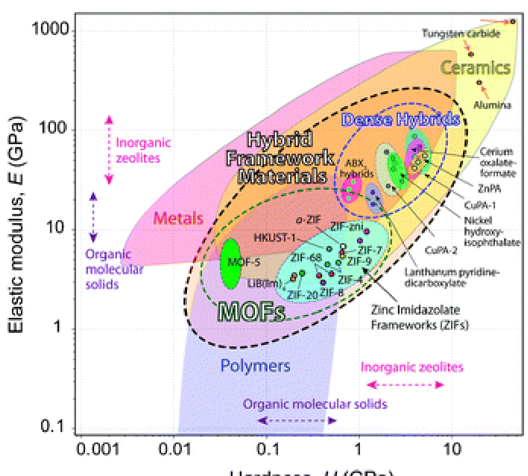
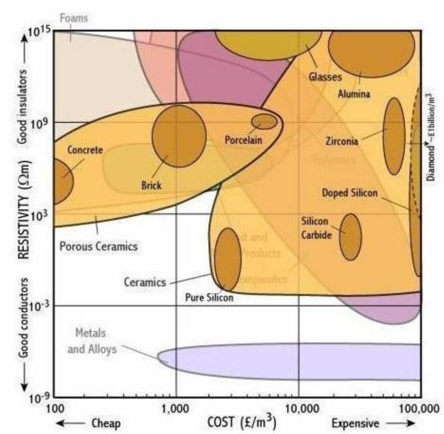
CHAPTER 6:
MECHANICAL PROPERTIES

ISSUES TO ADDRESS...

- **Stress and strain**: What are they and why are they used instead of load and deformation?
- **Elastic behavior:** When loads are small, how much deformation occurs? What materials deform least?
- **Plastic behavior:** At what point do dislocations cause permanent deformation? What materials are most resistant to permanent deformation?
- **Toughness and ductility**: What are they and how do we measure them?





Issu de "Material selection and processing"
http://www-materials.eng.cam.ac.uk/mpsite/

Vocabulary:
Mechanical Properties of Materials

- The changes in materials dimensions in response to mechanical forces is called **deformation.**

- If upon removal of load the material reverts back to its initial size – **elastic** deformation.

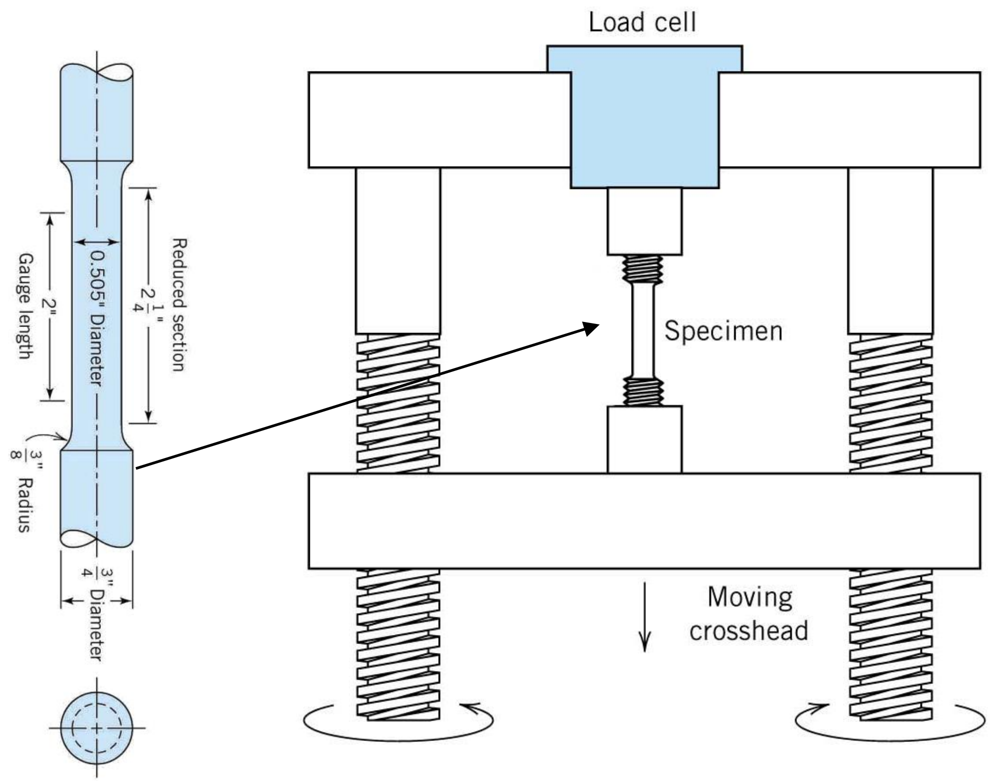
- If application and removal of the load results in a permanent material's shape change – **plastic** deformation.

- **Fracture** occurs when a structural component separates into two or more pieces.

- Material **failure*,*** i.e. an inability of a component to perform its desired function, may occur prior to fracture.

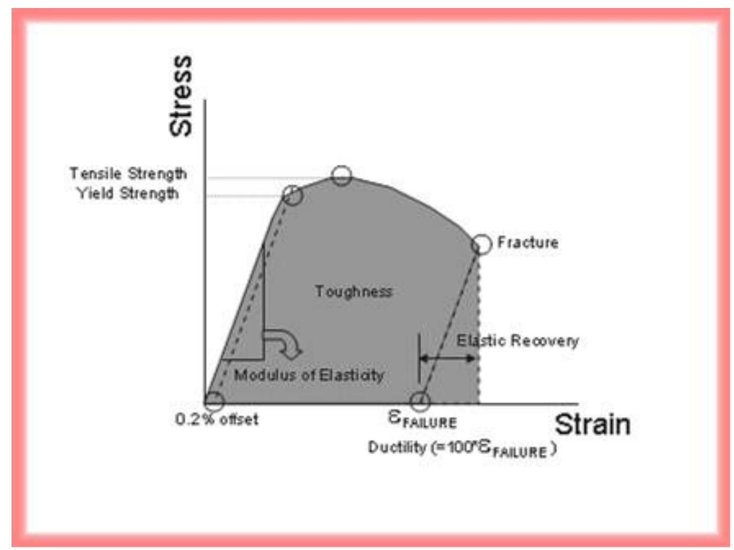
- Materials behavior (e.g. failure) depends on load ***nature*** its ***time*** schedule, and ***environmental conditions*** (e.g. temperature).

Tensile Stress-Strain Test



Conditions are defined by ASTM Standards:

- load-time schedule
- temperature
- sample shape



Strain-Stress Concept

To compare specimens of different sizes, the **load** is calculated **per unit area**.

