designing FIR fan filters suitable for the multibeam forming network of the wideband beamspace adaptive array.

Characteristics were approximated through the combination of spectral transformation and the window method so that those beam patterns that included the sidelobe characteristics of the resulting fan filters were virtually frequency independent.

Using the fan filters designed by the proposed method for the multibeam forming network, we demonstrated by computer simulation the possibility of suppressing wideband interference signals as well as a much faster convergence speed than the conventional Griffiths and Jim beamformer [9].

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Adaptive Interference Suppression for CDMA Systems with a Worst-Case Error Criterion

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***Abstract*—This correspondence proposes a novel design strategy for linear multiuser detection in CDMA systems to satisfy a deterministic worst-case error performance measure. Formulations with and without the use of training signals are considered, and a near-far resistant solution for each is presented. Further, computationally efficient adaptive algorithms are derived in this framework that allow for sharing of detectors among different users.**

I. INTRODUCTION

Multiuser detection (MUD) for direct-sequence code division multiple access (CDMA) systems deals with detection of a desired user in the presence of interfering users and ambient noise. An optimal maximum likelihood (ML) solution to the MUD problem, with an exponential complexity in the number of users, was was found in [10]. Much attention has been focused on linear multiuser detectors for which the decorrelating detector and the minimum mean-square error (MMSE) detector have been derived [5], [6], [9]. Uncertainty in the system model due to disturbances such as timing jitter and others can lead to significant performance degradation when detectors are designed with a nominal signal model in mind. Since we may not have precise statistical characterization of such modeling uncertainties, an alternative is to design estimators by imposing only deterministic constraints on the output error sequence. A possible solution explored in this work for estimation with nonstatistical constraints is to require a bounded worst-case norm of the error sequence. In general, this type of estimation is referred to as set-membership filtering (SMF) [2].

The need to know all the parameters of interfering users such as signature sequences, received amplitudes, etc. is alleviated by the use of adaptive schemes when codes with a short period are used in CDMA. However, since these adaptive methods require the use of training bits, blind adaptive multiuser detection, as proposed in [3], may be more appealing. A bottleneck arises in these adaptive detectors due to the tradeoff between achievable performance and computational complexity. Design of suitable low-complexity adaptive algorithms that converge to the optimal solution (for example, in the sense of MMSE) within a modest number of training signal sets is a critical problem in practical implementations of adaptive interference suppression for CDMA communications. It is shown in this work that by imposing an upper bound on the worst-case estimation error, we can develop adaptive algorithms that converge to the desired solution while achieving a significant saving in computation.

A similar bounded error-type performance criterion has also been studied in [1] and [7] for applications to adaptive equalization with

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training signals. The novel developments in this correspondence include the blind worst-case output constrained detector design methodology along with the low-complexity and fast converging blind adaptive algorithms.

The correspondence is organized as follows. Section II presents the bounded error performance criterion for linear interference suppression for CDMA systems. In this context, a blind design method is also presented. Closed form solutions for both blind and nonblind MUD are derived as are sufficient conditions for the existence of such a linear estimator. The blind formulation restricts the worst-case output magnitude to be bounded under a certain signal preserving constraint on the linear estimator. These are shown to be equivalent in some sense and are also shown to be near-far resistant [10]. Section III discusses adaptive solutions involving projection-based algorithms. Some convergence properties of the recursive estimates are also discussed. Section IV presents simulation results and discussion, and Section V concludes the paper.

II. Performance Criterion for Linear Multiuser Signal Estimation

For simplicity, we will consider a synchronous CDMA network of K users. It is pertinent to note that conceptually, a K-user asynchronous system can be viewed as an equivalent synchronous system with $(2K-1)$ users [8]. Throughout this paper, we assume that user 0 is the desired user to be demodulated. In a synchronous setup, *one-shot* detection is possible, that is, symbols are detected based on the received signal in one symbol period T. The received signal in the interval $[iT,\, iT+T)$ is given by

$$r(t) = A_0 b_0(i) s_0(t-iT) + \sum_{k=1}^{K-1} A_k b_k(i) s_k(t-iT) + n(t) \quad (1)$$

in which $A_k,\ b_k(i),\ s_k(t)$ are the received amplitude and ith symbol and signature sequence of the kth user, respectively. We further assume that *short codes* are used, although this assumption is needed only in Section III, where adaptive algorithms are discussed. This implies that the signature sequences are periodic with a period equal to the spreading gain N.

