THE VIRTUAL INEVITABILITY OF SINGULARITIES IN INFLATIONARY MODELS OF THE MACRO UNIVERSE: THE PROBABILITY OF CREATION OF THE MACRO UNIVERSE

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INTRODUCTION

I assume that many of you are familiar with the classical big bang model of cosmology which requires a singularity (temporal boundary) of the universe, approximately 13.7 billion years ago. Prior to 1980, the Hawking-Penrose Singularity was considered to be one of the most probative pieces of physical evidence of a limit to past time in the universe. In 1980, Hawking wrote, "a curvature singularity that will intersect every world line… [makes] general relativity predict a beginning of time."[1]

Quentin Smith, summarizing Hawking and Penrose, lists the five conditions necessitating a singularity according to standard GTR (General Theory of Relativity) model:

> (1) space-time satisfies the equations of GTR, (2) time travel into one's own past is impossible and the principle of causality is not violated (there are no closed time-like curves), *(3) the mass density and pressure of matter never become negative,* (4) the universe is closed and/or there is enough matter present to create a trapped surface, and (5) the space-time manifold is not too highly symmetric.[2]

Prior to the discovery of evidence for an inflationary era (and its vacuum energy), all five of these conditions were thought to be met in the universe. Quentin Smith summarized the consequences of this by noting:

> …it belongs analytically to the concept of the cosmological singularity that it is not the effect of prior physical events. The definition of a singularity that is employed in the singularity theorems entails that it is *impossible* to extend the space-time manifold beyond the singularity. The definition in question is based on the concept of inextendible curves [which must avoid implying infinite curvature and other similar mathematical paradoxes]…. This effectively rules out the idea that the singularity is an effect of some prior natural process.[3]

There have been several major adaptations to the classical big bang model since 1980 which have called into question the *necessity* of a singularity in that model. The most

[1] Hawking 1980, p.149.
[2] Smith 1993(a), p. 114.
[3] Smith 1993(a), p. 120.

compelling of these is the presence of vacuum energy at the initial stages of the GTR Universe which explains initial universal inflation, which in turn explains the conditions of the observable universe as we know it. If vacuum energy really did cause inflation at the inception of the observable universe, then it undermines the third condition elucidated by Hawking and Penrose in their list of five universal conditions necessitating a singularity.

If there was an inflationary period at the early phase of the universe, then there would exist a very strong pressure associated with the universe's vacuum energy which is equal to *minus* the density. Vacuum energy is different from mass energy in that it opposes gravity and exerts *repulsive* force in proportion to density. The presence of this energy would then exert a very strong negative pressure of matter, violating Hawking's and Penrose's third condition.

At first glance, the violation of this third condition might seem to avoid the need for a singularity (and a beginning of the universe). However, the work of Borde, Vilenkin, Guth, and others shows that singularities seem to be inevitable in *inflationary* models of the universe (see below Section II). This implies that the universe would have to have had an initial singularity and therefore a beginning.

This thesis will be explained in three parts:

(1) A brief examination of the evidence of universal inflation and the resultant models of that inflation,
(2) the necessity of a singularity in inflationary theory, and
(3) contemporary developments in the inflationary model.

I.
EVIDENCE FOR AN INFLATIONARY MODEL UNIVERSE

Contemporary astrophysicists have gathered significant evidence for an inflationary period at the inception of the universe. In brief, evidence from the Hubble telescope, the COBE satellite, and consequent computer modeling of the universe seems to indicate that the universe would have had to have gone through an inflationary or super-accelerating period (caused by the presence of vacuum energy) in order to arrive at its current state.

Andrei Linde details four pieces of physical evidence in favor of an inflationary period near the origin of the universe:[4]

a) A period of extremely rapid inflation predicts density perturbations affecting the distribution of matter in the universe. These predictions may be verified by the distribution of galaxies. Currently, galactic distribution resembles what would have been predicted by an inflationary scenario rather than a classical big bang model, which cannot explain this at all.

b) The above density perturbations also imply slight variations in the large scale uniform cosmic radiation. Thus, the temperature of the cosmic background radiation should vary slightly over

[4] Linde actually gives six pieces of evidence, but as will be discussed below, two of them are metaphysical (Linde 1998, pp. 98-104).