Engineering tensile stress: $\sigma = F / A_o$

F is load applied perpendicular to the specimen cross section;
A_0 is the cross-sectional area **before** application of the load

- **Engineering tensile strain:** $\varepsilon = \Delta l / l_o$ **(× 100 %)**

These definitions of stress and strain allow to compare test results for specimens of **different** cross-sectional area A_0 and of **different initial** length l_0

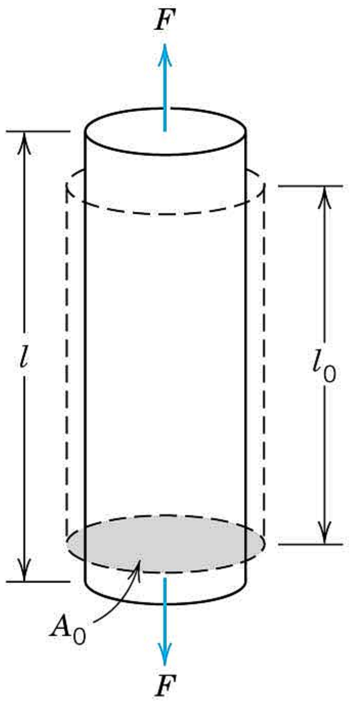
Stress and strain are positive for tensile loads and negative for compressive loads

Types of Deformation (1)

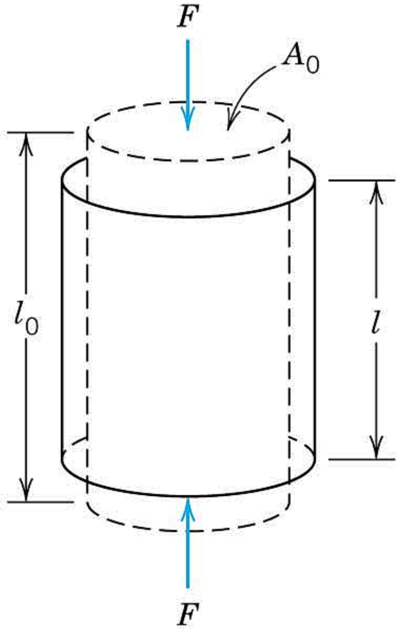
Types of loading are defined by the **direction of applied forces**

- **Elongation** – ***positive***
  linear ***strain***

- **Contraction** - ***negative***
  linear ***strain***



Forces applied ***normal*** to the sample surface



Tensile load

Compressive load

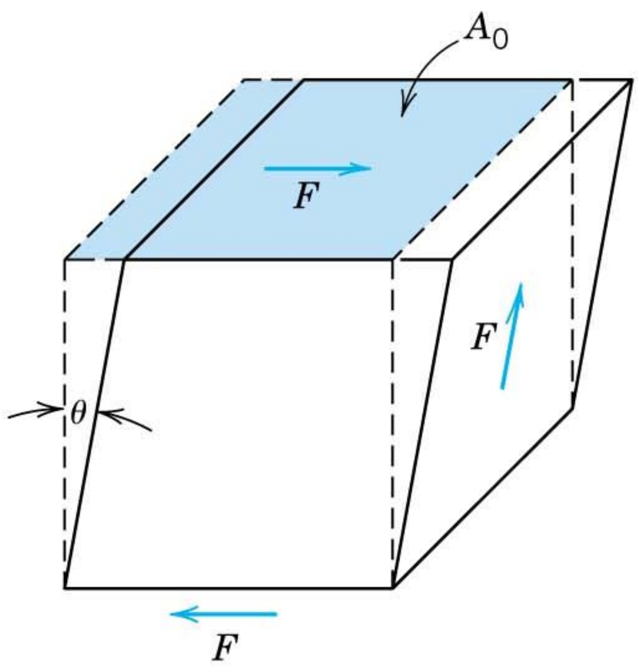
Types of Deformation (2)

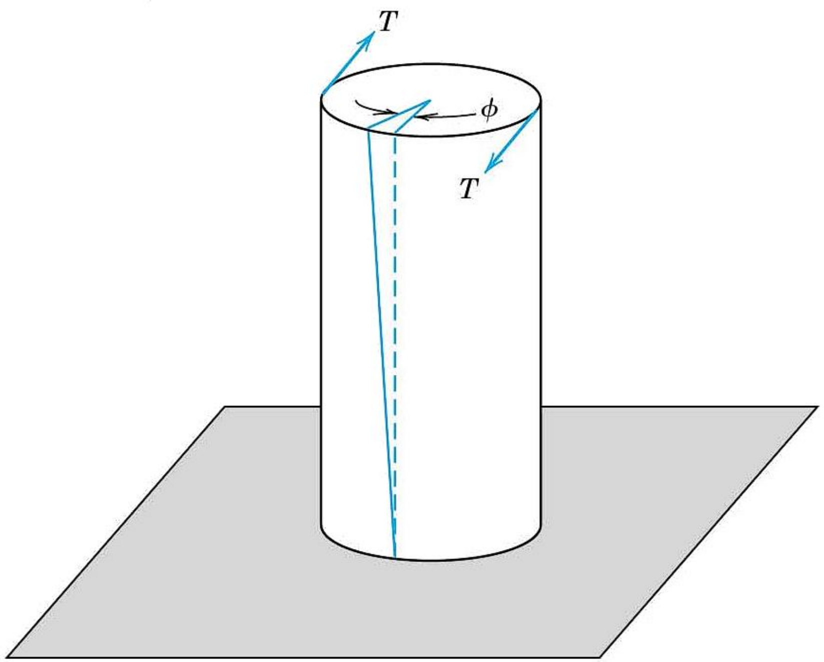
- Shear ***strain***

is ~ to the *strain* angle ***q***

- ***Torsional*** deformation

is ~ to the *twist* angle, f

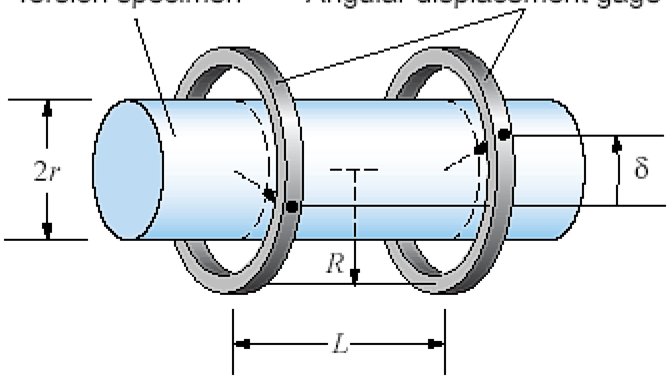




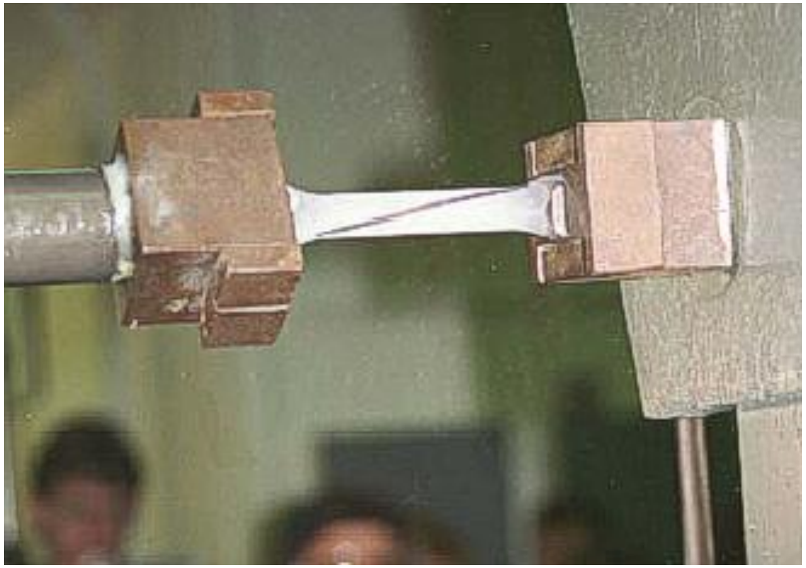
Pure shear load:
applied force is ***parallel***
to both sample faces