$$s_k(t) = \sum_{l=0}^{N-1} a_k(l)\psi(t-lT_c); \quad T = NT_c; \quad t \in [0,\, T)$$

and $s_k(t-iT) = s_k(t)$ for any t and i. The rectangular chip waveform $\psi(t)$ of duration T_c and $\{a_k\}$ is a binary-valued spreading code. The received signal is chip-matched filtered and sampled at the chip-rate ($1/T_c$), which yields a discrete-time model for the signal as

$$\boldsymbol{r}(i) = (\boldsymbol{AS})^T \boldsymbol{b}(i) + \boldsymbol{n}(i) \quad (2)$$

where $\boldsymbol{r}(i)$ is the $N \times 1$ vector of the sampled received values, $\boldsymbol{S}^T = [\boldsymbol{s}_0,\, \cdots,\, \boldsymbol{s}_{K-1}]$ is the $N \times K$ matrix of the spreading sequence, and $\boldsymbol{A} = \mathrm{diag}[A_0,\, A_1,\, \cdots,\, A_{K-1}]$ is the $K \times K$ matrix of received amplitudes.[1] In addition, $\boldsymbol{b}(i) = [b_0(i),\, \cdots,\, b_{K-1}(i)]^T$ is the $K \times 1$ vector of bits of the K users ($b_0(i)$ is the desired bit to be detected). In the nominal signal model, the $N \times 1$ vector $\boldsymbol{n}(i)$ is made up of samples from an independent and identically distributed (i.i.d.) zero-mean Gaussian random variable, each with variance σ^2.

The detection mechanism consists of a linear estimator $\boldsymbol{c}$ that linearly projects the received vector to the decision output, followed by a hard-limiter to produce the decision

$$z_0(i) = \langle \boldsymbol{c},\, \boldsymbol{r}(i)\rangle; \quad \hat{b}_0(i) = \mathrm{sgn}[z_0(i)] \quad \forall\, i \quad (3)$$

where $\hat{b}_0(i)$ is the decision on $b_0(i)$.

Common performance indices for the design of linear multiuser detectors include the zero-forcing (decorrelating) and the MMSE criteria. The decorrelating detector nulls out the contribution of the multiple access interference (MAI) in the output but results in noise enhancement. The MMSE detector achieves a balance between MAI and noise suppression by partially suppressing the MAI.

A. Constrained Output Error Formulation with Training

In this section, we propose a performance measure that ensures that the worst-case error achieved by the detector is bounded by a specified value. By doing so, we can be assured of good performance on a deterministic basis, i.e., of achieving bounded errors for a large subset of the possible received signal vectors. In some cases, we can also design the detector so that it results in error-free decisions for a subset of the received vectors. It can be shown that this detector is equivalent to the linear MMSE detector and the decorrelating detector, under some conditions.

The error term e for any detector $\boldsymbol{c}$ is expressed as

$$e \triangleq A_0 b_0 - \langle \boldsymbol{c},\, \boldsymbol{r}\rangle$$

where

$$\boldsymbol{r} = A_0 b_0 \boldsymbol{s}_0 + \sum_{k=1}^{K-1} A_k b_k \boldsymbol{s}_k + \boldsymbol{n}$$

and therefore

$$e = e(\boldsymbol{c},\, \boldsymbol{b},\, \boldsymbol{n}) = (1 - \langle \boldsymbol{c},\, \boldsymbol{s}_0\rangle)A_0 b_0 - \sum_{k=1}^{K-1} \langle \boldsymbol{c},\, \boldsymbol{s}_k\rangle A_k b_k - \langle \boldsymbol{c},\, \boldsymbol{n}\rangle$$

where we have noted explicitly the dependence of e on the estimator $\boldsymbol{c}$, the vector of transmitted bits $\boldsymbol{b}$, and the additive uncertainty vector $\boldsymbol{n}$. Our objective in this work is to design a linear estimator $\boldsymbol{c}_*$ such that the supremum of the error magnitude is bounded by a specified value $\gamma > 0$. Here, the worst case is taken with respect to certain sets of allowable noise realizations and transmitted bit vectors. Specifically, the bounded output error specification is imposed over a set of noise values that are bounded by some positive value, which is denoted by β. This noise set of interest $\mathcal{N}$ is defined as

$$\boldsymbol{N} = \{\boldsymbol{n}\colon \|\boldsymbol{n}\|_\infty \le \beta\}.$$

The transmitted bit vector is allowed to range in $\mathcal{B}^K$, where $\mathcal{B} = \{+1,\, -1\}$. With these definitions in place, we need to find any estimator $\boldsymbol{c}$ such that

$$\sup_{\boldsymbol{b}\in\mathcal{B}^K,\, \boldsymbol{n}\in\mathcal{N}} |e(\boldsymbol{c},\, \boldsymbol{b},\, \boldsymbol{n})| \le \gamma. \quad (4)$$

[1]Vectors are denoted by bold lowercase letters, e.g., $\boldsymbol{x}$, and matrices by bold uppercase letters, e.g., $\boldsymbol{W}$. $\|\boldsymbol{x}\|_1$, $\|\boldsymbol{x}\|_2$, and $\|\boldsymbol{x}\|_\infty$ denote the 1-, 2-, and infinity norms of $\boldsymbol{x} \in \mathbb{R}^N$. The inner product between two vectors $\boldsymbol{x},\, \boldsymbol{y} \in \mathbb{R}^N$ is denoted by $\langle \boldsymbol{x},\, \boldsymbol{y}\rangle \triangleq \boldsymbol{x}^T\boldsymbol{y}$.

