Singular Learning Theory

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Many difficult problems in machine learning are victims of the curse of singularities. As the big data becomes more important, the key mathematical issues need to be analyzed at a deeper level. In this course, we give a brief introduction to singular learning theory, a powerful geometric approach recently developed by Sumio Watanabe. No prior knowledge of statistics is required.

Day 1: Statistical Learning Theory

* Random Variables
  - Discrete Variables
  - Gaussian Variables

* Statistical Models
  - Likelihood Function
  - Maximum Likelihood
  - Kullback-Liebler Function
  - Mixture Models

* Bayesian Statistics
  - Likelihood Integral
  - Laplace Approximation
  - Bayesian Information Criterion

* Linear Regression
  - Least squares
  - Sparsity penalty

Day 2: Real Log Canonical Thresholds

* Integral Asymptotics
* Resolution of Singularities
* Real Log Canonical Thresholds
* Fiber Ideals
* Newton Polyhedra

Day 3: Singularities in Graphical Models

* Graphical Models
  - Causality vs Correlation
  - Directed vs Undirected
  - Discrete vs Gaussian

* Directed Discrete Tree Models
  - Binary Tree Cumulants

* Undirected Gaussian Models
  - Partial Correlation Hypersurfaces

* Neural Networks
  - Restricted Boltzmann Machines
  - Tropical Geometry of RBMs
  - Deep Learning
  - Sensor Networks