Torsion is a variation of pure shear
Torsion load produces a ***rotational motion***
(twist) around the axis of symmetry

Torsion Test



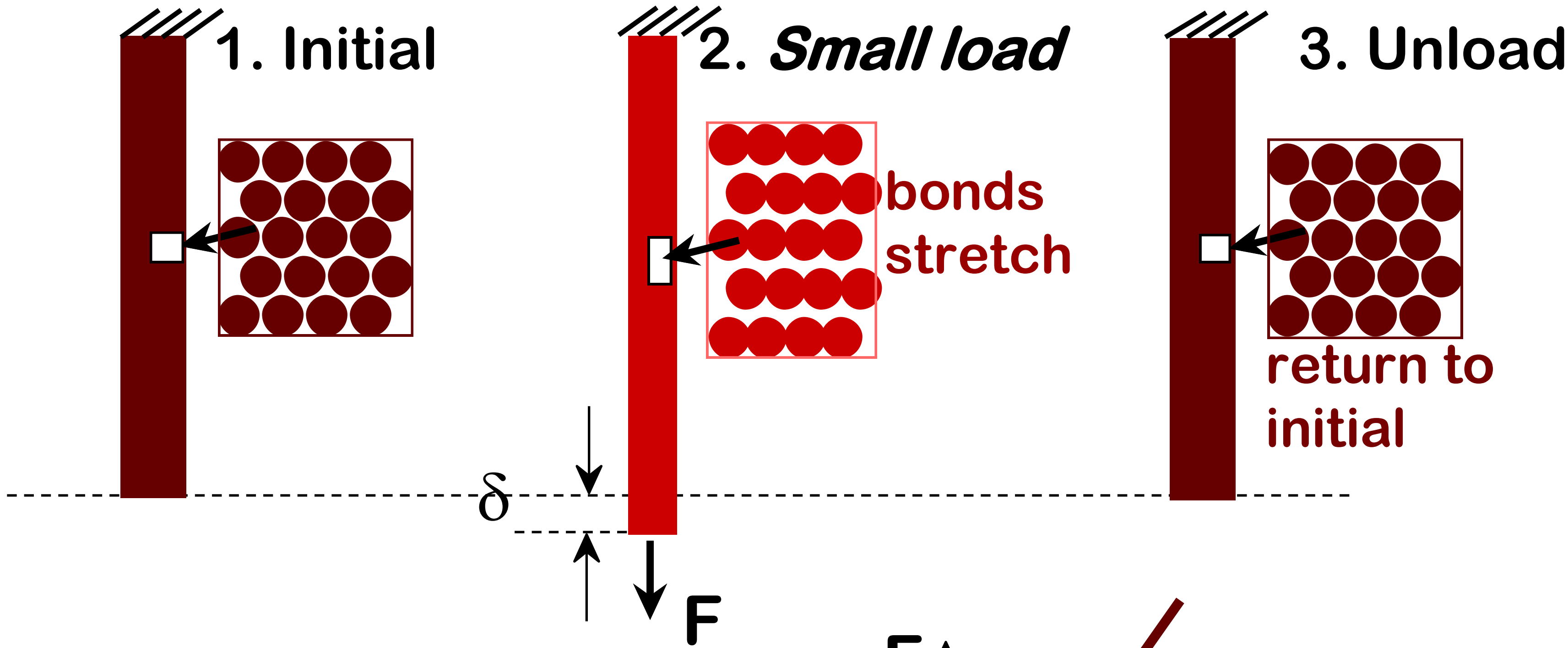
Geometry of angular displacement gage.



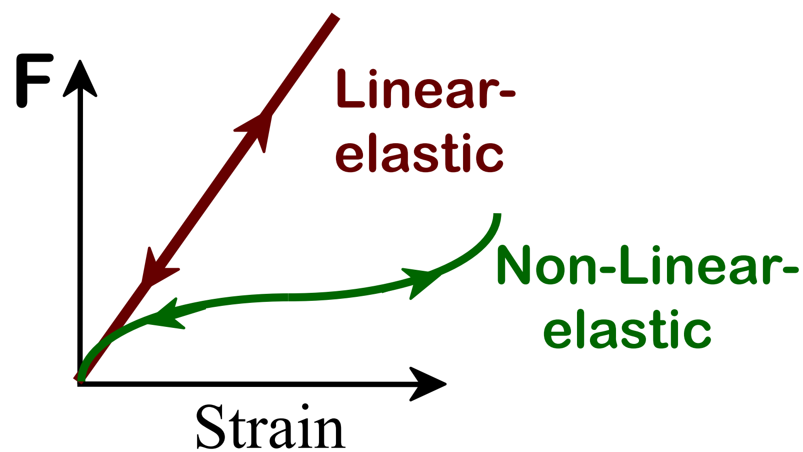
Angular Displacement Gage to measure the relative angle of rotation

The torsion machine is equipped with a linear variable-differential transformer to measure the twisting moment applied to the specimen