In general, the solution to the problem posed is not unique, and there exists a set of estimators that can achieve the error specification if the bound is not too stringent. This set is called the *feasibility set* $\mathcal{F}$ [2]. The necessary and sufficient condition for any $\boldsymbol{c}$ to belong to $\mathcal{F}$ is that

$$|(1-\langle \boldsymbol{c},\, \boldsymbol{s}_0\rangle)|A_0 + \sum_{k=1}^{K-1} |\langle \boldsymbol{c},\, \boldsymbol{s}_k\rangle|A_k + \beta\|\boldsymbol{c}\|_1 \le \gamma \tag{5}$$

since the left-hand side (LHS) of the equation is the supremum of the error in (4) and is achieved by $\boldsymbol{b}_* = [b_{0,*}, \cdots, b_{K-1,*}]^T \in \mathcal{B}^K$ and $\boldsymbol{n}_* \in \mathcal{N}$, where

$$\begin{aligned} b_{0,*} &= \operatorname{sgn}(1-\langle \boldsymbol{c},\, \boldsymbol{s}_0\rangle)\\ b_{k,*} &= -\operatorname{sgn}(\langle \boldsymbol{c},\, \boldsymbol{s}_k\rangle), \quad \mathrm{k}=1,\,2,\,\cdots \mathrm{K}-1\\ \boldsymbol{n}_* &= -\beta[1,\,1,\,\cdots,\,1]^T. \end{aligned}$$

Clearly, $-\boldsymbol{b}_*$ and $-\boldsymbol{n}_*$ also achieve the supremum of the error. If (5) does not hold for any $\boldsymbol{c}$, then $\mathcal{F}$ is empty, and there exists no estimator that meets the error specification.

In the following, a simple and verifiable sufficient condition for the existence of a nonempty feasibility set as well as a closed-form expression for a member of $\mathcal{F}$ is given. Notice that the LHS in (5) is a one-norm of the "augmented" vector $[A_0(1-\langle \boldsymbol{c},\, \boldsymbol{s}_0\rangle),\, A_1\langle \boldsymbol{c},\, \boldsymbol{s}_1\rangle,\, \cdots,\, A_K\langle \boldsymbol{c},\, \boldsymbol{s}_K\rangle,\, \boldsymbol{c}^T]^T$. In addition, for any $\boldsymbol{x} \in \mathbb{R}^N$, there exists a positive constant α such that $\|\boldsymbol{x}\|_1^2 \le \alpha\|\boldsymbol{x}\|_2^2$. Thus, we have that

$$\begin{aligned} &\left(|(1-\langle \boldsymbol{c},\, \boldsymbol{s}_0\rangle)|A_0 + \sum_{k=1}^{K-1} |\langle \boldsymbol{c},\, \boldsymbol{s}_k\rangle|A_k + \beta\|\boldsymbol{c}\|_1\right)^2\\ &\quad\le \alpha\left(A_0^2(1-\langle \boldsymbol{c},\, \boldsymbol{s}_0\rangle)^2 + \sum_{k=1}^{K-1} A_k^2\langle \boldsymbol{c},\, \boldsymbol{s}_k\rangle^2 + \beta^2\|\boldsymbol{c}\|_2^2\right). \end{aligned}$$

We can find the tightest upper bound by minimizing the right-hand side (RHS) over all $\boldsymbol{c}$. This yields a closed-form expression for a possible candidate $\boldsymbol{c}_* \in \mathcal{F}$ as

$$\begin{aligned} \boldsymbol{c}_* &= A_0^2\boldsymbol{R}^{-1}\boldsymbol{s}_0\\ \boldsymbol{R} &= \left(\boldsymbol{S}^T\boldsymbol{A}^2\boldsymbol{S} + \beta^2\boldsymbol{I}\right). \end{aligned} \tag{6}$$

The sufficient condition is obtained by verifying that $\boldsymbol{c}_*$ satisfies (5). Alternately, using (5) with $\boldsymbol{c} = \boldsymbol{c}_*$, we could choose the noise set $\mathcal{N}$, which is equivalent to choosing β such that at least one estimator would meet the error specification γ. It is interesting to note the following.

- The structure of the proposed detector is very similar to that of the MMSE detector. In fact, if (5) is met for $\boldsymbol{c}_*$ with $\beta = \sigma$, then $\boldsymbol{c}_*$ is the MMSE detector.
- In the noise-free case, as β approaches 0, $\boldsymbol{c}_*$ approaches the decorrelating detector. Since the decorrelating detector is near-far resistant, it follows that $\boldsymbol{c}_*$ is near-far resistant.
- By setting $\gamma \le 1$, we impose the constraint that the detector make no error whenever the noise vector belongs to $\mathcal{N}$.

