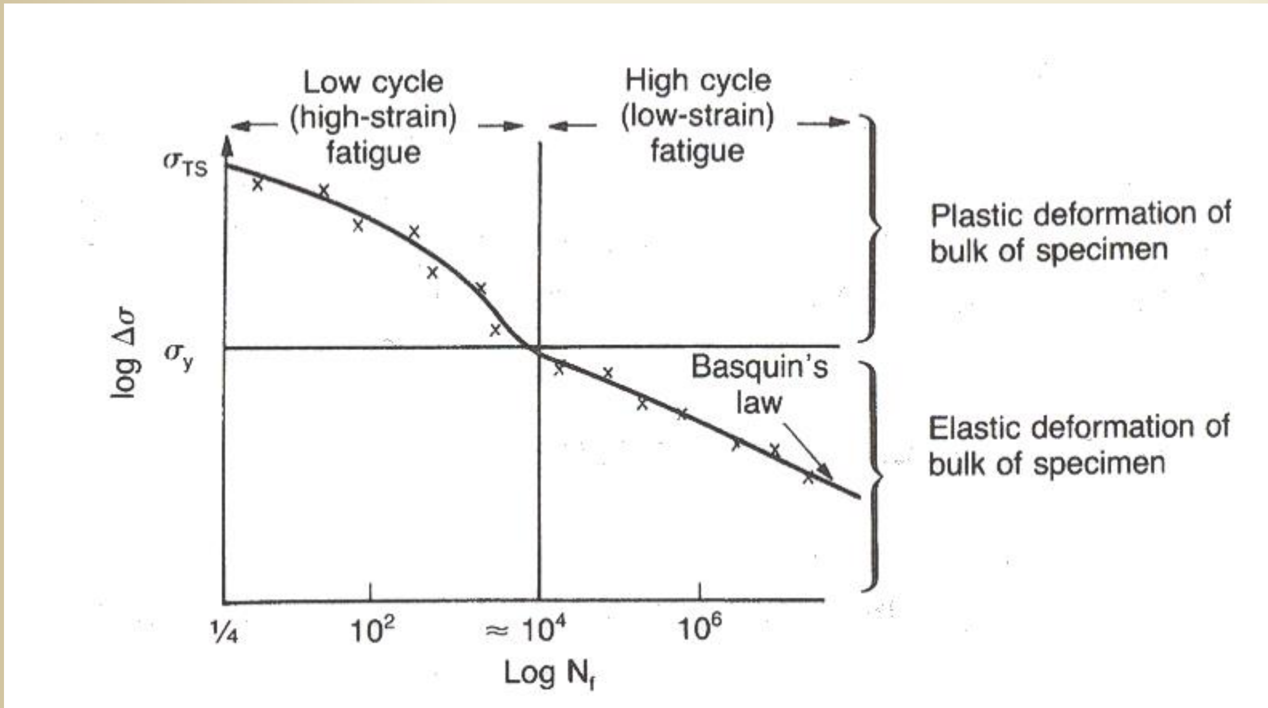
# Fatigue Failure: Classification



# Fatigue of Un-Cracked Component: high cycle fatigue




Both $\sigma_{max}$ and $/\sigma_{min}/$ are **below yield stress**.



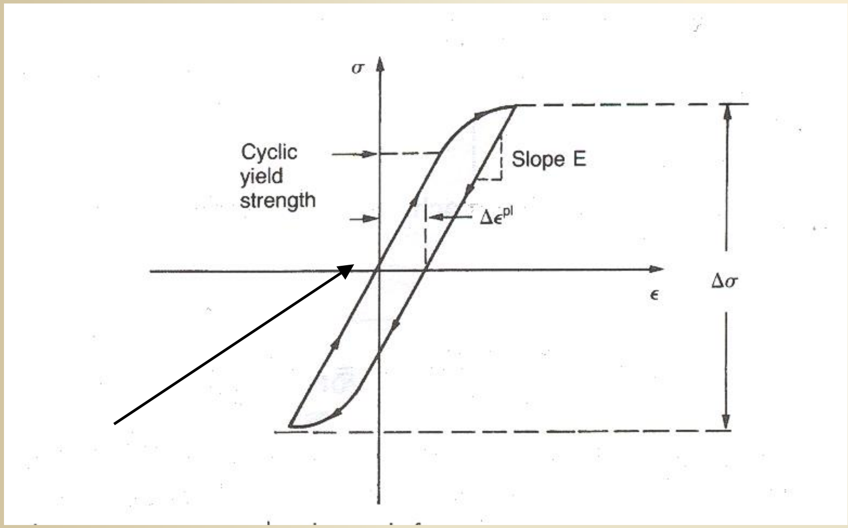

Empirical formula:

$\Delta\sigma N^a = C_1$

**Basquin's law**

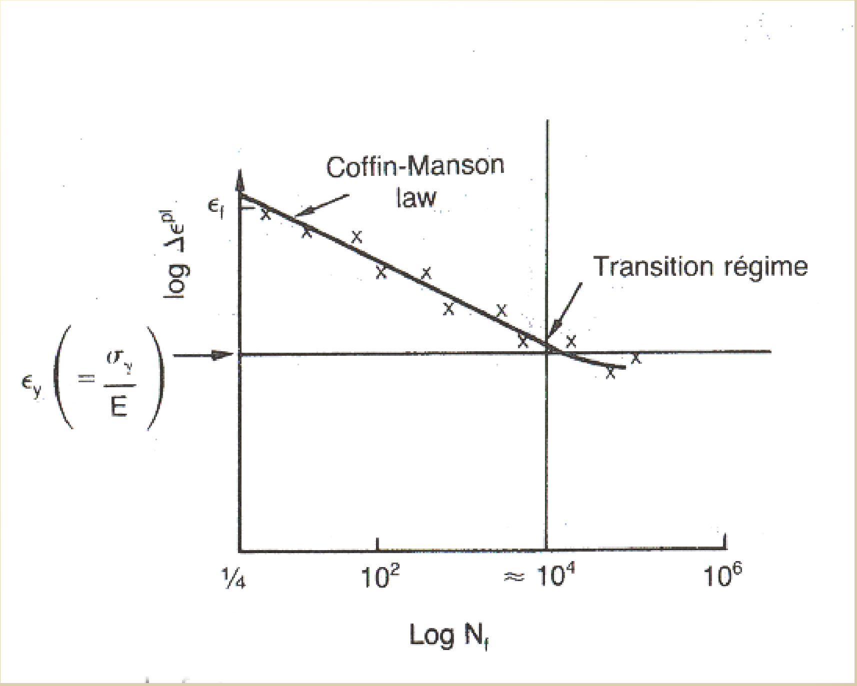
N - number of cycle
$C_1$ – constant
a - constant
$\Delta\sigma = (\sigma_{max} - \sigma_{min})/2$

# Fatigue of Un-Cracked Component: low-cycle fatigue




Plastic strain range $\Delta\varepsilon^{pl}$

$\sigma_{max}$ and/or $/\sigma_{min}/$ are **above yield stress**.



