

TROPICAL GEOMETRY PROBLEMS, DAY 4

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Definition 1. For the purpose of these problems, we will say that a tropical curve is *smooth* at a point v if v is contained in either 2 or 3 outgoing edges and all those edges have multiplicity 1. (By the balancing condition, it is not possible for v to be contained in only one outgoing edge.)

A tropical curve is *nodal* at a point v if a neighborhood of v is a union of two lines, again all having multiplicity 1.

In all of the following problems, K is an algebraically closed field with valuation. The rough heuristic principle is that under tropicalization, the singularities might get “worse” but never better: a smooth curve might or might not tropicalize to a smooth tropical curve, but a nodal curve will tropicalize to a tropical curve with nodal singularities or worse.

- (1) Give an example of a smooth irreducible curve in $(K^*)^2$ whose tropicalization has a node.
- (2) Give an example of a nodal irreducible curve in $(K^*)^2$ whose tropicalization has a node, but is otherwise smooth.
- (3) Prove that if C is a curve in $(K^*)^2$ and the tropicalization of C is smooth in the above sense, then C is also smooth.