B. Equivalent Formulation without Training

The MMSE counterpart for blind (anchored) detection is the constrained minimum output energy (MOE) criterion [3]. The advantage of the MOE formulation is that it does not require knowledge of the correlation between the received vector and the desired bit. It achieves this by exploiting knowledge of the space spanned by the desired signal. Thus, any adaptive implementation of the MOE detector does not require training. The objective of this section is to formulate a blind estimation problem whose solution is equivalent to that of the nonblind worst-case error estimation formulation.

To this end, it is instructive to note the following: For any anchored estimator $\boldsymbol{c}$ that has a unit projection along the desired signal direction, i.e., $\langle \boldsymbol{c},\, \boldsymbol{s}_0\rangle = 1$, we have

$$\sup_{\boldsymbol{b}\in\mathcal{B}^K,\boldsymbol{n}\in\mathcal{N}} |e(\boldsymbol{c},\, \boldsymbol{b},\, \boldsymbol{n})| = \sum_{k=1}^{K-1} |\langle \boldsymbol{c},\, \boldsymbol{s}_k\rangle|A_k + \beta\|\boldsymbol{c}\|_1.$$

In addition, for any $\boldsymbol{c} \in \mathcal{F}$

$$\sup_{\boldsymbol{b}\in\mathcal{B}^K,\boldsymbol{n}\in\mathcal{N}} |z| = A_0 + \sum_{k=1}^{K-1} |\langle \boldsymbol{c},\, \boldsymbol{s}_k\rangle|A_k + \beta\|\boldsymbol{c}\|_1 \le A_0 + \gamma \tag{7}$$

where $z = \langle \boldsymbol{c},\, \boldsymbol{r}\rangle$. Therefore, instead of imposing a bound γ on the worst-case estimation error magnitude, we can design an estimate that bounds the maximum output magnitude by $A_0+\gamma$ under the constraint $\langle \boldsymbol{c},\, \boldsymbol{s}_0\rangle = 1$. This formulation imposes a condition on the estimator output and, thus, would not need training in an adaptive implementation.

As before, by minimizing an upper bound on the LHS in (7) subject to the constraint $\langle \boldsymbol{c},\, \boldsymbol{s}_0\rangle = 1$, a sufficient condition for the existence of the feasibility set is given by

$$\sum_{k=1}^{K-1} |\langle \boldsymbol{c}_*,\, \boldsymbol{s}_k\rangle|A_k + \beta\|\boldsymbol{c}_*\|_1 \le \gamma \tag{8}$$

where

$$\boldsymbol{c}_* = \frac{1}{\boldsymbol{s}_0^T\boldsymbol{R}^{-1}\boldsymbol{s}_0}\boldsymbol{R}^{-1}\boldsymbol{s}_0$$

and $\boldsymbol{R}$ is defined as in (7). The detector is identical to the nonblind multiuser detector in Section II-A since they differ by only a scaling factor. This is analogous to the equivalence of the MMSE to the MOE detectors [3]. Thus, we have a blind formulation that gives the same performance as the nonblind worst-case bounded-error methodology.

III. Adaptive Algorithms

The previous section outlined possible off-line solutions for the problem posed. These require knowledge of the interference parameters, such as amplitude, timing, etc., which implies undesirable implementation overhead. In the following, adaptive algorithms are proposed for adaptation with and without training. These algorithms use a constrained least-squares-like cost function and are derived using ideas of projections onto convex sets. In each of the following cases, we will derive algorithms with automatic and variable gain assignment and sparse updating.

A. Case 1: Adaptation with Training Signals

Consider the case in which training bits are initially used for convergence of the detector, after which, the updating is performed in decision-directed mode. For the ith symbol, we receive the vector $\boldsymbol{r}(i)$, and the desired output is the bit that was transmitted: $b_0(i)$. To develop a recursive algorithm, assume that $\boldsymbol{c}(i-1)$ is the detector at time $i-1$, which was obtained using the values $[\boldsymbol{r}(k),\ b_0(k)]_{k=1}^{i-1}$. This method is similar to that proposed by the authors for a general SMF problem [7], namely, to construct a quadratic cost function at time i, which is given by

$$F(\boldsymbol{c}) = [\boldsymbol{c} - \boldsymbol{c}(i-1)]^T \boldsymbol{W}^{-1}(i-1)[\boldsymbol{c} - \boldsymbol{c}(i-1)] \tag{9}$$

where $\boldsymbol{W}(i-1)$ is a positive-definite weighting matrix. The estimate at the present time is obtained by minimizing this cost function subject to the constraint that the error not exceed γ

$$|b_0(i) - \langle \boldsymbol{c},\ \boldsymbol{r}(i)\rangle| \le \gamma. \tag{10}$$