large areas. This variation in the radiation was not discovered before 1992 because the variation was so slight compared with its large scale uniformity. However, in 1992, the Cosmic Background Explorer satellite (COBE) began to show evidence of this. There is still work to be done in confirming these "ripples in space," but as of the present moment, the COBE satellite and other extensive studies of the cosmic microwave background radiation, have not been able to disprove inflation.

c) Inflationary theory is the only one that will allow for either a "flat universe" or a large, homogeneous, open universe. Since these two scenarios are currently more explicative of universal conditions than the closed one, it would seem that inflationary theory is necessary to explain our universe.

d) Under a standard GTR assumption, the physics of the early universe entails phase transitions which would have, as their consequence, unusually heavy particles like monopoles. Such heavy particles should be easily detectable. Up to this point, no such particles have been discovered.[5]

A fifth and very convincing line of evidence was recently discovered by the microwave anisotropy probe (MAP) observatory, which placed the genesis of stars to a very early universal age. This finding can only be explained through an inflationary era in the first few seconds of the universe's existence.[6]

Some physicists believe that an inflationary condition exists even today. This current mild inflationary period followed a period of deceleration which occurred until the era marked by a redshift of approximately 0.5.

The occurrence of this inflationary (accelerating) universe is attributable to the effects of vacuum energy which negatively interacts with density and has a strong pressure associated with it. Current evidence suggests that this pressure exceeds gravitational attraction, causing an overall acceleration in the expansion of the universe.

In quantum theory, vacuum energy is evidenced in the "borrowing" of energy for virtual particle pair creation within a quantum system. It is different from mass energy in that it opposes gravity and actually increases in magnitude with distance. When Linde and others discovered the probability of an inflationary era, they also pointed to the probable existence of vacuum energy in the universe as a whole (in addition to the vacuum energy in quantum systems) to induce this inflation.

Alan Guth has elucidated some of the stages which are likely to have occurred at the inception of inflation:

> [a] a patch of [a special] form of matter [creating gravitational repulsion at high energies] existed in the early universe – it was probably more than a billion times

[5] Ibid Linde, 1998, p.98-104

[6] See the NASA February 11, 2003 press conference and principal investigator Bennett's explanation of it in footnote 18 (above) and at www.space.com/scienceastronomy/ map_discovery_030211.html.

> smaller than a single proton! The gravitational repulsion created by this material was the driving force behind the big bang. The repulsion drove it into exponential expansion, doubling in size every 10^{-37} second or so!
>
> [b]The repulsive-gravity material is unstable, so it decayed like a radioactive substance, ending inflation. The decay released energy, which produced ordinary particles, forming a hot, dense "primordial soup." Inflation lasted maybe 10^{-35} second. At the end, the region destined to become the presently observed universe was about the size of a marble.
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> [c] The "primordial soup" matches the assumed starting point of the standard big bang—the standard big bang description takes over. The region continues to expand and cool to the present day.[7]

Andrei Linde has proposed a model to explain the transition from the universe's initial state to its inflationary era. He gives a chaotic, fractal-like twist to traditional inflationary theory.[8] The theory is premised on the notion of scalar fields which mediate the interaction between electromagnetic and weak forces:

> If a scalar field interacts with the W and Z particles, they become heavy. Particles that do not interact with the scalar field, such as photons, remain light.[9]

Linde adapts this notion of scalar fields to the universe as a whole and shows how it could explain inflationary regions by various scalar fields having arbitrarily different values in different regions ("chaotic" – "fractal-like" inflation). This obviates the need for phase transitions, super-cooling, or even the standard assumption that the universe originally was hot, which complicated earlier models of inflationary theory.[10] For Linde, scalar fields can take on arbitrary values in the early universe. Some of these values will result in the universe expanding quite rapidly while others result in very little expansion.

Using chaotic inflation as a base (where different regions can have incredibly different volumes), he conjectures that each inflationary region can produce new regions (like the multiplication of a fractal):

> From this it follows that if the universe contains at least one inflationary domain of a sufficiently large size, it begins unceasingly producing new inflationary domains. Inflation in each particular point may end quickly, but many other places will continue to expand. The total volume of all these domains will grow without end. In essence, one inflationary universe sprouts other inflationary bubbles, which in turn produce other inflationary bubbles.[11]

He then makes recourse to multiple singularities expressing temporal beginnings and ends of specific regions of the universe:

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[7] Guth 2003 (web address).
[8] See Linde 1998 and 1994, and Guth 1997.
[9] Linde 1998, p. 101.
[10] Ibid., p. 102.
[11] Ibid., p. 103.