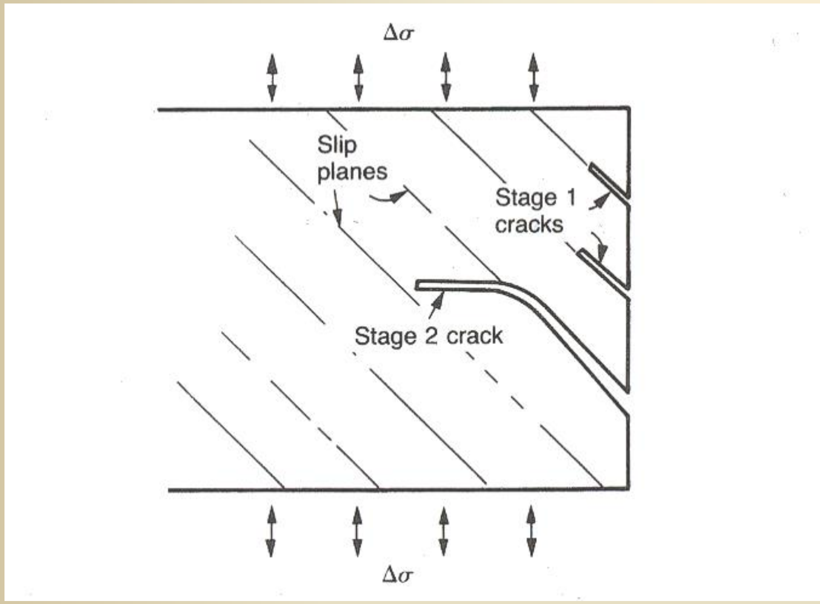

Empirical formula:

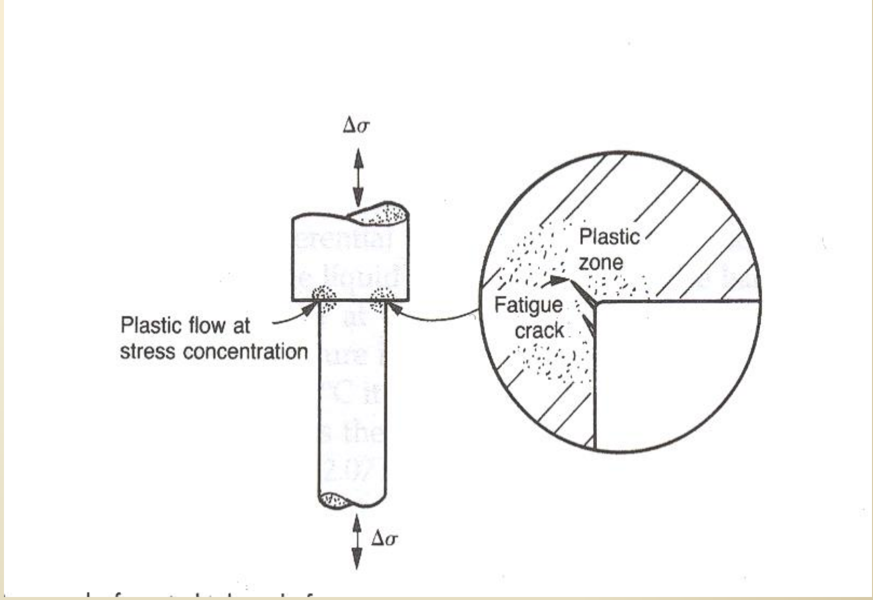
$$\Delta\varepsilon^{pl}N^{b}=C_2$$

**Coffin-Manson law**

N - number of cycle
$C_2$ – constant
b - constant

# Fatigue Mechanism: un-cracked structures

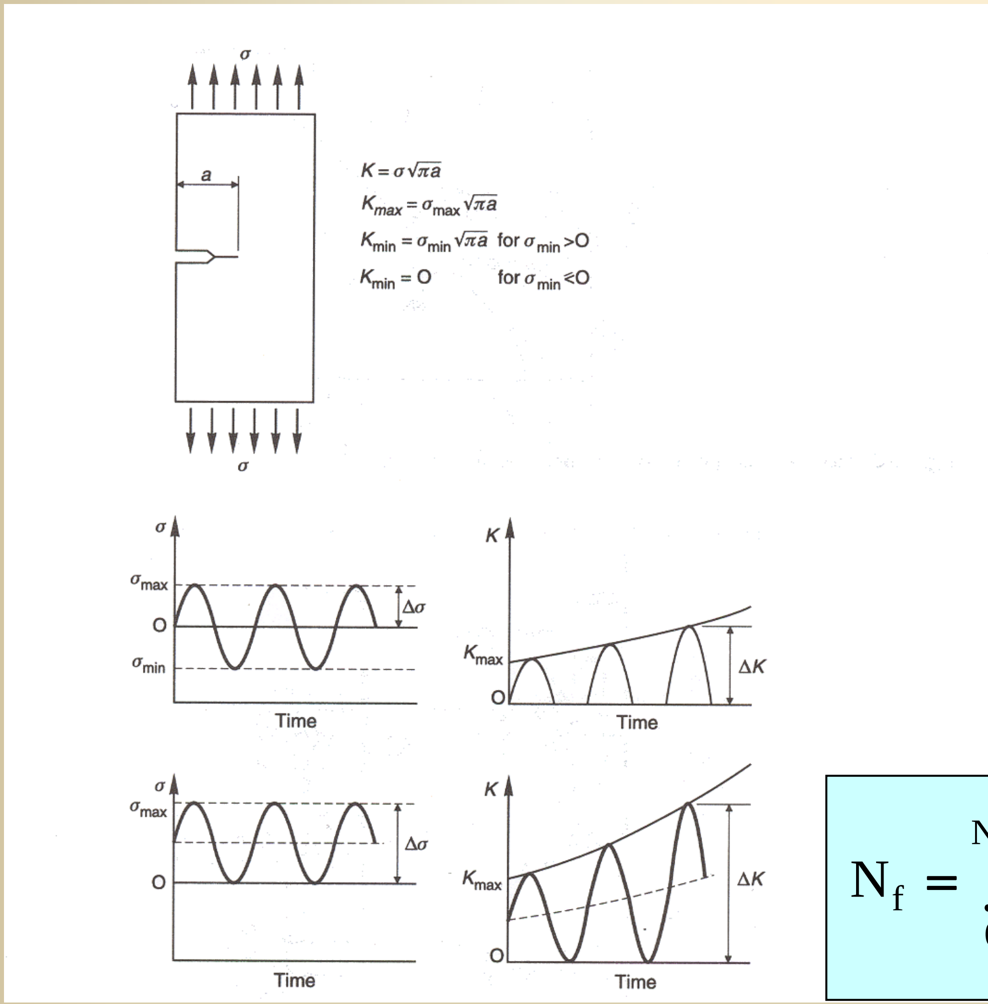



<u>Low-cycle fatigue</u>:
The general plasticity quickly roughens the surface and crack ***forms there***, first propagating along a ***slip plane*** and then normal to the tensile axis

<u>High-cycle fatigue</u>:
Stress is below $\sigma_y$, thus essentially all of the life is time up in ***initiating*** a crack in the place of local plasticity, which is related to the ***zones of stress concentrations***. Formed crack propagates slowly at first and then faster, until component fails.

