Generalized Ordered Logit Models

Midwest Sociological Meetings, Chicago
April 2, 2010
Richard Williams, Notre Dame Sociology, rwilliam@ND.Edu
gologit2 support page: http://www.nd.edu/~rwilliam/gologit2

Abstract

When dependent variables are ordinal rather than continuous, conventional OLS regression techniques are inappropriate. The ordered logit model, also known as the proportional odds model, is a popular method in such cases. However, in many instances, generalized ordered logit (gologit) models may be a superior alternative. Gologit models can be less restrictive than proportional odds models, whose assumptions are often violated, and more parsimonious than methods like multinomial logit that ignore the ordering of categories altogether. At the same time, the gologit model offers challenges of its own with regards to proper usage and interpretation. In this paper, we discuss the rationale behind the gologit model and show how it can be estimated using the `gologit2` routine in Stata. We also discuss potential problems that can occur with the model and review several different possible interpretations of parameters that are possible.

The Ordered Logit / Proportional Odds Model

Long and Freese (2006) present data from the 1977/1989 General Social Survey. Respondents are asked to evaluate the following statement: "A working mother can establish just as warm and secure a relationship with her child as a mother who does not work." Responses were coded as 1 = Strongly Disagree (1SD), 2 = Disagree (2D), 3 = Agree (3A), and 4 = Strongly Agree (4SA). Explanatory variables are yr89 (survey year; 0 = 1977, 1 = 1989), male (0 = female, 1 = male), white (0 = nonwhite, 1 = white), age (measured in years), ed (years of education), and prst (occupational prestige scale). Stata's `ologit` yields the following results.

```
. use "http://www.indiana.edu/~jslsoc/stata/spex_data/ordwarm2.dta"
(77 & 89 General Social Survey)
. * Ordered logit model
. ologit  warm yr89 male white age ed prst, nolog

Ordered logistic regression                     Number of obs   =       2293
                                                LR chi2(6)      =     301.72
                                                Prob > chi2     =     0.0000
Log likelihood = -2844.9123                     Pseudo R2       =     0.0504

------------------------------------------------------------------------------
        warm |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        yr89 |   .5239025   .0798988     6.56   0.000     .3673037    .6805013
        male |  -.7332997   .0784827    -9.34   0.000    -.8871229   -.5794766
       white |  -.3911595   .1183808    -3.30   0.001    -.6231815   -.1591374
         age |  -.0216655   .0024683    -8.78   0.000    -.0265032   -.0168278
          ed |   .0671728    .015975     4.20   0.000     .0358624    .0984831
        prst |   .0060727   .0032929     1.84   0.065    -.0003813    .0125267
```

```
-------------+------------------------------------------------------------------
       /cut1 |  -2.465362   .2389126                      -2.933622   -1.997102
       /cut2 |   -.630904   .2333155                      -1.088194    -.173614
       /cut3 |   1.261854   .2340179                       .8031873    1.720521
-------------------------------------------------------------------------------
```

These results are relatively straightforward, intuitive and easy to interpret. People tended to be more supportive of working mothers in 1989 than in 1977. Males, whites and older people tended to be less supportive of working mothers, while better educated people and people with higher occupational prestige were more supportive.

But, while the results may be straightforward, intuitive, and easy to interpret, are they correct? The `brant` command (part of Long and Freese's `spost` routines) indicates that the assumptions of the ordered logit/ proportional odds model are not met.

```
. use "http://www.indiana.edu/~jslsoc/stata/spex_data/ordwarm2.dta"
(77 & 89 General Social Survey)
. * Parallel Lines/ Proportional Odds Model (results already shown above)
. quietly ologit  warm yr89 male white age ed prst, nolog
.* Brant test shows assumptions are violated
. brant, detail

Estimated coefficients from j-1 binary regressions

             y>1         y>2         y>3
 yr89    .9647422   .56540626   .31907316
 male  -.30536425  -.69054232  -1.0837888
white  -.55265759  -.31427081  -.39299842
  age   -.0164704  -.02533448  -.01859051
   ed   .10479624   .05285265   .05755466
 prst  -.00141118   .00953216   .00553043
_cons   1.8584045   .73032873  -1.0245168

Brant Test of Parallel Regression Assumption

    Variable |      chi2   p>chi2    df
-------------+--------------------------
         All |     49.18    0.000    12
-------------+--------------------------
        yr89 |     13.01    0.001     2
        male |     22.24    0.000     2
       white |      1.27    0.531     2
         age |      7.38    0.025     2
          ed |      4.31    0.116     2
        prst |      4.33    0.115     2
---------------------------------------

A significant test statistic provides evidence that the parallel
regression assumption has been violated.
```