> Each particular part of the universe may stem from a singularity somewhere in the past, and it may end up in a singularity somewhere in the future…. The situation with the very beginning is less certain. There is a chance that all parts of the universe were created simultaneously in an initial big bang singularity. The necessity of this assumption, however, is no longer obvious.[12]

Borde, Vilenkin and Guth are not nearly as convinced as Linde about the non-obvious necessity of a singularity. As will be explained in Section II, these three physicists suggest that a singularity seems to be inevitable in inflationary models.

II.
THE NECESSITY OF A SINGULARITY IN INFLATIONARY THEORY

In a famous article written in 1993, Arvind Borde and Alexander Vilenkin gave a proof that all inflationary models which presume four universal conditions must begin with a singularity. If these four conditions governed the initial state of the universe (even a multiverse with many "pocket universes"), then that universe would have to begin with a singularity.[13] Borde and Vilenkin articulate the proof as follows:

> A spacetime cannot be past null geodesically *complete* [entailing a singularity] if it satisfies the following conditions: (A) It is past causally simple. (B) It is open. (C) Einstein's equation holds, with a source that obeys the weak energy condition (i.e., the matter energy density is non-negative). (D) There is at least one point *p* such that for some point *q* to the future of *p* the volume of the difference of the pasts of *q* and *p* is finite. Observe that geodesic incompleteness is being taken as a signal that there is a singularity. (A geodesic is incomplete if it cannot be continued to arbitrarily large values of its affine parameter.) This is the conventional approach in singularity theorems.[14]

Thus, if inflationary models require that all four conditions be fulfilled, inflationary models must be geodesically incomplete, and therefore, require a singularity. Borde and Vilenkin prove this by showing that the absence of an initial singularity necessitates a contradiction in the four assumptions:

> *Proof*–Suppose that a spacetime that obeys assumptions (A)-(D) is past null geodesically complete [i.e., that a singularity does not exist]. We show in two steps that a contradiction ensues. [15]

This contradiction necessitates that spacetime (which obeys assumptions (A)-(D)) cannot be past null geodesically complete, which means it must be incomplete which means a singularity is required. Borde and Vilenkin extend their conclusion to other views of gravity (such as quantum gravity) which will become relevant below:

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[12] Ibid., p. 105.

[13] Borde and Vilenkin 1993, pp. 3305-3308.

[14] Borde and Vilenkin 1993, p. 3305.

[15] Borde and Vilenkin 1993, p. 3306.

> Thus our results will remain true even in other theories of gravity, as long as the null convergence condition continues to hold. [16]

Borde and Vilenkin conclude that an initial singularity is required. They leave open the possibility that there could be many regions of the universe (not part of our observable universe) and many other "universes" or states of the multiverse which could have existed prior to our observable universe; but all of them would have to begin with a singularity, including the multiverse, if the four proposed conditions of an inflationary universe hold:

> The conclusion to be drawn from this argument is that inflation does not seem to avoid the problem of the initial singularity (although it does move it back into an indefinite ["though finite"] past). In fact, our analysis of assumption (D) suggests that almost all points in the inflating region will have a singularity somewhere in their pasts. As with most singularity theorems, our analysis tells us nothing about the nature of the singularity or about its precise location. In particular, we cannot tell whether or not the singularity occurs on a spacelike surface, like the big bang singularity of the standard Robertson - Walker cosmology. However, the fact that inflationary spacetimes are past incomplete forces one to address the question of what, *if anything*, came before.[17] (Italics mine).