# Fatigue of Cracked Component




$$\Delta K = K_{max} - K_{min} = \Delta\sigma(\pi a)^{1/2}$$

The cycle stress intensity ΔK increases with time (at constant S) because the crack growth increases tension.

For example in steady-state regime the crack growth rate is:

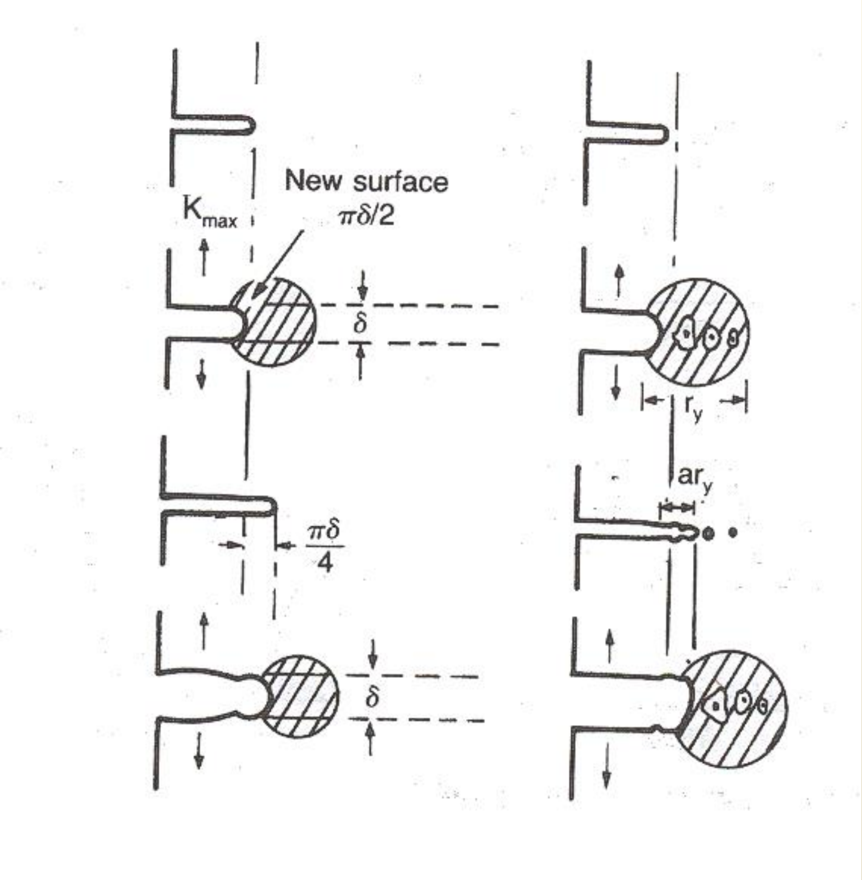
$$da/dN = A\ \Delta K^m$$

where **A** and **m** are materials' constant

$$N_f = \int_0^{N_f} dN = \int_{a_o}^{a_f} \frac{da}{A(\Delta K)^m} = \int_{a_o}^{a_f} \frac{da}{A[\Delta\sigma(\pi a)^{1/2}]^m}$$

Large, particularly ***welded structures*** (bridge, ships, nuclear reactors)

# Fatigue Mechanism: pre-cracked structures



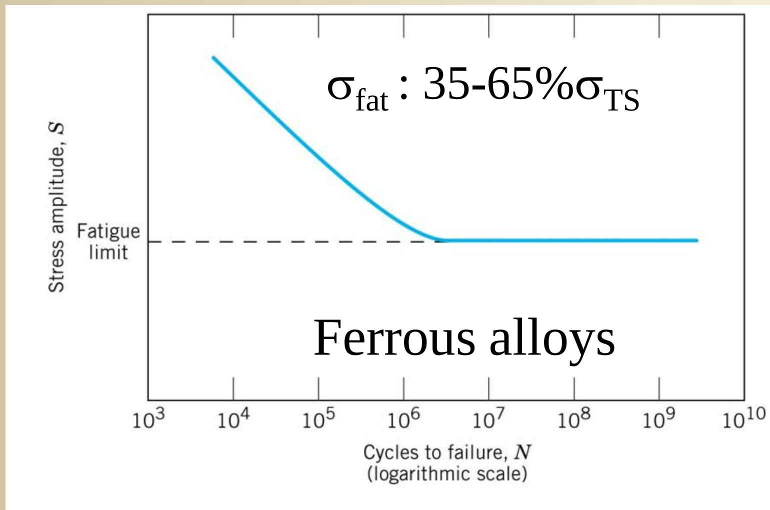

- In pure metals or polymers the tensile stress produces a plastic zone which makes the crack tip stretch open by the amount of $\delta$, creating **a new surface** there.

- As the stress is removed the crack closes and new surface folds forward extending crack ~ $\delta$.

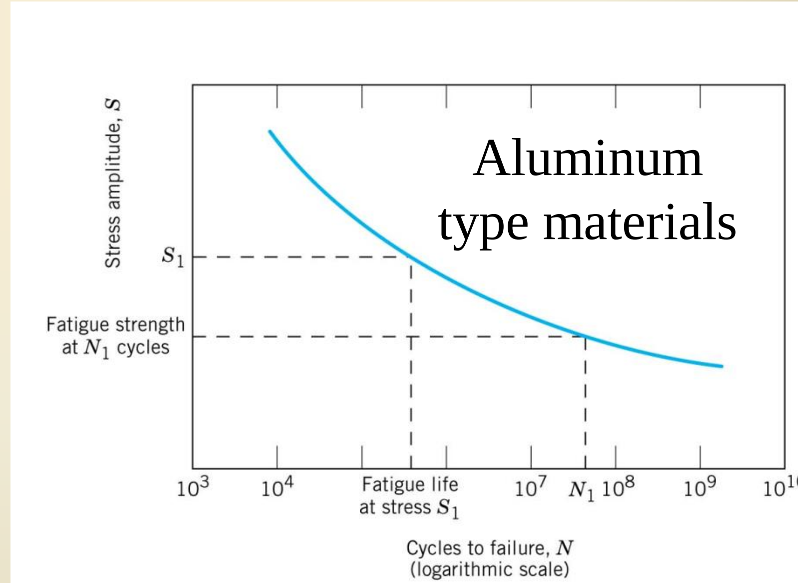
- On the next cycle the same happens again and **crack moves with rate** $da/dN \sim \delta$.