This constraint describes a pair of N-dimensional hyperplanes, the interior of which is denoted by $\mathcal{S}(i) = \{\boldsymbol{c}\colon |b_0(i) - \langle \boldsymbol{c},\ \boldsymbol{r}(i)\rangle| \le \gamma\}$—a convex set in $\mathbb{R}^N$. If $\boldsymbol{c}(i-1)$ belongs to $\mathcal{S}(i)$, then there is no need to update the estimate at time i. Otherwise, the new estimate lies on the bounding hyperplane nearest $\boldsymbol{c}(i-1)$, resulting in an *a posteriori* error magnitude of exactly γ [7]. The condition to be checked is whether $\boldsymbol{c}(i-1) \in \mathcal{S}(i)$, which is equivalent to checking whether $|\delta(i)| \le \gamma$, where $\delta(i)$ is the *a priori* error $\delta(i) = b_0(i) - \langle \boldsymbol{c}(i-1),\ \boldsymbol{r}(i)\rangle$. The updating equations for an arbitrary $W(i-1) > \mathbf{0}$ are

$$\boldsymbol{c}(i) = \begin{cases} \boldsymbol{c}(i-1), & |\delta(i)| \le \gamma \\ \boldsymbol{c}(i-1) - \frac{\lambda(i)\delta(i)\boldsymbol{W}(i-1)\boldsymbol{r}(i)}{(1+\lambda(i)G(i))}, & |\delta(i)| > \gamma \end{cases} \tag{11}$$

where $G(i) = \|\boldsymbol{r}(i)\|_{\boldsymbol{w}}^2$, and $\lambda(i) = 1/G(i)((|\delta(i)|/\gamma) - 1)$ is the time-varying gain.[2] The term $\lambda(i)$ is also the Lagrangian multiplier in the solution of the problem. Therefore, at each instant, the algorithm chooses to update the estimate if and only if the *a priori* error is less than γ in magnitude. If not, then the data at that instant is discarded, and no update is performed. After a sufficient number of training bits, we switch to a decision-directed mode of operation.

In general, the computational complexity of the algorithm above is of the order N^2 per sample. However, in practice, it is computationally less intensive due to selective updating. By setting $\boldsymbol{W}(i-1)$ equal to the inverse of the weighted correlation matrix of the received vectors, we obtain an RLS-type algorithm that is termed *a posteriori* error constrained recursive least squares (APEC-RLS). In this case, at time i, we have an additional update of the matrix $W(i-1)$ given by

$$\boldsymbol{W}(i) = W(i-1) - \frac{\lambda(i)\boldsymbol{W}(i-1)\boldsymbol{r}(i)\boldsymbol{r}^T(i)\boldsymbol{W}(i-1)}{(1+\lambda(i)G(i))}. \tag{12}$$

On the other hand, by setting $\boldsymbol{W}(i-1) = \boldsymbol{I}_N$ for all i, where $\boldsymbol{I}_N$ is the identity matrix of size N, we obtain an order N algorithm with selective updating, which is termed APEC-LMS, by analogy to the least mean squares (LMS) algorithm [2]. These algorithms belong to a general class of adaptive set-membership algorithms referred to as set-membership adaptive recursive techniques (SMART). At each time, the estimate in APEC-LMS is obtained such that the vector $\boldsymbol{c}(i) - \boldsymbol{c}(i-1)$ is orthogonal to the nearest bounding hyperplane [2]. This happens because the cost function (9) reduces to the Euclidean norm of the detector error. An important property of this algorithm is that, at each time, the estimate moves closer (in the two-norm) to every point in the feasibility set. Hence, the sequence of errors is monotone non-increasing. It can also be shown that asymptotically, the value of the prediction error becomes less than or equal to γ in magnitude and that the algorithms cease updating [2], [7].

For adaptation with training bits, the algorithms outlined above can be used to design detectors that guarantee the asymptotic worst-case error to be bounded from above by the specified γ. Recursive algorithms for blind adaptation are considered next.

B. Case 2: Blind Adaptation

Ideas similar to those used for deriving the algorithms above are employed in the development of blind recursive algorithms. For MOE-based blind adaptive MUD (i.e., algorithms that do not need training sequences), an LMS algorithm is proposed in [3], and a blind RLS algorithm has been proposed in [8] to suppress both MAI and narrowband interference. Blind adaptive MUD using subspace tracking algorithms is proposed in [11]. A blind MUD receiver in conjunction with beam forming via an antenna array is explored in [4]. This receiver uses a similar criterion for adaptive blind MUD to the one used here, although a different algorithm is proposed. The work in [4] is based on decomposing the detector as a sum of the desired user's signature sequence and a vector that spans the space orthogonal to the desired signal space. An optimal bounding ellipsoid algorithm is then used to adapt the coefficients of the estimator. Here, the algorithms are derived as blind adaptive solutions to the problem posed in Section II-B, which result in different updating strategies in contrast with the algorithm in [4].