In 1997, Borde and Vilenkin recognized a possible exception to their proof of a singularity in inflationary models. It arises out of the implications of quantum cosmology which allows for quantum fluctuations violating Einstein's weak energy condition (see above assumption (C)). This exception is relevant because a singularity would have to be the temporal boundary not only of inflation, but also of causal conditions of inflation which might include quantum gravity or strings or an unusual configuration of de Sitter spacetime. Indeed, if the vacuum energy (dark energy) which is thought to be the source of inflation is associated with an initial quantum condition (recall that in quantum theory, vacuum energy is evidenced in the "borrowing" of energy for virtual particle pair creation within a quantum system) then, the singularity would be the temporal boundary of this initial quantum cosmological condition. This leaves the door open for a violation of assumption (C) which would allow an exception to Borde's and Vilenkin's 1993 proof.

However, Borde, and Vilenkin were unable to show any realistic way of applying the violation of assumption (C) to the universe as we know it. They developed some hypothetical configurations of a steady-state inflationary model which might allow for a violation of condition (C) from string-theory conditions which might have caused inflation and from an unusual configuration of de Sitter spacetime. They found that these and other hypotheses linked to a violation of the weak-energy condition seem extremely improbable, because they do not even remotely fit into the expectations of any known realistic spacetime model. Furthermore, the probability of their extended occurrence (sufficient to have a significant effect in the universe) is also remote. They concluded that such violations of assumption (C) are difficult if not impossible:

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[16] Borde and Vilenkin p. 3306.

[17] Borde and Vilenkin 1993, p. 3307.

Cosmologies with a scale factor of the form $a(t) \sim (-t)^{-q}$, where $0<q\leq 1$ (and $t<0$), satisfy these conditions. Such a scale factor appears, for example, in the "pre-big-bang" stage of the proposed models of string cosmology [30-32]. These models do not, however, qualify as models of "steady-state" inflation. The Riemann tensor in such models decreases as $R^{\mu 1 \nu}{}_{\sigma\tau} \} t^{-2}$ when $t \rightarrow -\equiv$ indicating that the spacetime is asymptotically flat in the past direction. The Hubble parameter H also vanishes as $t \rightarrow -\equiv$. This behavior is very different from the quasiexponential expansion with H const that is characteristic of inflation at later times. Since the idea behind a steady-state model, and its chief attraction, is that the Universe is in more or less the same state at all times, models with very different behavior at early and late times are *not* viable as models of steady-state inflation.[18]

Borde and Vilenkin conclude that a violation of assumption (C) would not occur as a result of pre-big-bang string-theory configurations of the universe because they are too highly divergent from a subsequent steady-state inflation. Moreover, a violation of assumption (C) would not occur from unusual configurations of de Sitter spacetime because these configurations imply a contracting initial state of the universe which would diverge from an inflationary condition and would be fundamentally unstable:

In other models of inflation, we have shown here that there is a possibility for nonsingular models to exist, based on the violation of the weak energy condition that occurs in these models [assumption (C)]. Whether realistic models of this type can be constructed, however, remains open. The discussion of Sec. IV suggests that the construction of such models may be difficult, if not impossible.[19]

Alan Guth has added to Borde's and Vilenkin's work by showing that all current models of inflation (even those possessing pre-big-bang quantum cosmological and string-theory components) necessitate a singularity. The current inability to construct such non-singularity models causes him to question whether any such models will be able to be constructed in the future:

For the explicit construction of eternally (into the future) inflating models, the answer is clear. Such models start with a state in which there are no pocket universes at all, just pure repulsive-gravity material filling space. So there is definitely a beginning to the models that we know how to construct. [20]

He concludes from this:

At the present time, I think it is fair to say that it is an open question whether or not eternally inflating universes can avoid having a beginning. In my own opinion, it looks like eternally inflating models *necessarily* have a beginning. I believe this for two reasons. The first is the fact that, as hard as physicists have worked to try to construct an alternative, so far all the models that we construct have a beginning; they are eternal into the future, but not into the past. The second reason is that the technical assumption questioned in the 1997 Borde-Vilenkin paper does not seem important enough to me to change the *conclusion*, even

[18] Borde and Vilenkin 1997, p. 720.

[19] Borde and Vilenkin 1997, p. 722.

[20] Guth 1999, p. 13.

