Modeling with 1st order ODEs: an example with Newton's cooling law

I'm writing this note in order to finish an example I started in class on April 8. The problem was as follows:

A meteor falls some distance in front of you. It takes you an hour to reach the crater, at which point you measure the temperature of the meteor to be 150 degrees celsius. It's a fine spring evening with an outdoor temperature of 10 degrees celsius. Wondering what the temperature of the meteor was on impact, you recall Newton's law of cooling: *the rate at which the temperature of an object changes is proportional to the difference between its temperature and that of its surroundings.* So you take the meteor (miraculously intact and mercifully small enough to carry) back to your basement lab and determine after some experimentation that the thing cools in such a way that the difference between its temperature and that of its surroundings is cut in half every 23 minutes. So just what was the meteor's impact temperature?

Solution. Let $y(t)$ be the temperature of the meteor at time t and y_{amb} be the (fixed) temperature of the meteor's surroundings. Then Newton's cooling law can be written:

$$y' = -k(y - y_{amb})$$

where $k > 0$ is the (as yet unknown) proportionality constant. We know $y(60) = 150$ and $y_{amb} = 10$, and we seek to find $y(0)$. Our experiments also tell us that for any t, we have

$$y(t + 23) - y_{amb} = \frac{1}{2}(y(t) - y_a mb).$$

Now solving the above differential equation gives the formula:

$$y(t) - y_{amb} = Ae^{-kt}$$

for some constant A. Combining the solution of the differential equation with our experimental result, we find that

$$\frac{1}{2} = \frac{y(t + 23) - y_{amb}}{y(t) - y_{amb}} = \frac{Ae^{-k(t+23)}}{e^{-kt}} = e^{-23k}.$$

Hence $e^{-23k} = 1/2$, which implies that $k = \frac{\ln 2}{23}$. And using our initial condition and value for y_{amb}, we find that

$$140 = y(60) - y_{amb} = Ae^{-60k}.$$

Thus $A = 140e^{60k} = 140e^{(60 \ln 2)/23} \approx 854$, and therefore the impact temperature of the meteor is

$$y(0) = y_{amb} + A = 864 \text{ degrees celsius.}$$