- Inclusions make the crack propagate even **faster.**

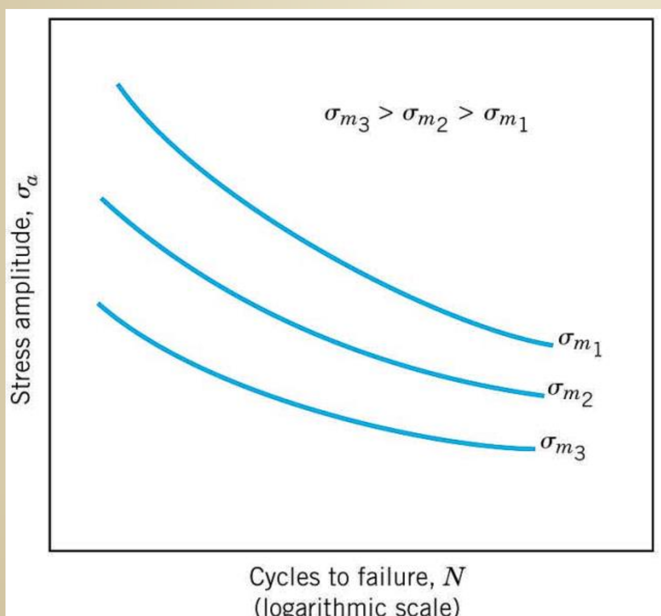
# S-N DIAGRAMS: FATIGUE DESIGN PARAMETERS




- Fatigue limit, $S_{fat}$: -no fatigue if $S < S_{fat}$




- Sometimes, the fatigue limit is **zero!**



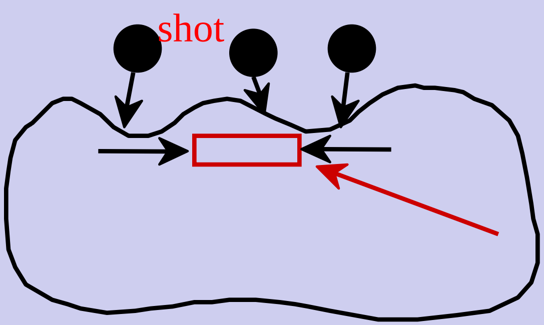

Factors affect fatigue life:

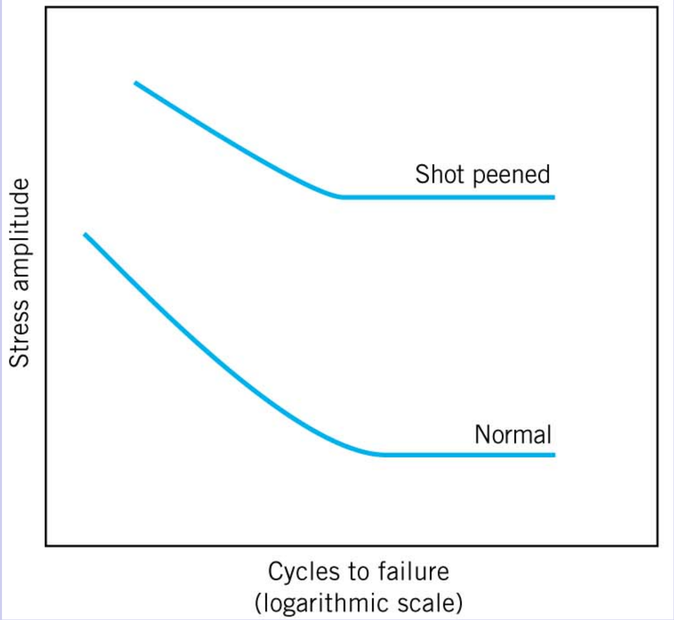
- increasing the **mean stress** leads to a decrease in fatigue life
- **surface effects:** design factor, surface treatment

# IMPROVING FATIGUE LIFE

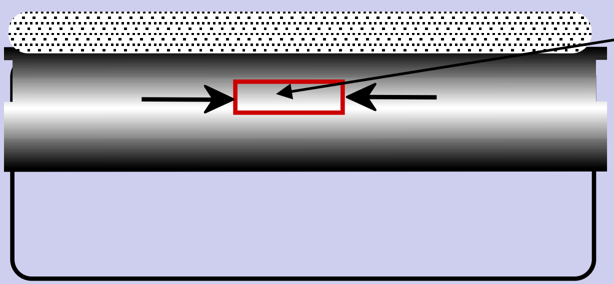
1. Impose a **compressive surface stress**: to suppress surface cracks from growing.

-Approach 1: **shot peening**

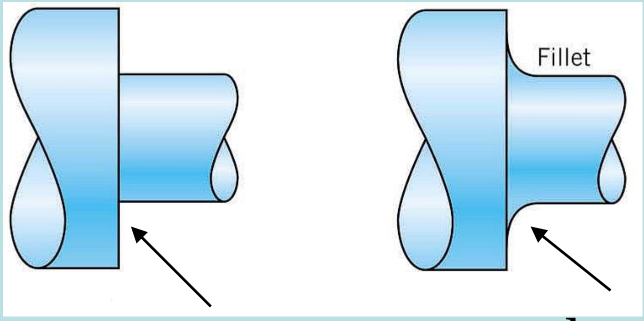



-Approach 2: **carburizing or nitriding**



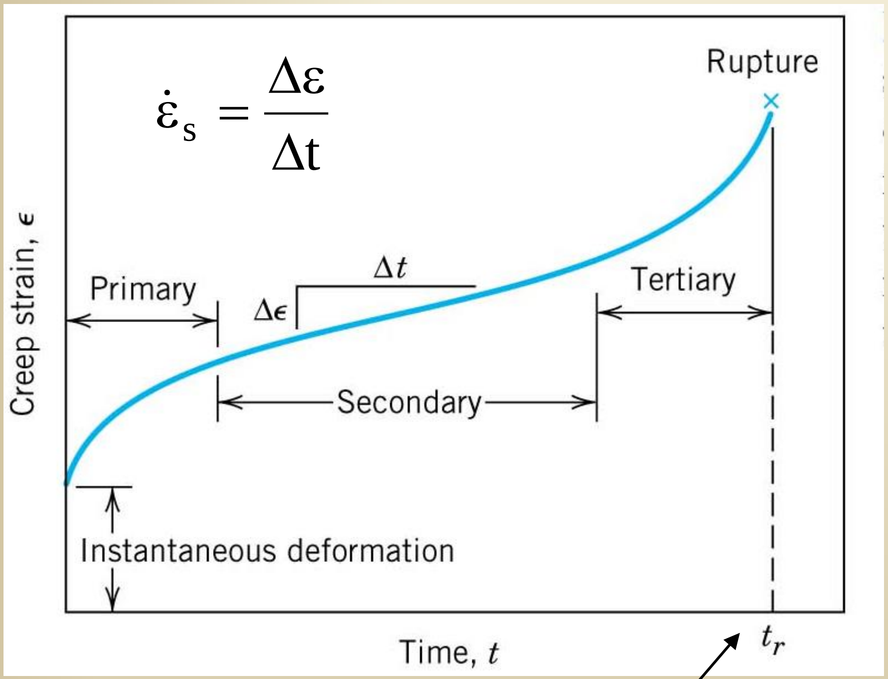

2. **Remove stress concentrators.**



# 3: CREEP

- **Creep** is deformation at elevated temperature ($T > 0.4\ T_{melt}$) but under static mechanical stress