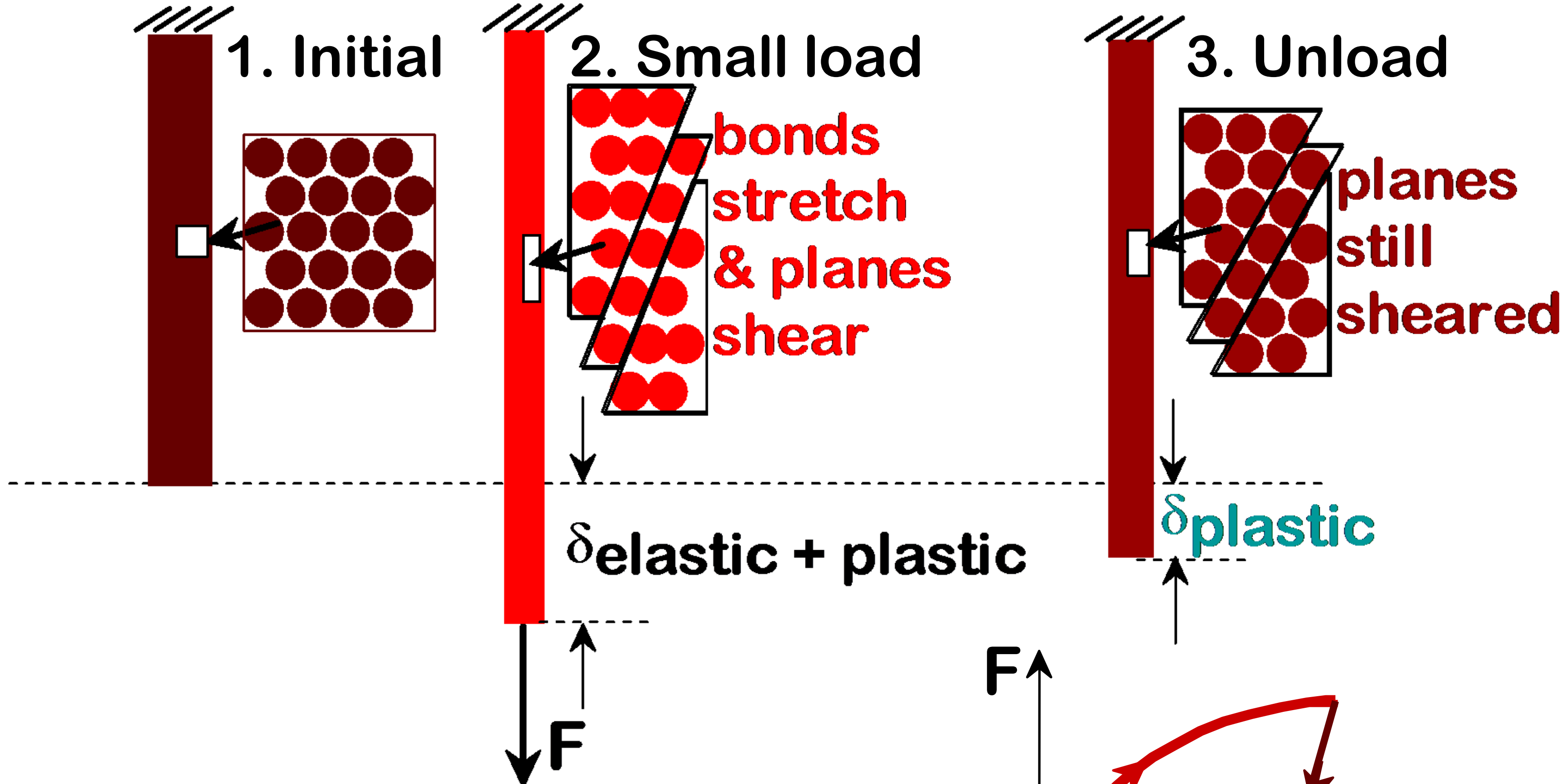
ELASTIC DEFORMATION



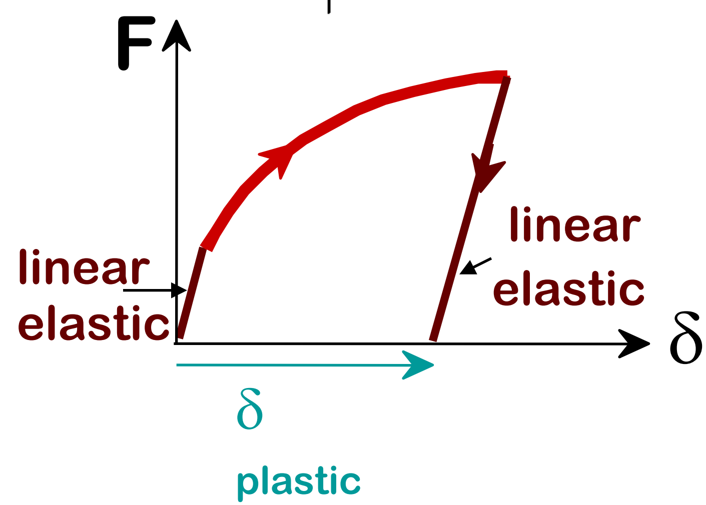
Elastic means reversible!



PLASTIC DEFORMATION (METALS)

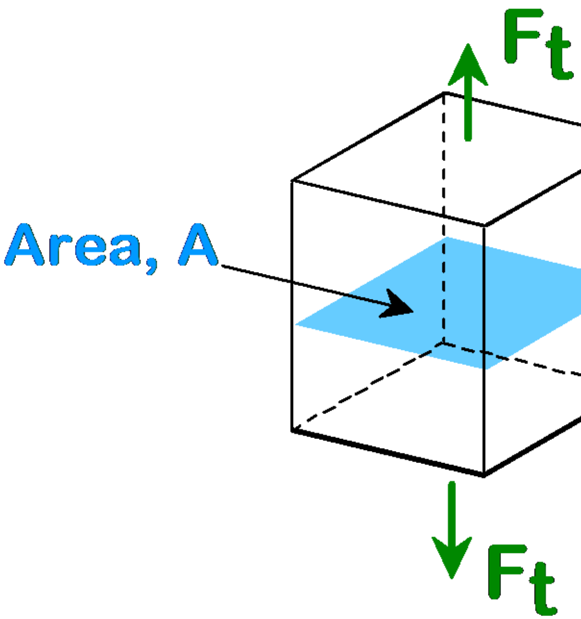


Plastic means permanent!



ENGINEERING STRESS

- **Tensile stress, σ:**

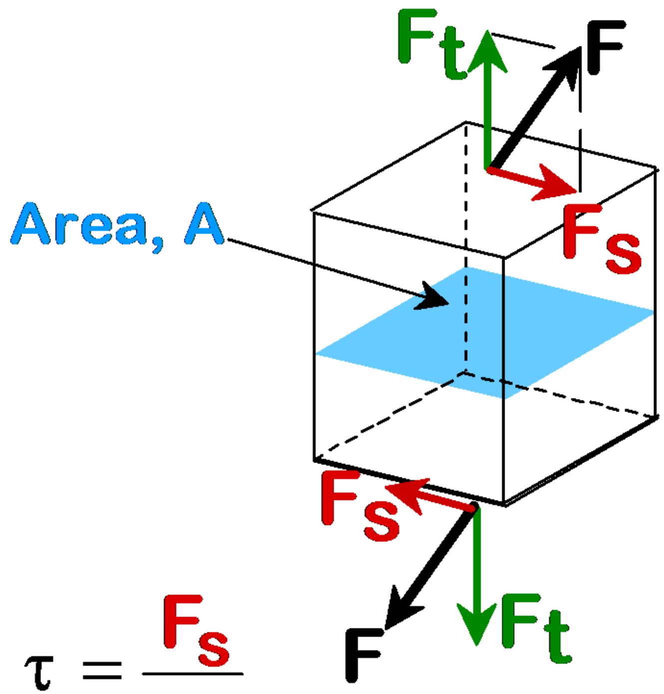


$$\sigma = \frac{F_t}{A_o}$$

original area before loading

Load applied *perpendicular* to the specimen cross section

- **Shear stress, τ:**



$$\tau = \frac{F_s}{A_o}$$

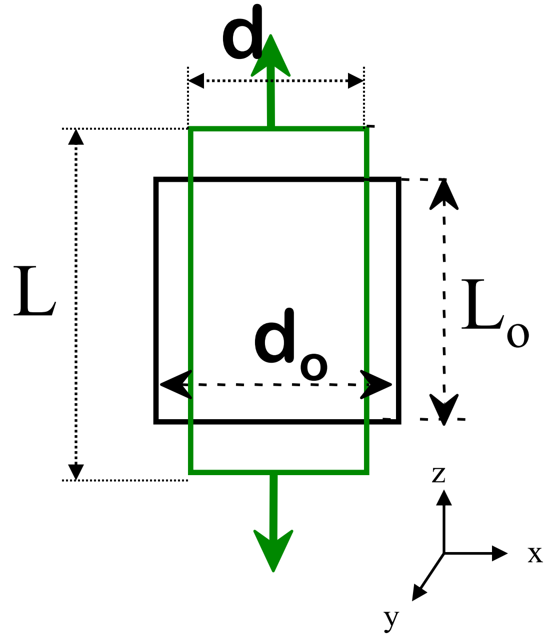
F_s is a load constituent *parallel* to the specimen cross section