Proportional Odds & Partial Proportional Odds/ Parallel Lines & Non-Parallel Lines

Model 0: Perfect Proportional Odds/ Parallel Lines

gender	attitude SD	D	A	SA	Total
Male	250	250	250	250	1,000
Female	100	150	250	500	1,000
Total	350	400	500	750	2,000

	1 versus 2, 3, 4	1 & 2 versus 3 & 4	1, 2, 3 versus 4
$Odds_M$	750/250 = 3	500/500 = 1	250/750 = 1/3
$Odds_F$	900/100 = 9	750/250 = 3	500/500 = 1
OR ($Odds_F$ / $Odds_M$)	9/3 = 3	3/1 = 3	1/ (1/3) = 3
Gologit2 Betas	1.098612	1.098612	1.098612

Gologit2 χ^2 (3 d.f.)	176.63 (p = 0.0000)
Ologit χ^2 (1 d.f.)	176.63 (p = 0.0000)
Ologit Beta (OR)	1.098612 (3.00)
Brant Test (2 d.f.)	0.0 (p = 1.000)
Comment	If proportional odds holds, then the odds ratios should be the same for each of the ordered dichotomizations of the dependent variable. Proportional Odds works perfectly in this model, as the odds ratios are all 3. Also, the Betas are all the same, as they should be.

Model 1: Partial Proportional Odds I

gender	attitude SD	D	A	SA	Total
Male	250	250	250	250	1,000
Female	100	300	300	300	1,000
Total	350	550	550	550	2,000

	1 versus 2, 3, 4	1 & 2 versus 3 & 4	1, 2, 3 versus 4
$Odds_M$	750/250 = 3	500/500 = 1	250/750 = 1/3
$Odds_F$	900/100 = 9	600/400 = 1.5	300/700 = 3/7
OR ($Odds_F$ / $Odds_M$)	9/3 = 3	1.5/1 = 1.5	(3/7)/(1/3) = 1.28
Gologit2 Betas	1.098612	.4054651	.2513144

Gologit2 χ^2 (3 d.f.)	80.07 (p = 0.0000)
Ologit χ^2 (1 d.f.)	36.44 (p = 0.0000)
Ologit Beta (OR)	.4869136 (1.627286)
Brant Test (2 d.f.)	40.29 (p = 0.000)
Comment	Gender has its greatest effect at the lowest levels of attitudes, i.e. women are much less likely to strongly disagree than men are, but other differences are smaller. The effect of gender is consistently positive, i.e. the differences involve magnitude, not sign.

Model 2: Partial Proportional Odds II

gender	attitude SD	D	A	SA	Total
Male	250	250	250	250	1,000
Female	100	400	250	250	1,000
Total	350	650	500	500	2,000

	1 versus 2, 3, 4	1 & 2 versus 3 & 4	1, 2 3 versus 4
$Odds_M$	750/250 = 3	500/500 = 1	250/750 = 1/3
$Odds_F$	900/100 = 9	500/500 = 1	250/750 = 1/3
OR ($Odds_F$ / $Odds_M$)	9/3 = 3	1/1 = 1	(1/3)/(1/3) = 1
Gologit2 Betas	1.098612	0	0