For blind adaptation, two constraints must be met at each instant. First, the output magnitude must be less than $A_0 + \gamma$. The other is that the component of the detector in the direction of the desired signature sequence be unity. As before, the update should be such that the cost function $F(\boldsymbol{c})$, which is defined in (9), be minimized subject to the following two constraints:

$$|\langle \boldsymbol{c},\ \boldsymbol{r}(i)\rangle| \le A_0 + \gamma \tag{13}$$

and

$$\langle \boldsymbol{c},\ \boldsymbol{s}_0\rangle = 1. \tag{14}$$

We denote by $\mathcal{S}(i)$ the set of all detectors that satisfy (13). Assume that $\boldsymbol{c}(i-1)$ satisfies (14); then, as before, if $|\delta(i)| = |\langle \boldsymbol{c}(i-1),\ \boldsymbol{r}(i)\rangle| \le A_0 + \gamma$, there is no need to update the estimate. However, if such is not the case, then the update should be such that $\boldsymbol{c}(i) \in \mathcal{S}(i) \cap \{\boldsymbol{c}\colon \langle \boldsymbol{c},\ \boldsymbol{s}_0\rangle = 1\}$. The solution for $\boldsymbol{c}(i)$ is obtained by the method of Lagrange multipliers and is given by

$$\boldsymbol{c}(i) = \begin{cases} \boldsymbol{c}(i-1) \\ \quad |\delta(i)| \le A_0 + \gamma \\ \boldsymbol{c}(i-1) - \kappa(i)\delta(i)\boldsymbol{W}(i-1)\boldsymbol{r}^{\perp}(i) \\ \quad |\delta(i)| > A_0 + \gamma \end{cases} \tag{15}$$

where

$$\boldsymbol{r}^{\perp}(i) = \boldsymbol{r}(i) - \frac{\langle \boldsymbol{r}(i), \boldsymbol{s}_0\rangle_{\boldsymbol{W}}}{\|\boldsymbol{s}_0\|_{\boldsymbol{w}}^2}\boldsymbol{s}_0 \tag{16}$$

$$\kappa(i) = \frac{\left(1 - \dfrac{A_0+\gamma}{|\delta(i)|}\right)\|\boldsymbol{s}_0\|_{\boldsymbol{w}}^2}{\|\boldsymbol{r}(i)\|_{\boldsymbol{w}}^2\|\boldsymbol{s}_0\|_{\boldsymbol{w}}^2 - \langle \boldsymbol{r}(i), \boldsymbol{s}_0\rangle_{\boldsymbol{w}}^2}. \tag{17}$$

Here, $\boldsymbol{r}^{\perp}(i)$ is the component of $\boldsymbol{r}(i)$ that is orthogonal to the desired signature $\boldsymbol{s}_0$ in the sense that $\langle \boldsymbol{s}_0,\ \boldsymbol{r}^{\perp}(i)\rangle_{\boldsymbol{W}} = 0$. As in the case of adaptation with training bits, the blind algorithm above updates at the ith time instant if and only if the magnitude of $\langle \boldsymbol{c}(i-1),\ \boldsymbol{r}(i)\rangle$ is greater

[2]The inner product between $\boldsymbol{x},\ \boldsymbol{y} \in \mathbb{R}^N$ with respect to a positive definite matrix $\boldsymbol{W} \in \mathbb{R}^{N\times N}$ is defined as $\langle \boldsymbol{x},\ \boldsymbol{y}\rangle_{\boldsymbol{w}} \triangleq \boldsymbol{x}^T\boldsymbol{W}\boldsymbol{y}$, and the two-norm induced by this inner product is $\|\boldsymbol{x}\|_{\boldsymbol{w}} \triangleq \sqrt{\boldsymbol{x}^T\boldsymbol{W}\boldsymbol{x}}$

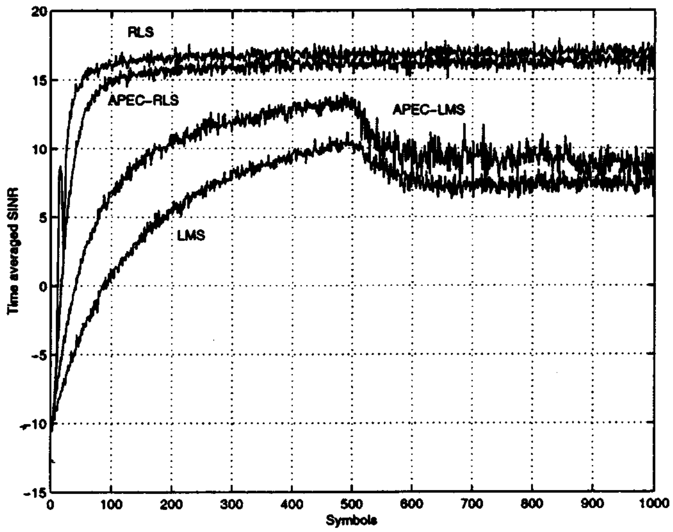

Fig. 1. Performance with training signals.