Stress has units: N/m^2 or lb/in^2

ENGINEERING STRAIN

- **Tensile strain:**

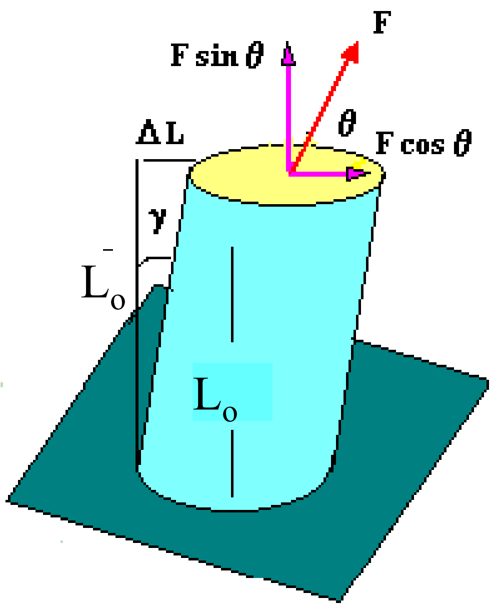
$$\varepsilon_z = \frac{L - L_o}{L_o} \times 100\%$$



- **Lateral strain:**

$$\varepsilon_{x,y} = \frac{d - d_0}{d_0} \times 100\%$$

- **Shear strain:**

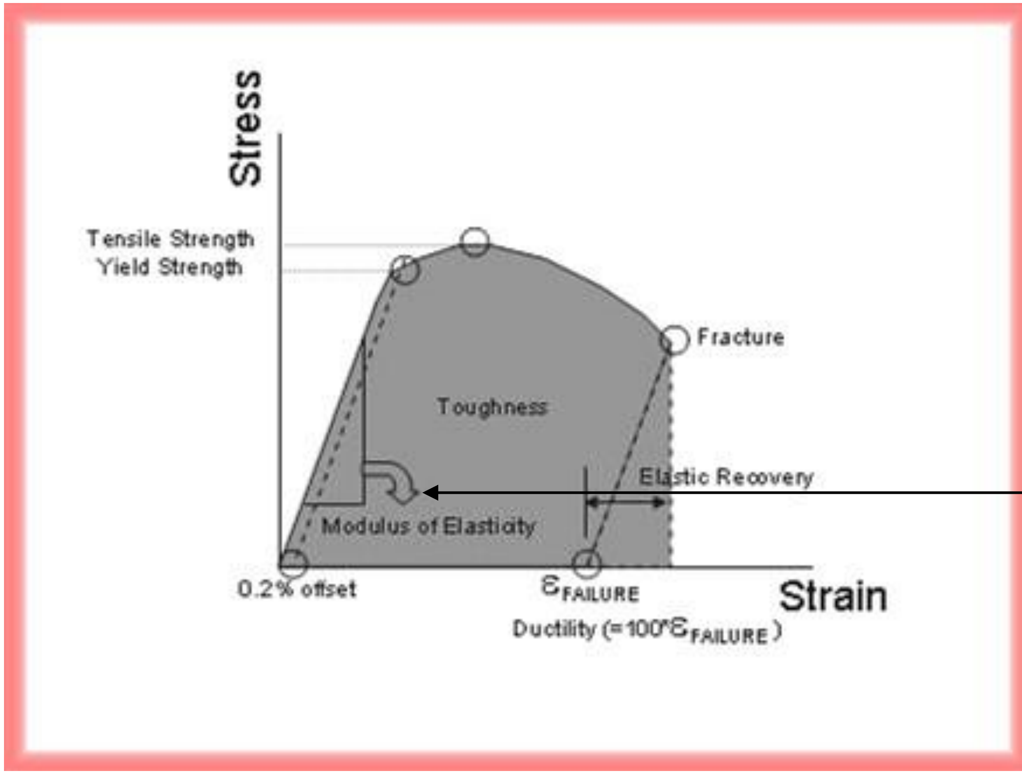


$$\gamma = \frac{\Delta L}{L_o} \times 100\% = \tan \gamma \times 100\%$$

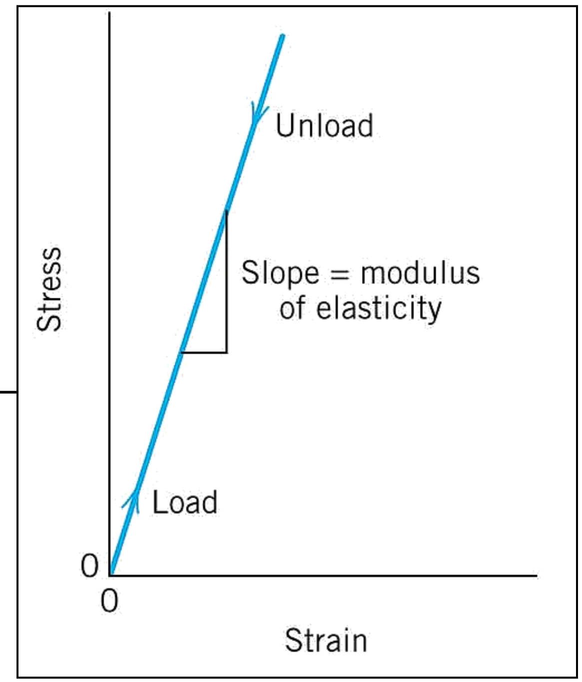
Strain is always dimensionless

Stress Versus Strain: Elastic Deformation

Typical Stress-Strain Diagram
for one-dimensional tensile test



Elastic Region

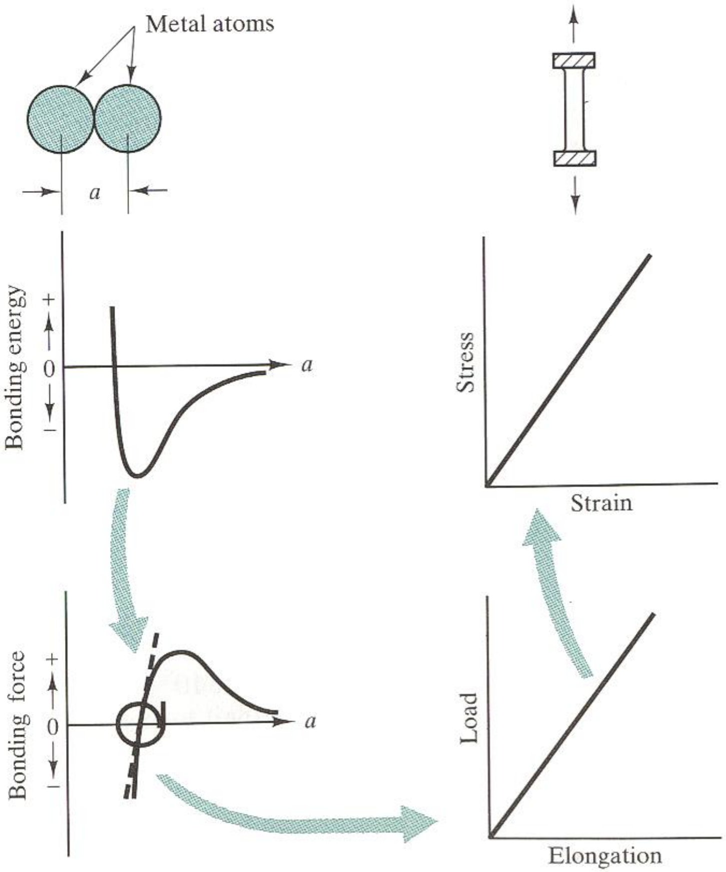


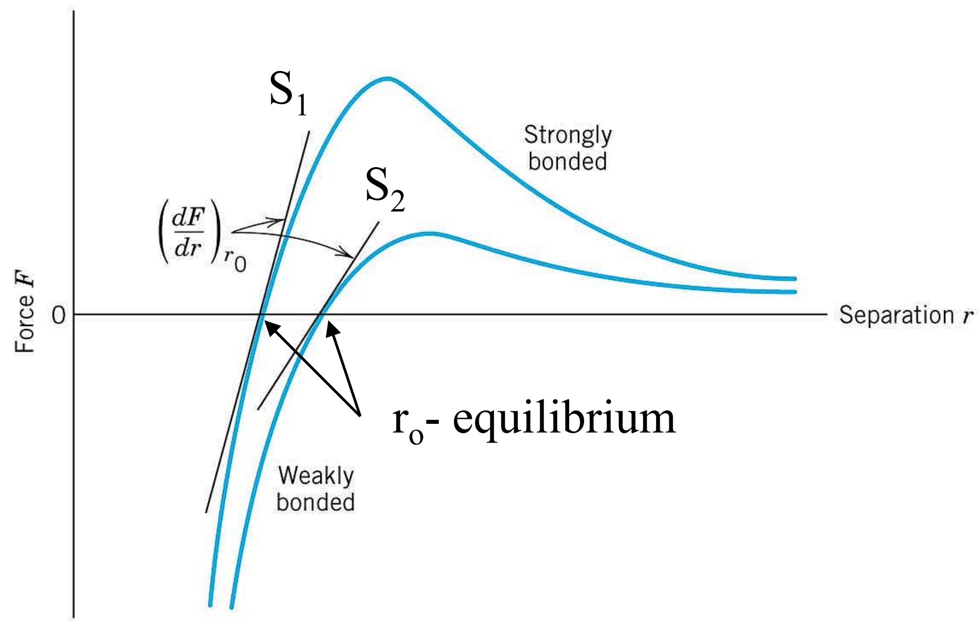
Hooke's Law: $\sigma = E\,\varepsilon$

E [N/m²; GPa] is **Young's modulus** or **modulus of elasticity**

Reminder:

Atomic Mechanism of Elastic Deformation



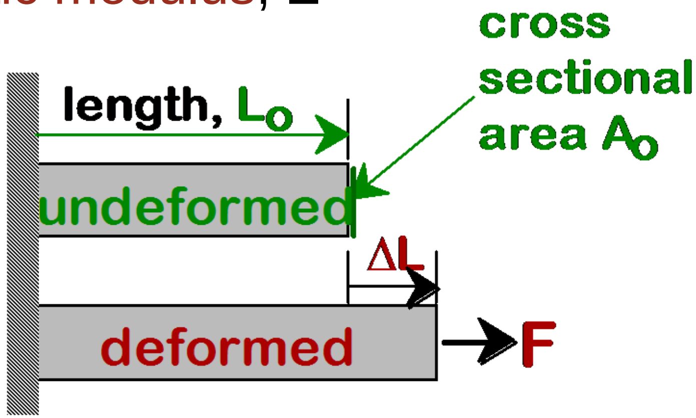


$$E=1/r_o \cdot (dF/dr)_{r_o}$$

Weaker bonds – the atoms easily move out from equilibrium position

Reminder: PROPERTIES FROM BONDING

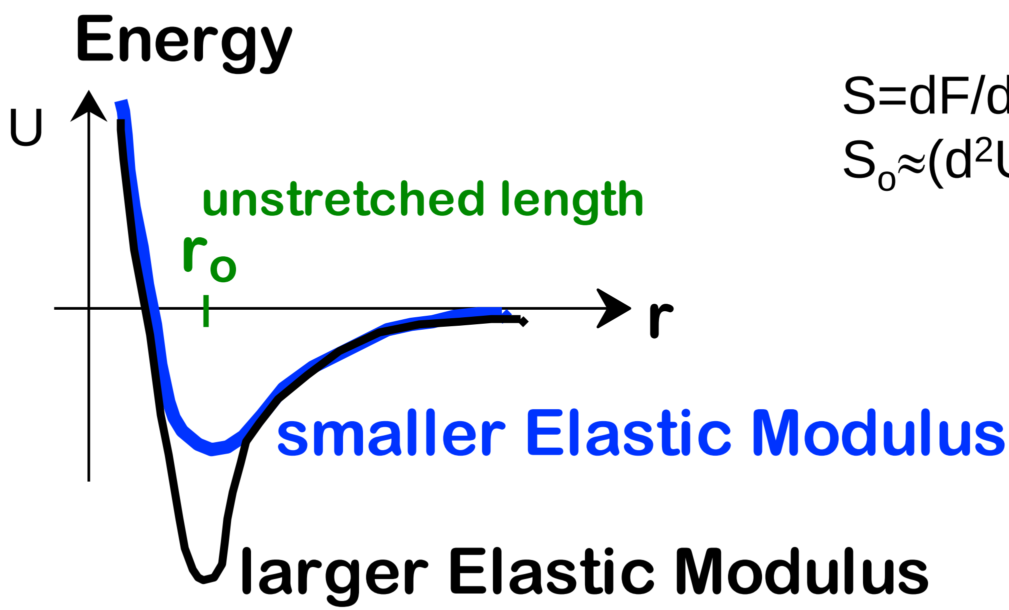
- Elastic modulus, E



Elastic modulus

$$\frac{F}{A_o} = E\,\frac{\Delta L}{L_o}$$