Gologit2 χ^2 (3 d.f.)	101.34 (p = 0.0000)
Ologit χ^2 (1 d.f.)	9.13 (p = 0.0025)
Ologit Beta (OR)	.243576 (1.275803)
Brant Test (2 d.f.)	83.05 (p = 0.000)
Comment	Gender has its greatest – and only – effect at the lowest levels of attitudes, i.e. women are much less likely to strongly disagree than men are. But, this occurs entirely because they are much more likely to disagree rather than strongly disagree. Other than that, there is no gender effect; men and women are equally likely to agree and to strongly agree. The ologit estimate underestimates the effect of gender on the lower levels of attitudes and overestimates its effect at the higher levels.

Model 3: Partial Proportional Odds III

gender	attitude SD	D	A	SA	Total
Male	250	250	250	250	1,000
Female	100	400	400	100	1,000
Total	350	650	650	350	2,000

	1 versus 2, 3, 4	1 & 2 versus 3 & 4	1, 2, 3 versus 4
$Odds_M$	750/250 = 3	500/500 = 1	250/750 = 1/3
$Odds_F$	900/100 = 9	500/500 = 1	100/900 = 1/9
OR ($Odds_F$ / $Odds_M$)	9/3 = 3	1/1 = 1	(1/9)/(1/3) = 1/3
Gologit2 Betas	1.098612	0	-1.098612

Gologit2 χ^2 (3 d.f.)	202.69 (p = 0.0000)
Ologit χ^2 (1 d.f.)	0.00 (p = 1.0000)
Ologit Beta (OR)	0 (1.00))
Brant Test (2 d.f.)	179.71 (p = 0.000)
Comment	The effect of gender varies in both sign and magnitude across the range of attitudes. Basically, women tend to take less extreme attitudes in either direction. They are less likely to strongly disagree than are men, but they are also less likely to strongly agree. The ologit results imply gender has no effect while the gologit results say the effect of gender is highly significant. Perhaps the current coding of attitudes is not ordinal with respect to gender, e.g. coding by intensity of attitudes rather than direction may be more appropriate. Or, suppose that, instead of attitudes, the categories represented a set of ordered hurdles, e.g. achievement levels. Women as a whole may be more likely than men to clear the lowest hurdles but less likely to clear the highest ones. If men are more variable than women, they will have more outlying cases in both directions. Use of ologit in this case would be highly misleading.

A Parsimonious Alternative: Generalized Ordered Logit/ Partial Proportional Odds

1. Unconstrained Gologit Model. All betas are free to differ across levels of j.

$$P(Y_i > j) = \frac{\exp(\alpha_j + X_i\beta_j)}{1+[\exp(\alpha_j + X_i\beta_j)]}, \text{ j} = 1, 2, ..., \text{M} - 1$$

2. Special Case: Proportional Odds. All betas the same across levels of j.

$$P(Y_i > j) = \frac{\exp(\alpha_j + X_i\beta\ )}{1+[\exp(\alpha_j + X_i\beta\ )]}, \text{ j} = 1, 2, ..., \text{M} - 1$$