> though it does undercut the *proof*. Specifically, we could imagine approximating the laws of physics in a way that would make them consistent with the assumptions of the earlier Borde-Vilenkin paper, and eternally inflating models would still exist. Although those modifications would be unrealistic, they would not drastically change the behavior of eternally inflating models, so it seems unlikely that they would change the answer to the question of whether these models require a beginning. [21]

It is worthwhile repeating Guth's observation that even if such a non-singular model of steady-state inflation (like the one suggested in the 1997 Borde and Vilenkin paper) could be constructed, it would only serve to undercut Borde's and Vilenkin's *proof* of the impossibility of a non-singular model of inflation; it would *not* show the possibility of a non-singularity condition within the expected conditions of our universe.

III.
CONTEMPORARY DEVELOPMENTS IN THE INFLATIONARY MODEL

Since the writing of Guth's article, important work has been done by Linde and others to establish compatibility between string theory and the inflationary model. This compatibility allows a "string-condition universe" to be causally related to inflation. This does not mean that a string-condition universe gave rise to inflation, but only that it could have. The singularity predicted by Borde, Vilenkin, and Guth could have been the temporal boundary of a string-condition universe or some other non-string universe prior to or during inflation, for example, a quantum cosmological pre-big-bang universe or another kind of universe which could accommodate the vacuum energy necessary for initial inflation.

The reconciliation between string theory and inflationary models is very complex and allows for an extraordinary number of possible universes and landscapes. As Davide Castelvecchi notes:

> …"stringy" inflation seems to require a very complicated fine turning, as Linde found in a paper with seven other authors. Linde says that was a record number for him. The reason is, it took eight authors to fine-tune the parameters to get a nice inflation, he says. … [some physicists] find it troubling that the results of Linde and the string theorists seem to point to a "landscape" of possible universes—again, a multiverse of unpredictable bubbles.[22]

Of course this "multiverse of bubbles" is not radically different from Linde's bubbles or Guth's pocket universes. The sheer number of possibilities allowed by stringy inflation should not be overwhelmingly troublesome. It certainly does not constitute a reason for discounting Borde's, Vilenkin's, and Guth's conclusion of the very probable occurrence of an initial singularity, for a singularity is still required in a stringy inflationary model and in a multiverse.

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[21] Guth, 1999, p. 13.
[22] Castelvecchi 2004-05 p.5

Recall that a singularity has metaphysical implications, entailing a beginning of the universe or multiverse (concomitantly implying a creative power transcending universal space-time asymmetry). Quentin Smith's words are worth repeating here:

> …it belongs analytically to the concept of the cosmological singularity that it is not the effect of prior physical events. The definition of a singularity that is employed in the singularity theorems entails that it is *impossible* to extend the space-time manifold beyond the singularity. The definition in question is based on the concept of inextendible curves [which must avoid implying infinite curvature and other similar mathematical paradoxes]…. This effectively rules out the idea that the singularity is an effect of some prior natural process.[23]

These metaphysical consequences of a singularity have provoked some theorists to develop a new model of the universe which avoids inflation and therefore avoids a singularity and its concomitant implications for a creation of the universe or a multiverse. For example, Turok and Steinhardt have developed a periodic big bang model which is very speculative, and dependent on unverified universal constituents (such as "two parallel three-dimensional membranes separated by a tiny gap in the fourth dimension"[24] and cyclic collisions of these two membranes which release enough energy to re-start the big bang). It also depends on some seemingly paradoxical contentions such as the need for the big bang to be "tractable such that things can pass through it from before to after."[25]

Why assemble all this speculation? To avoid a singularity. As Castelvecchi notes:

> A periodic big bang, [Steinhardt] says, would solve the "singularity problem," the question of what came before the big bang. That's something inflation can't do, because it starts with a singularity, a point in time where the laws of physics break down.[26]

At present, the inflationary model appears to be the most realistic (least speculative) explanation of the effects of density perturbations and the absence of heavy particles (such as magnetic monopoles). The inflationary model would also seem to be more empirically verifiable (because there may be ways of testing for the presence or remnants of vacuum energy/dark energy in the universe).

If the inflationary model proves to best describe the conditions of our observable universe, then we are confronted with the extreme likelihood of a singularity which would imply a beginning of the universe, implying, in turn, the action of a causative power transcending space-time asymmetry (which could be viewed as God).

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[23] Smith 1993(a), p. 120.
[24] Castelvecchi 2004-05 p.5
[25] Castelvecchi 2004-05 p.5
[26] Castelvecchi 2004-05 p.5

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