- E ~ curvature of U(r) at r_o



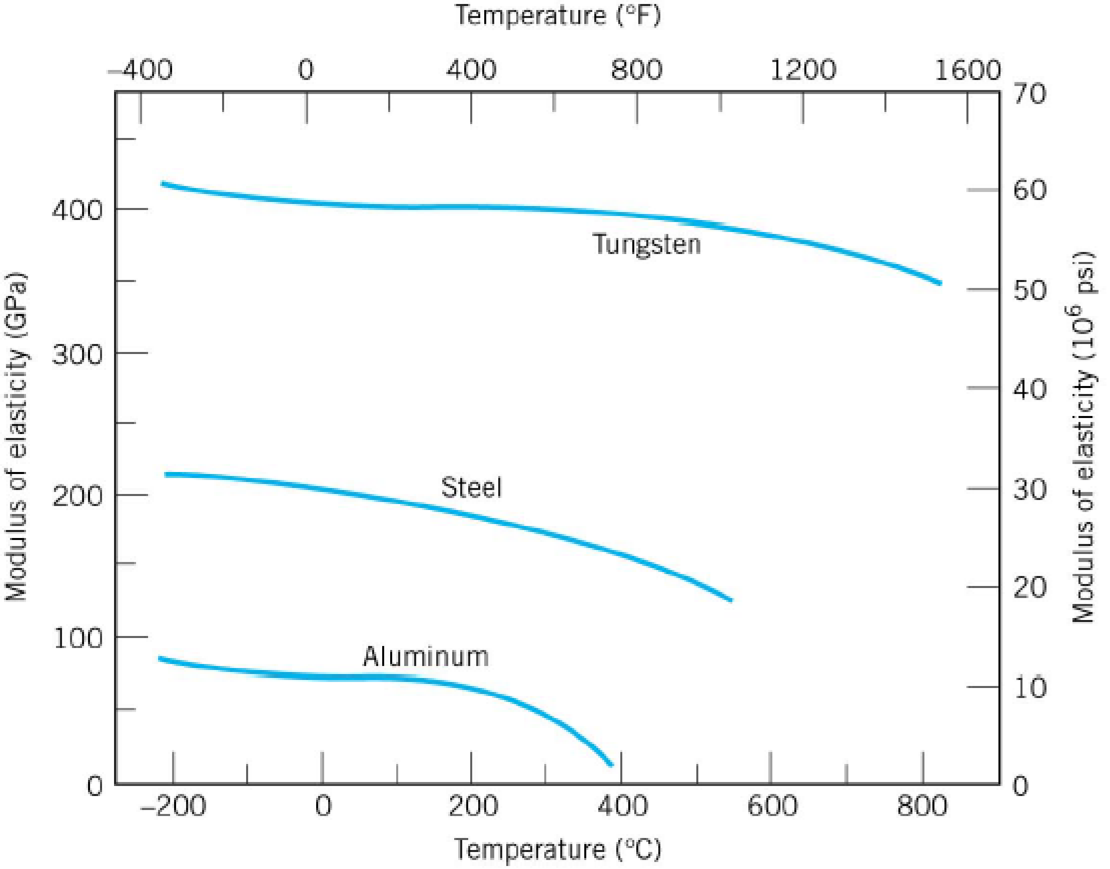
The **"stiffness"** of the bond is given by:

$S = dF/dr = d^2U/dr^2$

$S_o \approx (d^2U/dr^2\ )_{ro}$

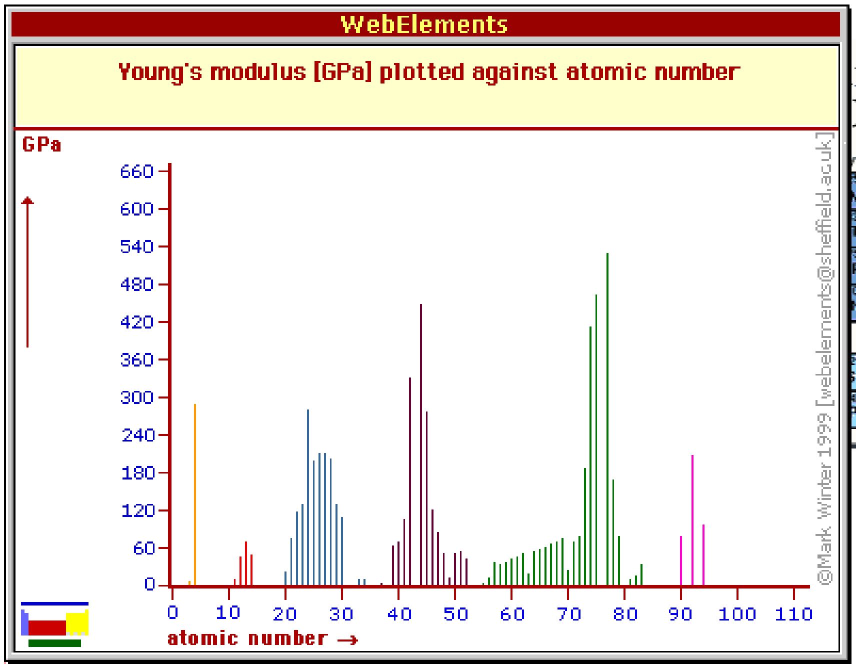
$E = (S_o/r_0)$

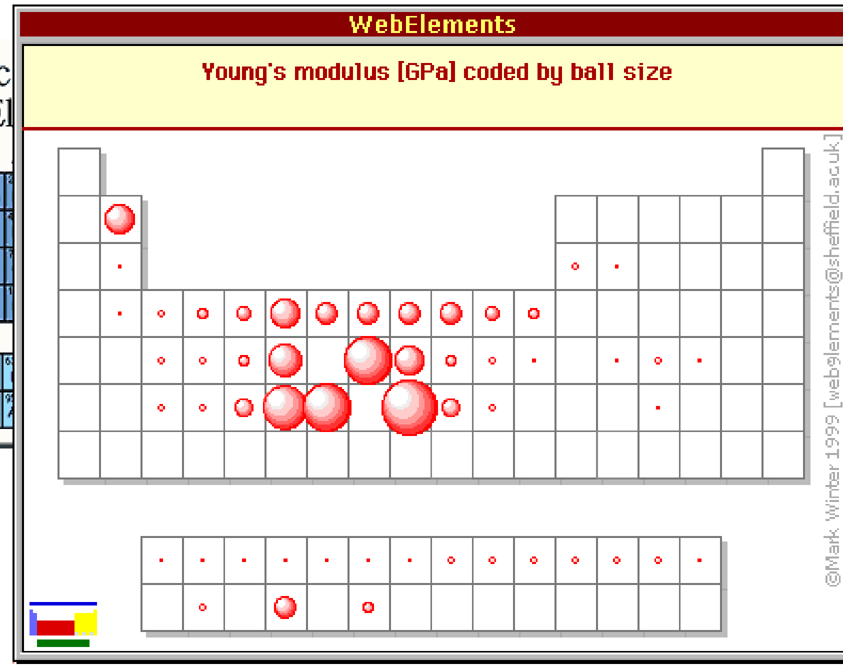
Modulus of Elasticity for Different Metals