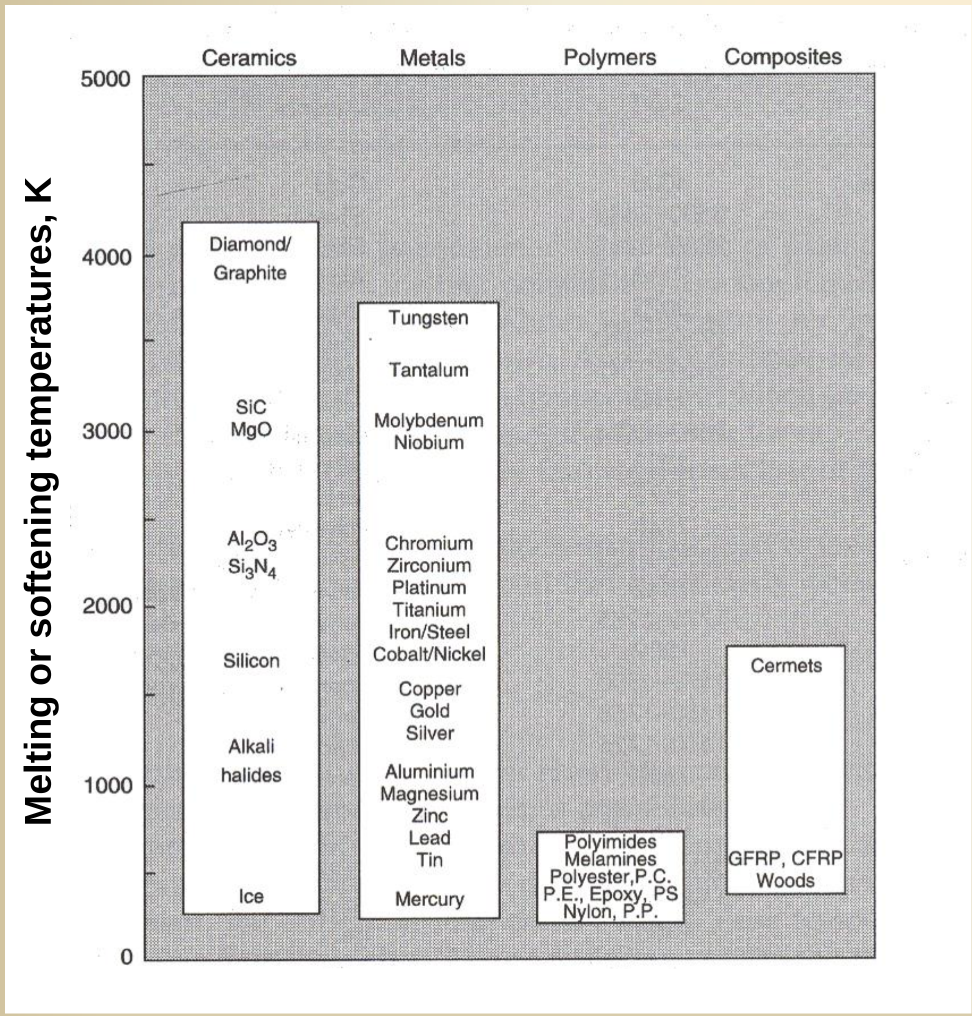
- But in this case deformation changes with time!!




$t_r$, **Creep rapture time**

- Three characteristic regions:
  - **primary creep**: creep rate decreases
  - **secondary creep**: steady-state creep,
  - **tertiary creep**: creep rate acceleration

# What is low T and what is high T?

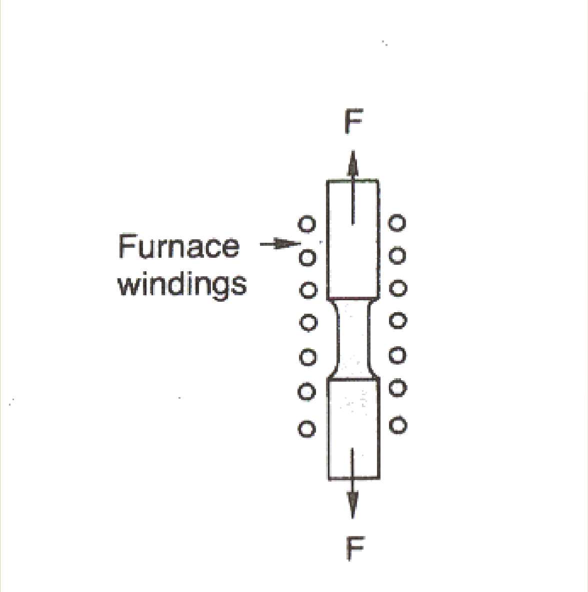



Examples:
Tungsten ~ 3000 °C
Lead ~ 400° C
Ice ceramic 0 °C - !!

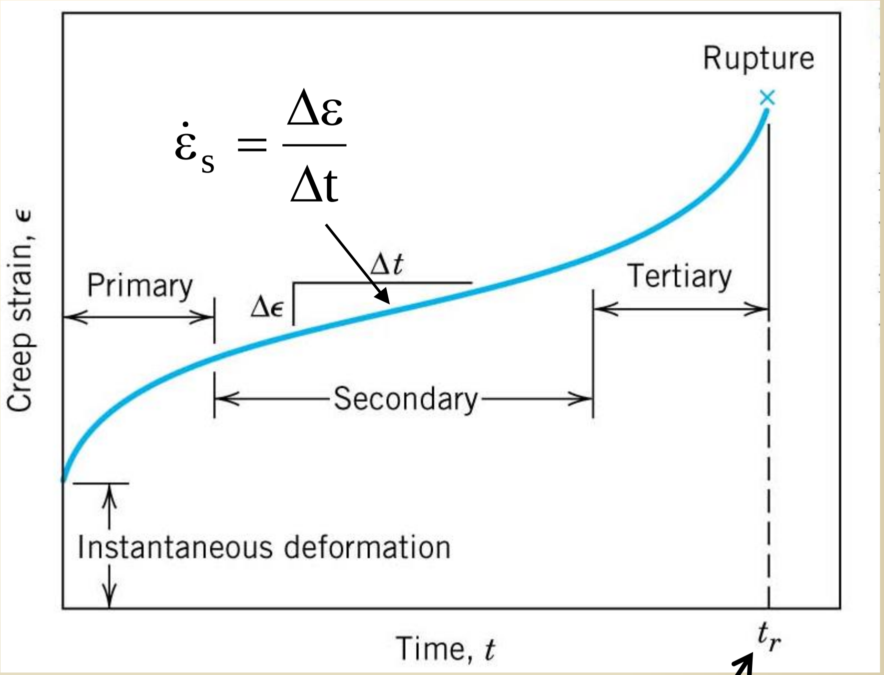
T> 0.3 to 0.4 $T_M$ for **metals**
T> 0.4 to 0.5 $T_M$ for **ceramics**

**Polymers** also creep – many of them do so at room temperature

# Creep Curve




**Schematics of Creep Curve Measurements**




**$t_r$, Creep rapture time**

- Three characteristic regions:

  - **primary creep**: creep rate decreases

  - **secondary creep**: steady-state creep,

  - **tertiary creep**: creep rate acceleration

# Steady-State (Secondary) Creep

- Typically the **longest duration** stage of creep
- **Strain rate is constant** at a given T and $\sigma$, because **strain hardening** is **balanced** by **recovery**!!!

stress exponent (material parameter)

