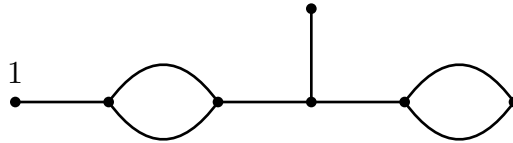


TROPICAL GEOMETRY PROBLEMS, DAY 3

DUSTIN CARTWRIGHT

In these problems, *curve* will always mean abstract tropical curve.

- (1) What is the genus of the following curve with one marked point?



All edges have length 1. What is the graph's stabilization?

- (2) Give an example of a stable curve of genus 3 with 1 marked point.
- (3) If you have learned about simplicial homology, prove that the genus $g = E - V + 1$ of a curve G is equal to the dimension of the rational homology $H_1(G, \mathbb{Q})$.
- (4) Let G be a genus 0 curve with 4 marked points. Suppose you know the distances between all pairs of marked points. How do you tell the combinatorial type of G from the distances?
- (5) Let G be a genus 0 curve with n marked points. For any 4 of the markings, you can forget the other marked points, and stabilize the curve to get a genus 0 curve with 4 marked points. How do you determine the combinatorial type of the resulting curve from the distances on the original curve?
- (6) Consider the following set of distances on a curve of genus 0 with 5 marked points:

$$\begin{array}{lll}
 d_{12} = -5 & d_{23} = -2 & d_{34} = -6 \\
 d_{13} = -5 & d_{24} = -6 & d_{35} = -6 \\
 d_{14} = -5 & d_{25} = -6 & d_{45} = -6 \\
 d_{15} = -3 & &
 \end{array}$$

What is the curve that these (negative) distances came from?

- (7) Can you reconstruct the combinatorial type of a genus 0 curve if you know the combinatorial types of the stabilizations of the restrictions to any 4 marked points?

Definition 1. A *weighted abstract tropical curve* is an abstract tropical curve together with a weight $g(v) \in \mathbb{Z}_{\geq 0}$ for every unmarked vertex v . The genus of a weighted curve G is $E - V + 1 + \sum_{v \in G} g(v)$. A weighted abstract tropical curve is *stable* if each unmarked vertex either has degree at least 3 or has positive weight.

The weights arise when you have a loop in your graph and you take the limit as the length goes to zero. For the rest of the problems, curve will mean stable weighted abstract tropical curve.

- (8) What are the combinatorial types of genus 1 curves with 1 marked point?
- (9) What do you think the moduli space for genus 1 curves with 1 marked point should be? (Hint: it's not a tropical variety)
- (10) What are the combinatorial types of genus 1 curves with 2 marked points?
- (11) What about the moduli space of genus 1 curves with 2 marked points?