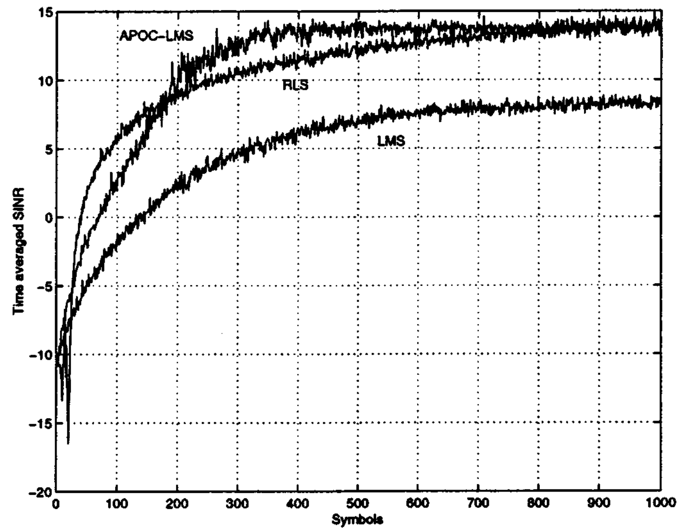

Fig. 2. Performance without training signals.

than $A_0 + \gamma$. By setting $\boldsymbol{W}(i-1) = \boldsymbol{I}_N$, we obtain an order-$N$ algorithm called *a posteriori* output constrained least mean squares (APOC-LMS).

IV. Simulation Studies and Discussion

A ten-user synchronous CDMA system with a spreading gain of 20 was simulated. The interference amplitudes were all chosen to be five times the desired user's amplitude to simulate a severe near-far situation. The spreading sequence was generated randomly for all the users and was varied from trial to trial for all the interferers, keeping the signature sequence of the desired user invariant through the trials. This approximately models the effect of arbitrary delays in an asynchronous system. The signal-to-background noise ratio was 20 dB after despreading.

The time-averaged signal-to-interference-plus-noise ratio (SINR) was used as a performance measure of the adaptive algorithms. The off-line detector of (7) resulted in a SINR of 18 dB, which is the same as that achieved by the MMSE detector. On the other hand, a matched filter resulted in an unacceptable −8-dB SINR. The plots of the performance of the proposed algorithms for adapting with and without training are shown in Figs. 1 and 2, respectively. The results are averaged over $M = 500$ trials. The time-averaged SINR at time i is given by

$$\text{SINR}(i) = \frac{\sum_{k=1}^{M} \left(\boldsymbol{c}_k(i-1)^T \boldsymbol{s}_0\right)^2}{\sum_{k=1}^{M} [\boldsymbol{c}_k(i-1)^T(\boldsymbol{r}_k(i) - b_{0,k}(i)\boldsymbol{s}_0)]^2}$$

where the subscript k indicates the kth trial.

In Fig. 1, training bits were used for the first 500 symbols, after which, adaptation switched to a decision-directed mode. The value of γ was set to 0.4. The APEC-RLS updated in 110 symbols out of the total of 1000, whereas the APEC-LMS updated 370 times. It can be seen that the APEC-LMS and the standard LMS (with step size = 0.001) have not achieved steady state in 500 symbols, resulting in a degradation in performance in the decision-directed mode. On the other hand, the APEC-RLS performs comparably with the traditional RLS (with forgetting factor of 0.995) while using only 10% of the symbols for updating.

In Fig. 2, it can be seen that the APOC-LMS does as well as the forgetting factor RLS of [8] (with a forgetting factor = 0.99) while updating for 460 symbols out of 1000 and outperforms the traditional LMS algorithm. The value of γ here was 0.1. A higher γ would result in reduced updating, but there would be some loss in performance and vice versa. The designer can adjust the value of γ to trade off between the resources and the performance.

The performance of the training-based adaptive algorithms over a flat-fading channel with a coherence time of 40 symbols is shown in Fig. 3. This plots and compares the output interference power arising from the APEC-RLS, APEC-LMS, RLS, and normalized-LMS (NLMS) algorithms. As before, the results are averaged over $M = 500$ trials. The mean power of the interfering users was five times that of the desired user. APEC-RLS updated 10% of the time, whereas APEC-LMS used 30% of the data for updating. The performance of APEC-RLS ($\gamma = 0.3$) compares favorably with that of the RLS algorithm (with a forgetting factor of 0.98), even though it is computationally far less complex. It was found that the standard LMS algorithm without normalization was not effective in suppressing the interference. On the other hand, NLMS performed better, although not as well as the APEC-LMS algorithm. It should be noted that the APEC-LMS algorithm has a computational complexity less than that of the traditional LMS-type algorithms. Further, although both the LMS and APEC-LMS algorithms did not perform as well in the decision-directed mode (i.e., after 500 iterations), APEC-LMS is seen to be able to recover from the initial errors and to converge to the mean-squared error performance of the RLS-type algorithms. As expected, the convergence speed of the RLS and APEC-RLS algorithms was far superior to that of their LMS counterparts.

