Differentiation Formulas

As we did with limits and continuity, we will introduce several properties of the derivative and use them along with the derivatives of some basic functions to make calculation of derivatives easier.

Constant Functions and Power Functions

Derivative of a Constatnt: $\frac{d}{dx}(c) = 0$, if c is a constant.

Power Rule : If n is a positive integer, then $\frac{d}{dx}(x^n) = nx^{n-1}$.
Example If $g(x) = 2, \quad f(x) = x^3$, find $f'(x)$ and $g'(x)$.

Just as with limits, we have the following rules:
Constant Multiple Rule : $\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$, where c is a constant and f is a differentiable function.
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Sums and Differences

The Sum Rule if f and g are both differentiable at x, then $f + g$ is differentiable at x and

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

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Example Find the derivative of the function $f(x) = x^2 + 2x + 4$.

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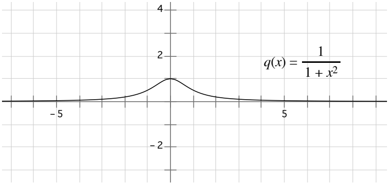
Special Case of The quotient Rule

Let g be differentiable and non-zero at x, then $\frac{d}{dx}\left(\frac{1}{g(x)}\right) = -\frac{g'(x))}{(g(x))^2}$

Example The curve $y = 1/(1+x^2)$ is called a **Witch of Maria Agnesi**. Find the equation of the tangent line to the curve at the point $(-1, 1/2)$.



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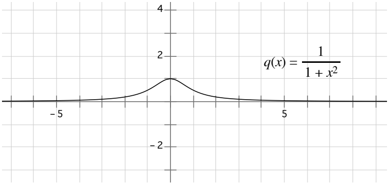

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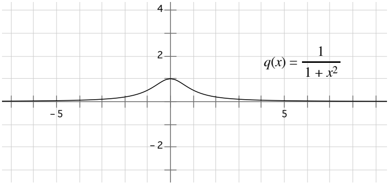

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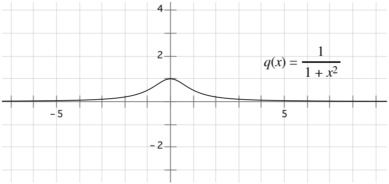

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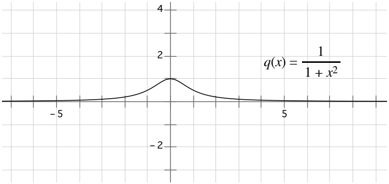

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- Since $(-1, 1/2)$ is a point on the tangent, we have the equation of the tangent to the curve at $x = -1$ is given by

$$y - 1/2 = \frac{1}{2}(x+1) \quad \text{or} \quad y = \frac{1}{2}x + \frac{1}{2} = +\frac{1}{2} \quad \text{or} \quad y = \frac{1}{2}x + 1.$$

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If f and g are differentiable at x and $g(x) \neq 0$, then $\frac{f}{g}$ is differentiable at x and

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The Quotient Rule

We can combine the above rules to get the quotient rule:
If f and g are differentiable at x and $g(x) \neq 0$, then $\frac{f}{g}$ is differentiable at x and

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}.$$

We can rewrite this as

$$\frac{d(\frac{u}{v})}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \quad \text{or} \quad \left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}.$$

Example Let $K(x) = \frac{x^3+x^2+1}{x^4+1}$, find $K'(x)$. What is $K'(1)$?

- let $f(x) = x^3 + x^2 + 1$ and $g(x) = x^4 + 1$.
- The rule above says $K'(x) = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2} = \frac{(x^4+1)\frac{d}{dx}[x^3+x^2+1] - (x^3+x^2+1)\frac{d}{dx}(x^4+1)}{[(x^4+1)]^2}$
- $= \frac{(x^4+1)(3x^2+2x) - (x^3+x^2+1)4x^3}{[(x^4+1)]^2}$
- $K'(1) = \frac{(2)(5) - (3)4}{[(2)]^2} = \frac{-2}{4} = \frac{-1}{2}$

Example

Here's a way to remember the quotient rule:
"low d-high minus hi d-low, square the bottom and away we go"
or
Low D High minus High D Low, all over the square of what's below.

Note we should see if we simplify a function with cancellation before we rush into using the quotient rule.

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- $L'(x) = 4x^3 + 2x$.

General Power Functions

When n is a positive integer $x^n = x \cdot x \cdot x \cdot \cdots \cdot x$, where the product is taken n times. We define $x^0 = 1$ and $x^{-n} = 1/x^n$. We can use the quotient rule to show that. If n is a positive integer

$$\frac{d}{dx}(x^{-n}) = -nx^{-n-1}$$

In fact it can be shown that if k is any real number

$$\frac{d}{dx}(x^k) = kx^{k-1}.$$

Example If $H(x) = 2/x^2 + 3/x^3 + 4/x^4 + 1$ find $H'(x)$.

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- $f'(x) = (\frac{1}{2} - \sqrt{2})x^{(-\frac{1}{2}-\sqrt{2})} + (2 - \sqrt{2})x^{(1-\sqrt{2})} + (\frac{1}{3} - \sqrt{2})x^{(-\frac{2}{3}-\sqrt{2})}$