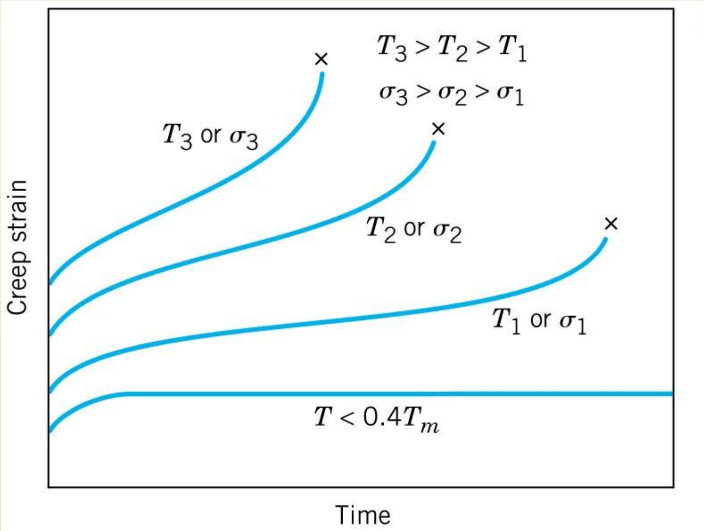
$$\dot{\varepsilon}_s = K_2 \sigma^n \exp\left(-\frac{Q_c}{RT}\right)$$

activation energy for creep (material parameter)

strain rate

material const.