3. Special Case: Partial Proportional Odds. Some betas differ across levels of j but others do not.

$$P(Y_i > j) = \frac{\exp(\alpha_j + X1_i\beta1 + X2_i\beta2 + X3_i\beta3_j)}{1+[\exp(\alpha_j + X1_i\beta1 + X2_i\beta2 + X3_i\beta3_j)]}, \text{ j} = 1, 2, ..., \text{M} - 1$$

```
. * Partial proportional odds - relax the pl assumption when it is violated
. use "http://www.indiana.edu/~jslsoc/stata/spex_data/ordwarm2.dta", clear
. gologit2  warm yr89 male white age ed prst, auto lrf store(gologit2)
--------------------------------------------------------------------------------
Testing parallel lines assumption using the .05 level of significance...

Step  1:  Constraints for parallel lines imposed for white (P Value = 0.7136)
Step  2:  Constraints for parallel lines imposed for ed (P Value = 0.1589)
Step  3:  Constraints for parallel lines imposed for prst (P Value = 0.2046)
Step  4:  Constraints for parallel lines imposed for age (P Value = 0.0743)
Step  5:  Constraints for parallel lines are not imposed for
          yr89 (P Value = 0.00093)
          male (P Value = 0.00002)

Wald test of parallel lines assumption for the final model:

 ( 1)  [1SD]white - [2D]white = 0
 ( 2)  [1SD]ed - [2D]ed = 0
 ( 3)  [1SD]prst - [2D]prst = 0
 ( 4)  [1SD]age - [2D]age = 0
 ( 5)  [1SD]white - [3A]white = 0
 ( 6)  [1SD]ed - [3A]ed = 0
 ( 7)  [1SD]prst - [3A]prst = 0
 ( 8)  [1SD]age - [3A]age = 0

           chi2(  8) =   12.80
         Prob > chi2 =    0.1190

An insignificant test statistic indicates that the final model
does not violate the proportional odds/ parallel lines assumption

If you re-estimate this exact same model with gologit2, instead
of autofit you can save time by using the parameter
```

```
pl(white ed prst age)

------------------------------------------------------------------------------

Generalized Ordered Logit Estimates              Number of obs   =       2293
                                                 LR chi2(10)     =     338.30
                                                 Prob > chi2     =     0.0000
Log likelihood = -2826.6182                      Pseudo R2       =     0.0565

 ( 1)  [1SD]white - [2D]white = 0
 ( 2)  [1SD]ed - [2D]ed = 0
 ( 3)  [1SD]prst - [2D]prst = 0
 ( 4)  [1SD]age - [2D]age = 0
 ( 5)  [2D]white - [3A]white = 0
 ( 6)  [2D]ed - [3A]ed = 0
 ( 7)  [2D]prst - [3A]prst = 0
 ( 8)  [2D]age - [3A]age = 0
------------------------------------------------------------------------------
        warm |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
1SD          |
       yr89 |     .98368   .1530091     6.43   0.000     .6837876    1.283572
       male |  -.3328209   .1275129    -2.61   0.009    -.5827417   -.0829002
      white |  -.3832583   .1184635    -3.24   0.001    -.6154424   -.1510742
        age |  -.0216325   .0024751    -8.74   0.000    -.0264835   -.0167814
         ed |   .0670703   .0161311     4.16   0.000     .0354539    .0986866
       prst |   .0059146   .0033158     1.78   0.074    -.0005843    .0124135
      _cons |    2.12173   .2467146     8.60   0.000     1.638178    2.605282
-------------+----------------------------------------------------------------
2D           |
       yr89 |    .534369   .0913937     5.85   0.000     .3552406    .7134974
       male |  -.6932772   .0885898    -7.83   0.000    -.8669099   -.5196444
      white |  -.3832583   .1184635    -3.24   0.001    -.6154424   -.1510742
        age |  -.0216325   .0024751    -8.74   0.000    -.0264835   -.0167814
         ed |   .0670703   .0161311     4.16   0.000     .0354539    .0986866
       prst |   .0059146   .0033158     1.78   0.074    -.0005843    .0124135
      _cons |   .6021625   .2358361     2.55   0.011     .1399323    1.064393
-------------+----------------------------------------------------------------
3A           |
       yr89 |   .3258098   .1125481     2.89   0.004     .1052197       .5464
       male |  -1.097615   .1214597    -9.04   0.000    -1.335671   -.8595579
      white |  -.3832583   .1184635    -3.24   0.001    -.6154424   -.1510742
        age |  -.0216325   .0024751    -8.74   0.000    -.0264835   -.0167814
         ed |   .0670703   .0161311     4.16   0.000     .0354539    .0986866
       prst |   .0059146   .0033158     1.78   0.074    -.0005843    .0124135
      _cons |  -1.048137   .2393568    -4.38   0.000    -1.517268   -.5790061
------------------------------------------------------------------------------
```