Appendix D: The Monty Hall Controversy

Let's Make a Deal
Prepared by Rich Williams, Spring 1991
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You are playing *Let's Make a Deal* with Monty Hall. You are offered your choice of door #1, door #2, or door #3. Monty tells you that goats are behind two of the doors; but, behind the other door is a brand new car. You choose door #1. Monty, who knows what is behind each door, then opens door #3, showing that it contains a goat. He then offers you the choice of either keeping your own door, #1, or else switching to Monty's remaining door, #2. Should you switch?

SOLUTION. The answer to this question is not straightforward. There are several different sets of assumptions that might or might not be valid, and the correct answer depends on which ones actually hold. Let us consider 4 possibilities:

ASSUMPTIONS I. As stated, Monty Hall knows what is behind each door. In addition, assume that Monty Hall follows this same procedure with <u>every</u> contestant, i.e. the contestant chooses a door, Monty then opens up one of the <u>losing</u> doors, and then offers the contestant the option to switch. Also assume that, when both of Monty's doors are losers, he randomly chooses which one to open.

I suspect that most people think that these assumptions are valid, and I told you to make these assumptions when I first gave you the problem. Indeed, Marilyn vos Savant, allegedly the world's smartest person, made these assumptions when she first addressed this question in her column[1]. Most people probably also think that the odds of either door winning are 50-50, but as Savant pointed out, most people are wrong. The odds are 2 to 1 in favor of switching.

The easiest way to see this is by using the complements rule. Let A = switch, $\bar{A}$ = doesn't switch. Note that resolving not to switch is the same as not having the option to switch. When you first pick, you have a 1/3 chance of winning; ergo, if you don't switch, your probability of winning stays at 1/3. Hence, you have a 2/3 chance of winning if you switch and only 1/3 if you don't switch. So, switch.

If you are still not convinced, consider this: Suppose Monty was really generous, and instead of revealing a losing door, he instead offered you both of his doors. Obviously, 2 times out of 3 you would be better off switching. But, offering you both doors (when one of them contains a goat) is in effect the same thing as showing you a door which has a goat and offering you the other. The fact that Monty opens a door just creates the illusion that these two variations are different from each other.

[1] More recently, the problem was discussed in Chapter 101 of the 2003 best-selling book, <u>The Curious Incident of the Dog in the Night-Time</u>, by Mark Haddon. The book tells the story of an autistic boy who tries to solve the murder mystery of the neighbor's dog. I found out about this book when Roy Palmer (from Ireland!) came across an earlier version of these notes and recommended it to me. Great book.

For those who remain unconvinced and simply must see things with their own 2 eyes: Let us take 6 trials of this experiment - 2 in which door #1 is the winner, 2 in which door #2 is the winner, 2 in which door #3 is the winner. Following are the different possibilities

Door # 1	Door # 2	Door # 3	Monty reveals	Correct decision
W	L	L	Door # 2	Stay
W	L	L	Door # 3	Stay
L	W	L	Door # 3	Switch
L	W	L	Door # 3	Switch
L	L	W	Door # 2	Switch
L	L	W	Door # 2	Switch

Hence, 2 times out of 3 you will do better by switching.

To put it another way:

If Monty opens door #2, the possible elementary events are (each letter tells whether the corresponding door 1, 2, or 3 is the winner):

W - L - L
L - L - W
L - L - W

If Monty opens door # 3:

W - L - L
L - W - L
L - W - L

Either way, the odds are 2 to 1 against you if you don't switch.

What confuses people is that Monty's 1 remaining door is pitted against your door, so people assume it is an even choice. The problem is that Monty's door is twice as likely to win as yours is. Before Monty opens a door, each of his doors has a 1/3 chance of being a winner. After he opens a door, the probability of that door being a winner drops to 0, but the probability of his other door being a winner jumps to 2/3!

ASSUMPTIONS II: But are the above assumptions valid? Cecil Adams, in his syndicated column The Straight Dope, argued that they were not (although, after initially attacking her, he did grudgingly admit that she was right when the above conditions did hold). If Monty always follows this procedure, sooner or later contestants will catch on, and Monty will lose a bundle because everybody will switch. So, suppose instead he does things slightly differently. Whenever the contestant has picked a winning door, he always reveals one of his losers and offers the option to switch. But, when the contestant has picked a loser, only half the time will Monty reveal a door and offer the option to switch. Under these conditions, if you are offered the option to switch, should you take it?

Let's again take 6 trials:

Door # 1	Door # 2	Door # 3	Monty reveals	Correct decision
W	L	L	Door # 2	Stay
W	L	L	Door # 3	Stay
L	W	L	Door # 3	Switch
L	W	L	Nothing	No choice
L	L	W	Door # 2	Switch
L	L	W	Nothing	No Choice

Under these conditions, a third of the time you are not offered the option to switch, so there is no decision to make. But, the 2/3 of the time you are offered the option, half the time you already have the winning door and the other half of the time Monty has the winner. So, the odds of winning are the same whether you switch or not.

ASSUMPTIONS III. Suppose Monty wants to minimize his losses, so he only offers you the option of switching if you have picked the winning door. Should you switch? Obviously not! Adams argued that Monty would not operate this way, since people would catch on and never take him up on his offer. On the other hand, Monty doesn't lose anything by doing this, and might get lucky and get somebody who doesn't realize they shouldn't switch.

ASSUMPTIONS IV. Monty is a nice guy and feels sorry for losers. Or, even if he isn't so nice, Monty might figure that having big winners on his show helps him more financially than paying for the prizes hurts him. Ergo, Monty will only offer the option to switch when the contestant has picked a loser. Under these conditions, you should obviously accept his offer to switch.

MORAL. This problem is a lot more complicated than it looks. In order to make the correct decision, you have to know what procedures Monty is following. Does Monty want to load the odds in your favor, in his favor, or does he want it to be a 50-50 proposition? Does Monty benefit more from keeping his prize money for himself, or does he do better by having lots of big winners that attract lots of viewers and advertisers? And, is Monty all that rational in choosing a strategy, and does Monty really have to worry about contestants being smart enough to figure out what his strategy is?

So, who has the more reasonable case? Marilyn vos Savant or her critics? Who better to address that question than the master himself. In an interview in the July 21, 1991 New York Times, Monty Hall declared, “Her answer’s right. You should switch.” But, after thinking about it a bit, Monty qualified his response: “If the host is required to open a door all the time and offer you a switch, then you should take the switch. But if he has the choice whether to allow a switch or not, beware. Caveat emptor. It all depends on his mood.”

Marilyn’s column attracted thousands of letters and reactions, with 92% of the writers saying she was wrong. Here are a few of the comments she received.

You’re wrong, but look at the positive side. If all those Ph.D.s were wrong, the country would be in very serious trouble.

-- *Letter to Marilyn from Everett Harman, Ph.D., U.S. Army Research Institute*

Our brains are just not wired to do probability problems very well, so I'm not surprised there were mistakes.

-- *Persi Diaconis, Harvard University professor specializing in probability (a Marilyn supporter)*

Maybe women look at math problems differently than men.

-- *Don Edwards, Sunriver, Oregon*

Give it up gang…Still don’t get it? Then at least have the sense to keep quiet about it.

-- *Cecil Adams’ advice to those who still thought Marilyn was wrong even after he [grudgingly] defended her.*

But on the Other Hand… When she was 8 years old, my daughter Bethy came up with the best counter-argument I have ever heard. When I told her the problem, she said she would not switch. I told her, “Bethy, you’ll probably win a goat if you do that.” Her response was, “But I want a goat!” ☺