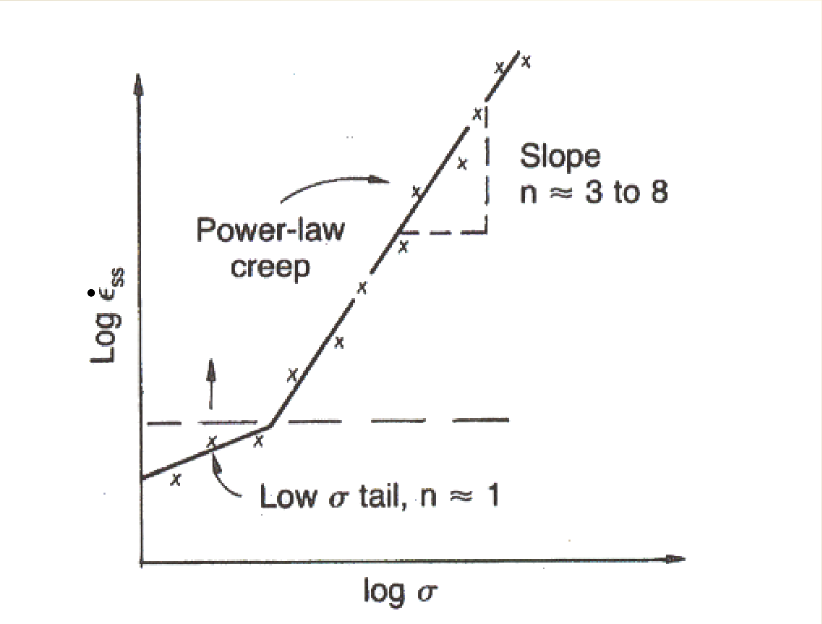
applied stress




**the strain rate increases for larger T, $\sigma$**

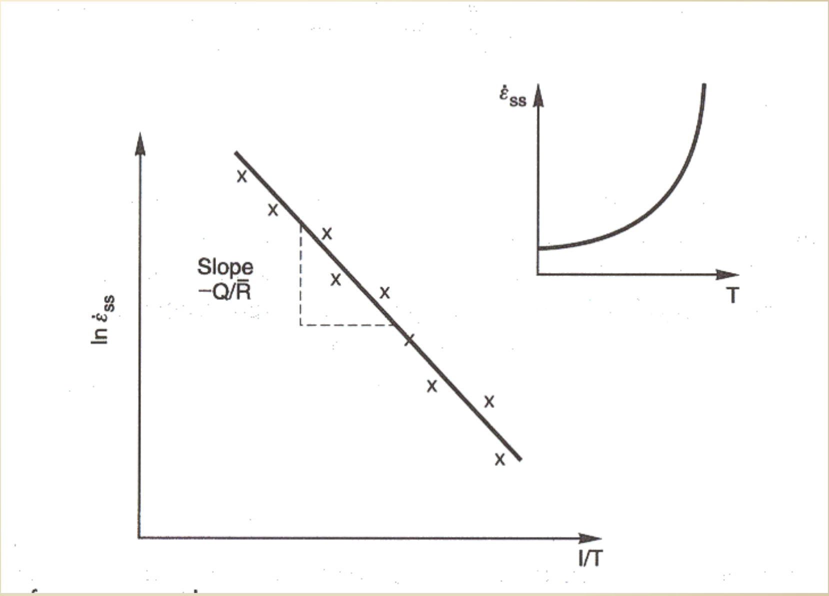
# Steady-State Creep

**Stress Dependence**



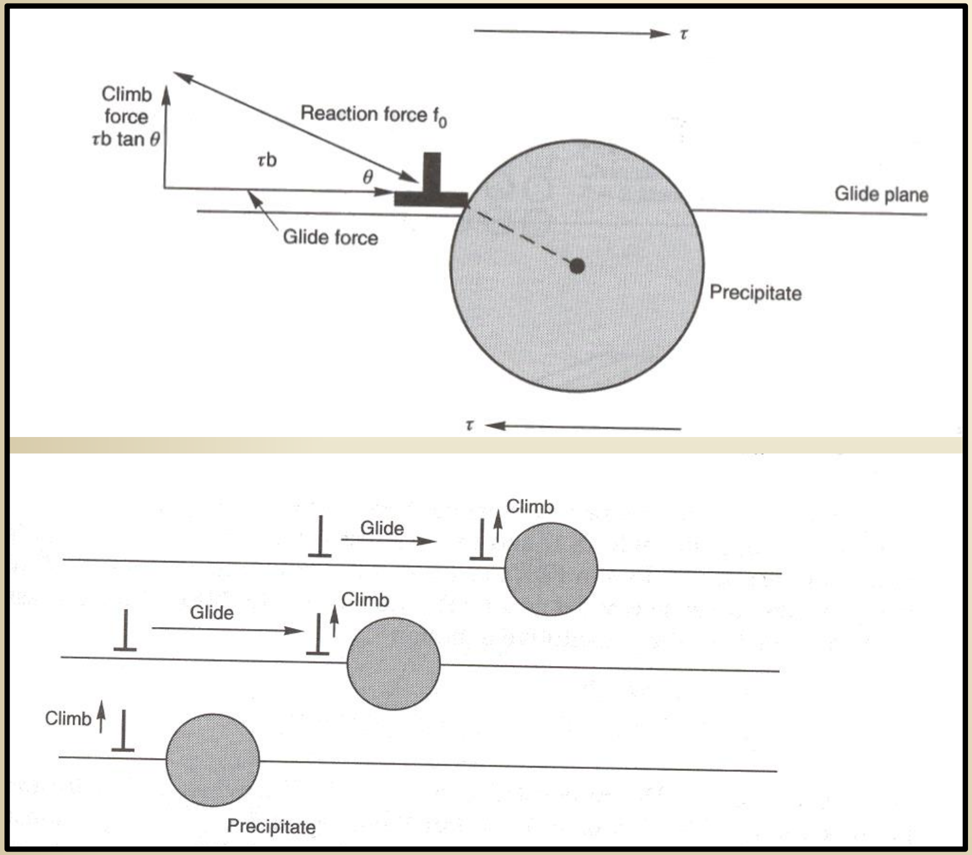

$$\dot{\varepsilon}_{ss} = B\sigma^{n}$$

**Temperature Dependence**




$$\dot{\varepsilon}_{ss} = Ce^{-(Q/RT)}$$

# Creep Mechanisms: Dislocation Creep



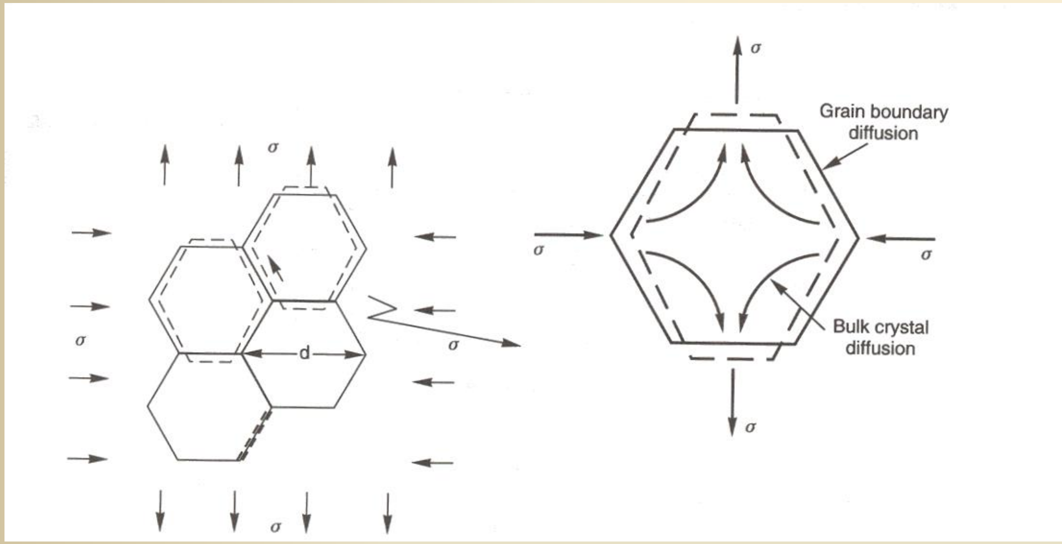

Climb unlocks dislocations
from precipitate which pin them
And further slip may take place!!

$D=D_o \exp(-Q/RT)$

$$\dot{\varepsilon}_{ss} = A\sigma^n e^{-(Q/RT)}$$

- At lower end of the creep regime (0.3-0.5Tm) **core diffusion** tends to dominate; at the higher end (0.5-0.9TM) it is **bulk diffusion**

# Creep Mechanisms: Diffusion Creep



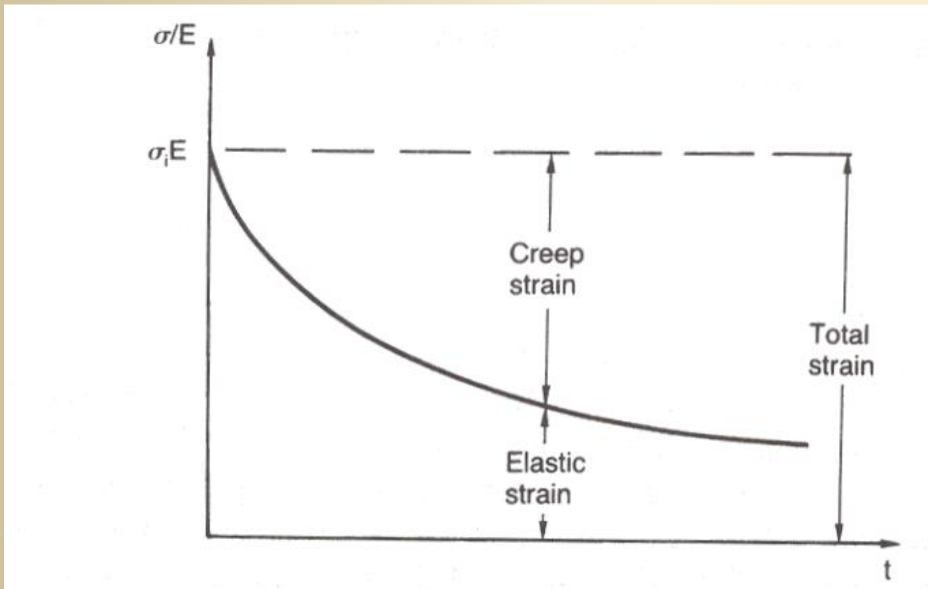

At **lower stresses** the rate of power –law creep falls quickly (from n =3 – 8 to **n=1**). Creep does not stop, but rather proceeds by an **alternative mechanism**, i.e. a polycrystal can extend in response to applied stress by **grain elongation**!

In this case **atoms diffuse** from one set of the grain faces to the another and dislocations are not involved.

$$\dot{\varepsilon}_{ss} = CD\sigma / d_2 = C'\sigma e^{-(Q/RT)} / d^2$$

**grain size**

# Creep Relaxation




<u>At constant displacement</u>, **creep causes stresses to relax** with time!

<u>Example</u>: Bolts in hot turbine must be regularly tightened; plastic paper-clips are not, in the long term, as good as a steel ones.

The relaxation time (arbitrary defined as time taken for the stress to relax to half of original value) can be estimated as follows:

$$\varepsilon^{total} = \varepsilon^{elastic} + \varepsilon^{creep}$$

$$\varepsilon^{el} = \sigma / e$$

$$\dot{\varepsilon}^{cr} = B\sigma^{n}$$

$$\varepsilon^{total} = const$$

$$\frac{1}{e} \cdot \frac{d\sigma}{dt} = -B\sigma^{n}$$

Integrating from $\sigma = \sigma_{in}$ to $\sigma = \sigma_{in} / 2$

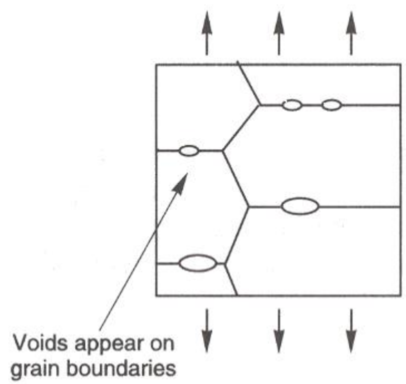
$$t_r = \frac{(2^{n-1} - 1)}{(n-1)BE\sigma_{in}^{n-1}}$$