It has been shown in [1] that the sparse updating can be advantageous when multiple filters need to be adapted. Since such is the case here, where we have one detector for each user, an updator-shared scheme [1] is possible by which the number of updators needed is drastically reduced. This leads to significant savings in the hardware requirements in a practical system. For example, in a general filtering set up, it has been shown that only 17 updators are required to estimate the coefficients of a bank of 100 filters without compromising the performance [1].

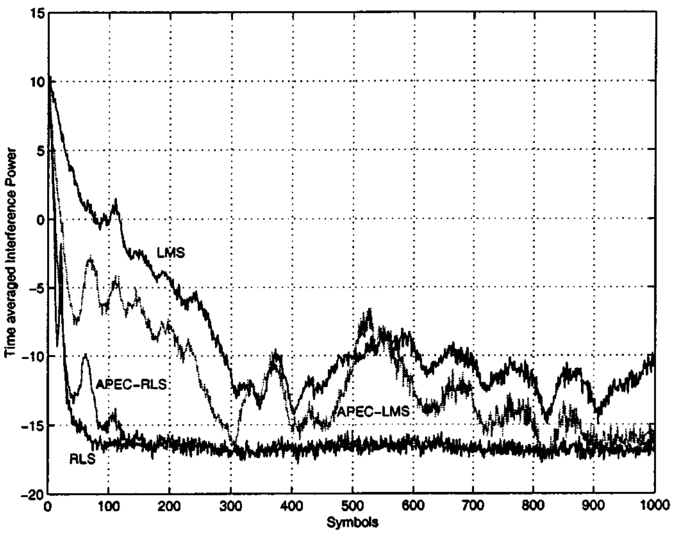

Fig. 3. Performance with training over fading channels.

V. CONCLUSION

This correspondence presented a new bounded estimation-error design criterion along with adaptive algorithms for linear multiuser detection in CDMA systems that ensure bounded estimation errors. Near-far resistant solutions to this problem for systems that require training and otherwise were presented. Equivalence to the MMSE and decorrelating detector were shown in special cases. Adaptive algorithms were presented that feature a selective-updating capability. It was shown via simulations that these algorithms can serve as a means to significantly reduce the computational load while rendering excellent performance.

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On the Stability of a Tank and Hopfield Type Neural Network in the General Case of Complex Eigenvalues

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***Abstract*—The stability of a Tank and Hopfield-type neural network is investigated for the general case of practically encountered complex eigenvalues s_D of the matrix product $D_g^T D_f$, where D_g and D_f are approximations of the connection matrix D on the signal and constraint sides of the neural net, respectively. A stability criterion in the form of an analytic expression is derived, thus generalizing the results obtained by Yan for the special case of purely real eigenvalues.**

***Index Terms*—Connection matrix, eigenvalues, stability, Tank and Hopfield neural net.**

I. INTRODUCTION

Culhane *et al.* presented an electric circuit for computing the discrete Hartley transform (DHT) and discrete Fourier transform (DFT) [1]. This circuit, shown in Fig. 1 is a modified Tank and Hopfield linear programming neural network [2] and has the nice feature of computing the DHT and DFT within RC time constants of the order of nanoseconds. Several variants of this circuit model have been used to solve linear and nonlinear programming problems [3]–[7]. With some modifications, the circuit can be used to solve linear least squares error (LSE) problems [8]. The neural net of Fig. 1 has signal and constraint sides denoted by the subscripts g and f, respectively. D_g and D_f are $M \times N$ interconnection conductance matrices, $\tau_g = R_g C_g$ and $\tau_f = R_f C_f$ are the relaxation time constants; u_g, v_g, and α_g are, respectively, the input, output, and gain of an operational amplifier on the signal side; u_f, v_f, and α_f are their counterparts on the constraint side. Culhane *et al.* [1] proved the stability of the circuit under the two assumptions that $\tau_f \ll \tau_g$ and $D_g^T D_f$ is positive definite. Yan [8] showed that the first assumption is not necessary and that the second assumption can be relaxed to

$$s_D > \frac{-1}{(\alpha_g R_g)(\alpha_f R_f)} \tag{1}$$

where s_D is an eigenvalue of $D_g^T D_f$. However, the criterion was derived under the assumption that all the eigenvalues s_D are purely real. The main objective of this correspondence is to generalize the stability condition (1) to the case of complex eigenvalues s_D of the matrix product $D_g^T D_f$, which will be done in Section II. In Section III, the results of Yan for the computational speed of the neural net will be correspondingly generalized. In Section IV, an illustrative example will be presented, where the matrices D_g and D_f are approximations to the matrix D of the discrete Hartley transform in the nonideal case and where the eigenvalues s_D of $D_g^T D_f$ will turn out to be complex, thus demonstrating the importance of treating the general case of complex eigenvalues presented in this correspondence.

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