10350 Algebra Quiz

1a. Express the following as a single fraction in its simplest form.

$$\frac{4}{2x-1}-\frac{3}{x+2}\stackrel{?}{=} \frac{4(x+2)-3(2x-1)}{(2x-1)(x+2)} = \frac{4x+8-6x+3}{2x^2-x+4x-2}$$

$$= \frac{-2x+11}{2x^2+3x-2} \quad \text{or} \quad \frac{-2x+11}{(2x-1)(x+2)}$$

1b. If $f(x)=2x^2+1$ simplify the following expressions assuming that $x \neq 3$ and $h \neq 0$.

$$\frac{f(x)-f(3)}{x-3}\stackrel{?}{=} \frac{2x^2+1-(2(9)+1)}{x-3} = \frac{2x^2+1-19}{x-3}$$

$$= \frac{2x^2-18}{x-3} = \frac{2(x^2-9)}{x-3}$$

$$= \frac{2\cancel{(x-3)}(x+3)}{\cancel{(x-3)}} = 2(x+3).$$

$$\frac{f(h+1)-f(1)}{h}\stackrel{?}{=} \frac{2(h+1)^2+1-(2+1)}{h} = \frac{2(h+1)^2+1-3}{h}$$

$$= \frac{2(h^2+2h+1)-2}{h} = \frac{2(h^2+2h)+2-2}{h}$$

$$= \frac{2h^2+4h}{h} = \frac{\cancel{h}(2h+4)}{\cancel{h}} = 2h+4$$

2. If $f(x) = \dfrac{x-2}{2x+3}$ evaluate $f\left(\dfrac{1}{3}\right)$.

$$f\left(\frac{1}{3}\right) \stackrel{?}{=} \frac{\frac{1}{3} - 2}{\frac{2}{3} + 3} = \frac{\frac{1 - 6}{3}}{\frac{2 + 9}{3}} = \frac{\frac{-5}{3}}{\frac{11}{3}}$$

$$= -\frac{5}{3} \times \frac{3}{11} = -5/11$$

3. Let $g(n) = \dfrac{2^{2n}\sqrt{x^{n+1}}}{3^{n+2}}$. Find the expression $\dfrac{g(n+2)}{g(n+1)}$.

You should collect all like terms. The final answer should have no radicals and no negative exponents.

$$\frac{g(n+2)}{g(n+1)} \stackrel{?}{=} g(n+2) \div g(n+1) = \frac{2^{2(n+2)}\sqrt{x^{n+3}}}{3^{n+4}} \div \frac{2^{2(n+1)}\sqrt{x^{n+2}}}{3^{n+3}}$$

$$= \frac{2^{2n+4}\sqrt{x^{n+3}}}{3^{n+4}} \times \frac{3^{n+3}}{2^{2n+2}\sqrt{x^{n+2}}}$$

$$= \frac{2^{2n+4-2n-2}}{3^{n+4-n-3}} \cdot \sqrt{\frac{x^{n+3}}{x^{n+2}}} = \frac{2^2}{3}\sqrt{x^{n+3-n-2}}$$

$$= \frac{4}{3}\sqrt{x} = \frac{4}{3}x^{1/2}$$

4. Write the following expression as a single logarithmic expression.

$3 \ln x - \ln(2x) + \ln(4) \stackrel{?}{=} \ln x^3 - \ln(2x) + \ln(4)$

$= \ln\left(\frac{x^3 \cdot 4}{2x}\right) = \ln(2x^2)$

5. If $\ln 2 = a$ and $\ln 5 = b$ write the following expressions in terms of a and b.

5a. $\ln(50) \stackrel{?}{=} \ln(2 \times 25) = \ln(2 \times 5^2)$

$= \ln 2 + \ln 5^2 = \ln 2 + 2 \ln 5$

$= a + 2b$

5b. $\ln\sqrt{\frac{5}{2}} \stackrel{?}{=} \ln\left(\frac{5}{2}\right)^{1/2} = \frac{1}{2}\ln\left(\frac{5}{2}\right)$

$= \frac{1}{2}\left[\ln 5 - \ln 2\right] = \frac{1}{2}\left[b - a\right]$

6. Rationalize $\frac{\sqrt{2}+2}{\sqrt{2}-1}$ and write in the form $a+b\sqrt{2}$ where a and b are numbers.

$$\frac{\sqrt{2}+2}{\sqrt{2}-1} \stackrel{?}{=} \qquad \frac{\sqrt{2}+2}{\sqrt{2}-1} \cdot \frac{\sqrt{2}+1}{\sqrt{2}+1}$$

(use conjugate of $\sqrt{2}-1$)

$$= \frac{(\sqrt{2}+2)(\sqrt{2}+1)}{(\sqrt{2})^2 - 1^2} =$$

$$= \frac{\sqrt{2}\cdot\sqrt{2} + 2\sqrt{2} + \sqrt{2} + 2}{2-1}$$

$$= \frac{2 + 3\sqrt{2} + 2}{1} = 4 + 3\sqrt{2}$$