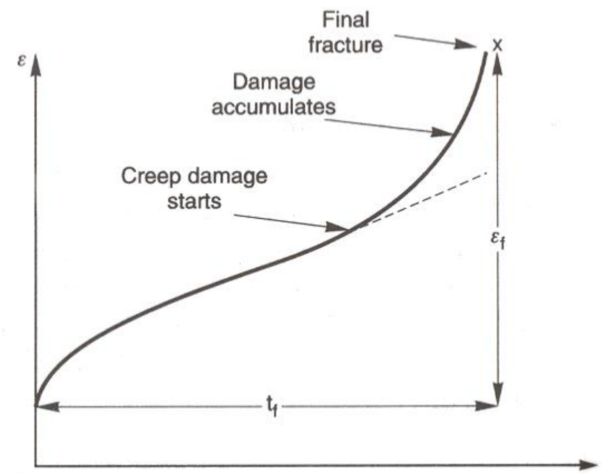
# Creep Damage and Creep Fracture

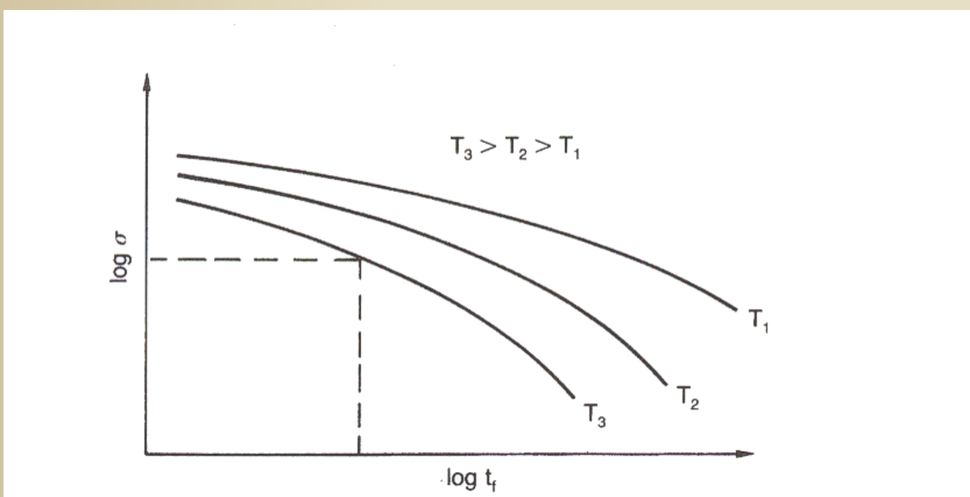
Damage in the form of internal cavities, accumulates during Tertiary Stage of creep

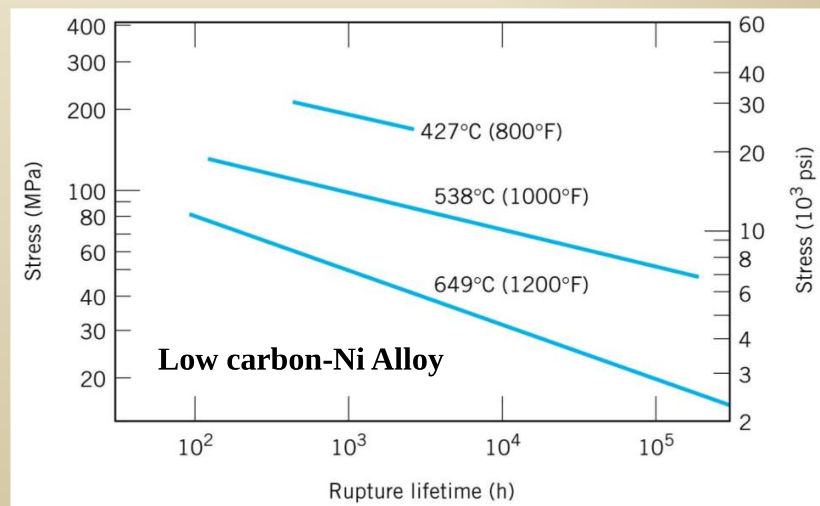
Time-to-failure is described by equation which looks like that for creep itself:

$$t_f = A'\sigma^{-m} e^{Q/RT}$$



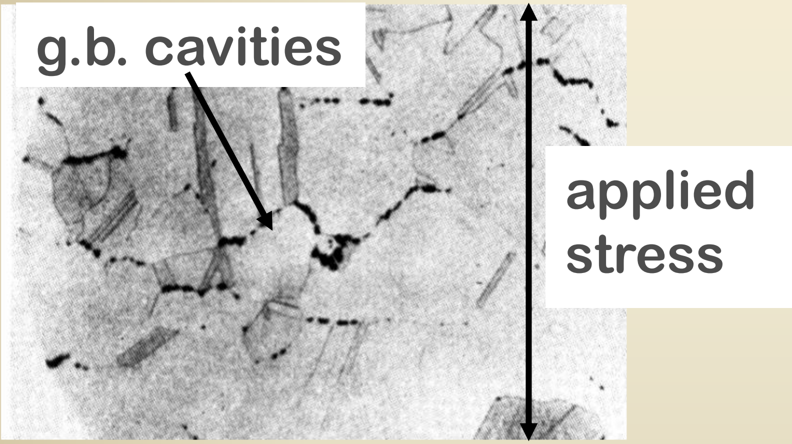



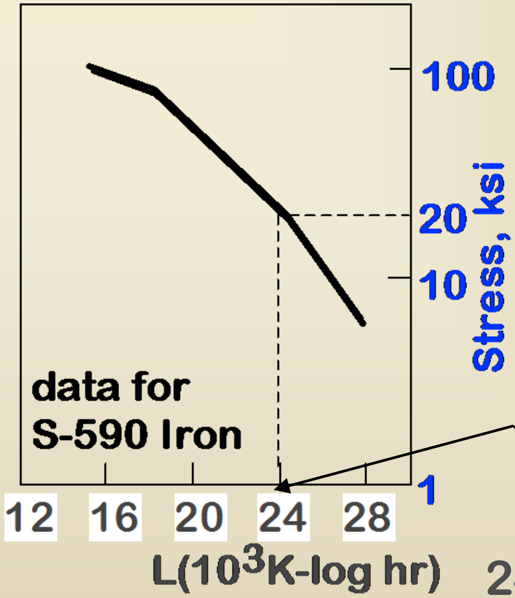

- the rapture time diminishes for larger T, σ

# CREEP FAILURE

- **Failure:**
  **along grain boundaries.**




- **Estimate rupture time**
  **S-590 Iron, T = 800ºC, σ = 20 ksi**




- Time to rupture, $t_r$

$$\underbrace{T}_{\text{temperature}}(20 + \log \underbrace{t_r}_{\text{time to failure (rupture)}}) = \underbrace{L}_{\text{function of applied stress}}$$

temperature

function of
applied stress

time to failure (rupture)

$$\underbrace{T}_{1073K}(20 + \log t_r) = \underbrace{L}_{24\times10^3 \text{ K-log hr}}$$

1073K

**Answer: $t_r$ = 233hr**

# SUMMARY

- Engineering materials don't reach theoretical strength.
- **Flaws** produce **stress concentrations** that cause premature failure.
- Sharp corners also produce large stress concentrations and premature failure.

- **Failure type depends on stress and T:**

  -for non-cyclic $\sigma$ and $T < 0.4T_m$, failure stress decreases with:

  increased maximum flaw size, decreased T,

  -for cyclic:

  cycles to fail decreases as $\Delta\sigma$ increases.

  -for higher T ($T > 0.4T_m$):

  time to fail decreases as $\sigma$ or T increases.