Math 20580: Information about exam 1
Fall 2010

Disclaimer. The following review sheet represents a sincere attempt to help you prepare for the first exam. It is not in any sense a practice exam. Nor is it necessarily complete. There might be things on the exam that are not mentioned directly here, problems that look different than the review problems suggested here, etc. Nonetheless, we think it's better than nothing...

Practical details concerning the first exam. The exam runs from 8-9:15 Tuesday, 9/21 in 101 Jordan. There will be a review session Monday 9/20 from 7-8 PM in Hayes-Healy 127. Tutorials will take place as usual on 9/22, but there will be no quiz that day. Homework, including the group assignment will be collected on 9/23.

The exam will cover Chapters 1 and 2 (only the sections covered in homework) from Lay's book. *No calculators, books, or notes will be allowed.* Some sharp pencils, a good eraser, and a good night's sleep will be all you need. There will be both multiple choice and partial credic problems on the exan. No partial credit will be given on multiple choice problems.

Things to know

Terminology. As with all areas of mathematics, and perhaps even more so than many, it's important to be comfortable with the *language* of linear algebra. Following are some words you should understand thoroughly for the first exam.

- **regarding linear systems:** System of linear equations, augmented matrix for a linear system, (in)consistent system, elementary row operations, row reduction, row equivalence, row-echelon form, reduced echelon form, pivot, (in)homogeneous system, parametric form for the solution of a linear system.
- **regarding vectors:** (column) vectors, $\mathbf{R}^n$, vector addition, scalar multiplication, linear combination, span, linear (in)dependence, standard basis for $\mathbf{R}^n$, subspace, basis and dimension of a subspace.
- **regarding matrices:** $m \times n$ matrix, ith row/jth column/ij-entry of a matrix, invertible matrix, column and null spaces of a matrix, rank. linear transformation, domain, codomain, image, range, one-to-one, onto.

Computational skills. We have learned essentially one computational technique for solving problems in this class: row reduction of a matrix. Hence the difficulty is generally not in deciding *whether* to use row reduction but rather *how* to use it. Among the problems that can be solved via row reduction are: solving a linear system or determining how many solutions exist, determining whether a vector lies in the span of some set of other vectors, determining whether a set of vectors is linearly independent, determining whether a matrix is invertible, finding the inverse of a matrix, determining whether a given linear transformation is one-to-one or onto, finding bases for the column and null spaces of a matrix, and determining the rank of a matrix.

There are a couple of exceptions to the above—i.e. computations that do not necessarily require row reduction. Among these are matrix multiplication, determining the matrix for a linear transformation.

Different ways of presenting a linear system. Assuming you've got a firm grasp of the things above, another thing crucial to understanding what's going on this class is to remember that there are several equally valid ways to express a linear system of equations. To illustrate this, we let A be an $m \times n$ matrix with jth column $\mathbf{a}_j \in \mathbf{R}^m$ and ij entry $a_{ij} \in \mathbf{R}$, and we let $\mathbf{x} \in \mathbf{R}^n$, $\mathbf{b} \in \mathbf{R}^m$ be vectors with ith entries x_i, b_i respectively. Let $T : \mathbf{R}^n \to \mathbf{R}^m$ be the linear transformation $T(\mathbf{x}) = A\mathbf{x}$. Then the following equations all say exactly the same thing.

m equations in n unknowns.:

$$\begin{array}{cccc} a_{11}x_1 + a_{12}x_2 + & \dots & a_{1n}x_n & = & b_1. \\ a_{21}x_2 + a_{22}x_2 + & \dots & a_{2n}x_n & = & b_2. \\ \vdots & & & & \vdots \\ a_{m1}x_2 + a_{m2}x_2 + & \dots & a_{mn}x_n & = & b_m. \end{array}$$

vector equation:

$$x_1\mathbf{a_1} + x_2\mathbf{a_2} + \dots + x_n\mathbf{a_n} = \mathbf{b}.$$

matrix/vector equation:

$$A\mathbf{x} = \mathbf{b}.$$

linear transformation equation:

$$T(\mathbf{x}) = \mathbf{b}.$$

Computations involving any of these equations will likely involve row reduction of the augmented matrix

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} & b_1 \\ a_{21} & a_{22} & \dots & a_{2n} & b_2 \\ \dots & & & & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} & b_m \end{pmatrix}$$

For instance, the questions "is there a vector $\mathbf{x}$ such that $T(\mathbf{x}) = \mathbf{b}$" (i.e. is $\mathbf{b}$ in the range of T)? and "does $\mathbf{b}$ belong to $\text{span}\{\mathbf{a}_1, \dots, \mathbf{a}_n\}$?" are the same as determining whether the linear system above is consistent. So-called 'theorems' in the book (such as the neverending *invertible matrix theorem* that begins on page 129 and is continued in various places later in the book) that consist of long lists of equivalent statements are mostly just straightforward consequences of thinking about a linear system from as many of the above points of view as possible.