Young's modulus

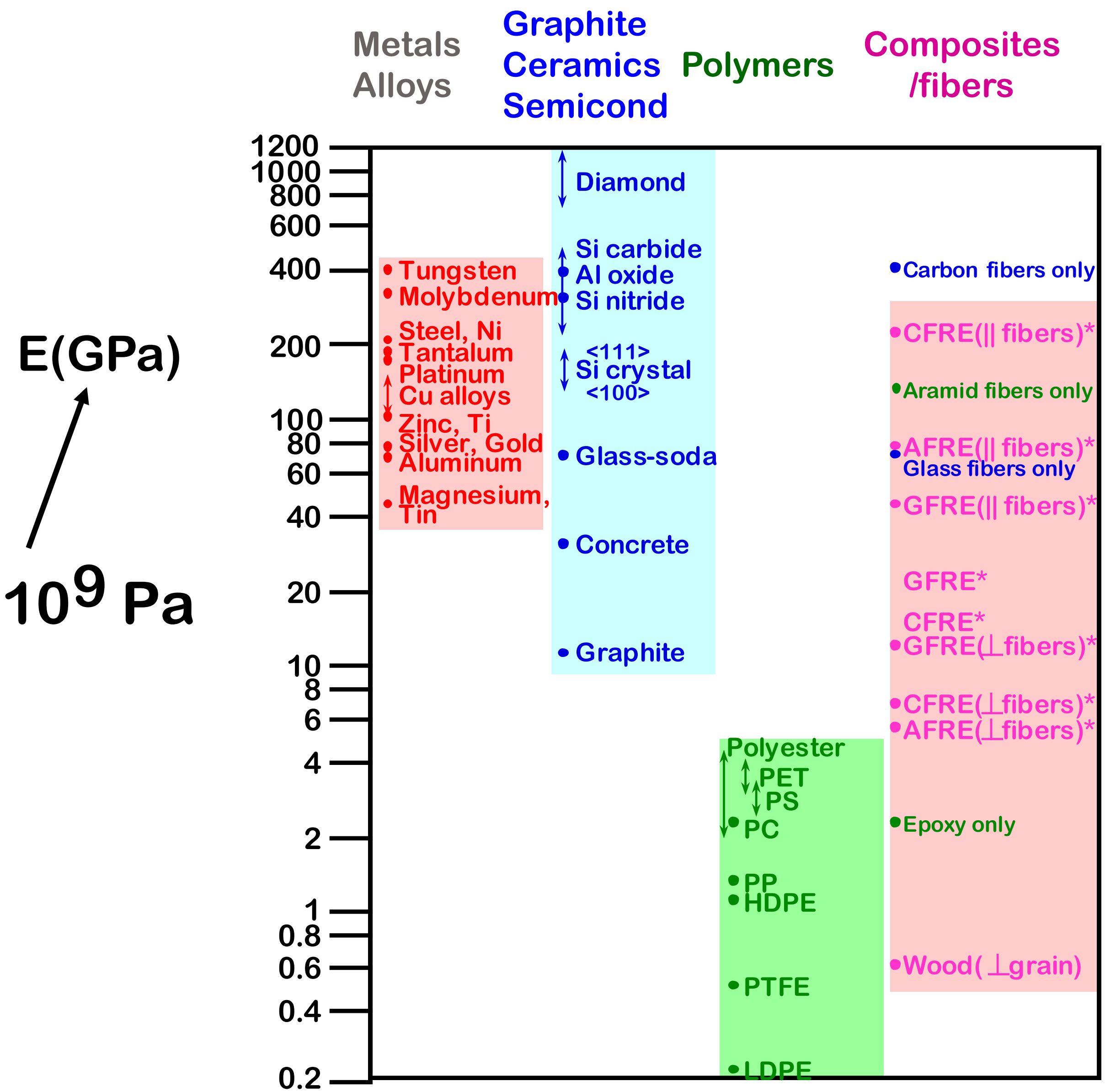
Young's modulus is a numerical constant, named for the 18th-century English physician and physicist **Thomas Young**, that describes the elastic properties of a solid undergoing tension or compression in only one direction.





Higher E – higher "stiffness"

YOUNG'S MODULI: COMPARISON



$E_{ceramics}$
$> E_{metals}$
$>> E_{polymers}$

Stress Versus Strain:
Constitutive Relations

Cauchy generalized Hooke's law to three dimensional elastic bodies and stated that the 6 components of stress are linearly related to the 6 components of strain. The stress-strain relationship written in matrix form, where the 6 components of stress and strain are organized into column vectors:

$$\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \varepsilon_{yz}\\ \varepsilon_{zx}\\ \varepsilon_{xy}\end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16}\\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26}\\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36}\\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46}\\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56}\\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66}\end{bmatrix}\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{yz}\\ \sigma_{zx}\\ \sigma_{xy}\end{bmatrix}$$

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{yz}\\ \sigma_{zx}\\ \sigma_{xy}\end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16}\\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26}\\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36}\\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46}\\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56}\\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \varepsilon_{yz}\\ \varepsilon_{zx}\\ \varepsilon_{xy}\end{bmatrix}$$

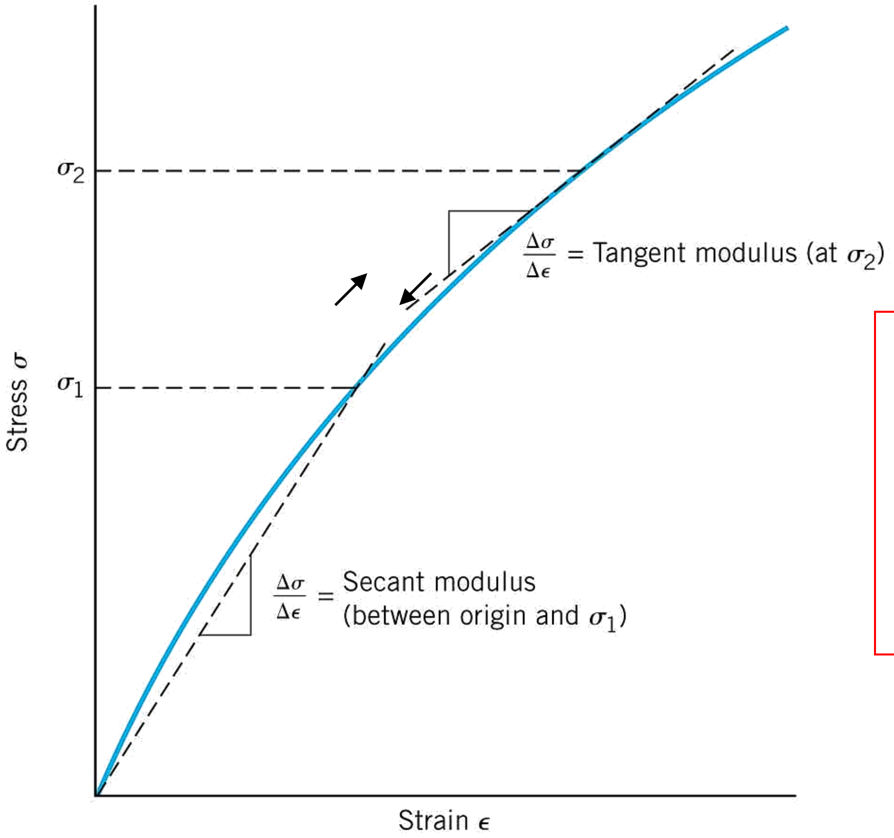
$$\boldsymbol{\varepsilon} = \mathbf{S}\cdot\boldsymbol{\sigma}$$

$$\boldsymbol{\sigma} = \mathbf{C}\cdot\boldsymbol{\varepsilon}$$

where **C** is the **stiffness matrix**, **S** is the **compliance matrix** ($\mathbf{S} = \mathbf{C}^{-1}$)

There are 36 stiffness matrix components. However, the conservative materials possess a strain energy density function and as a result, the stiffness and compliance matrices are symmetric and thus only 21 stiffness components are actually independent in Hooke's law.

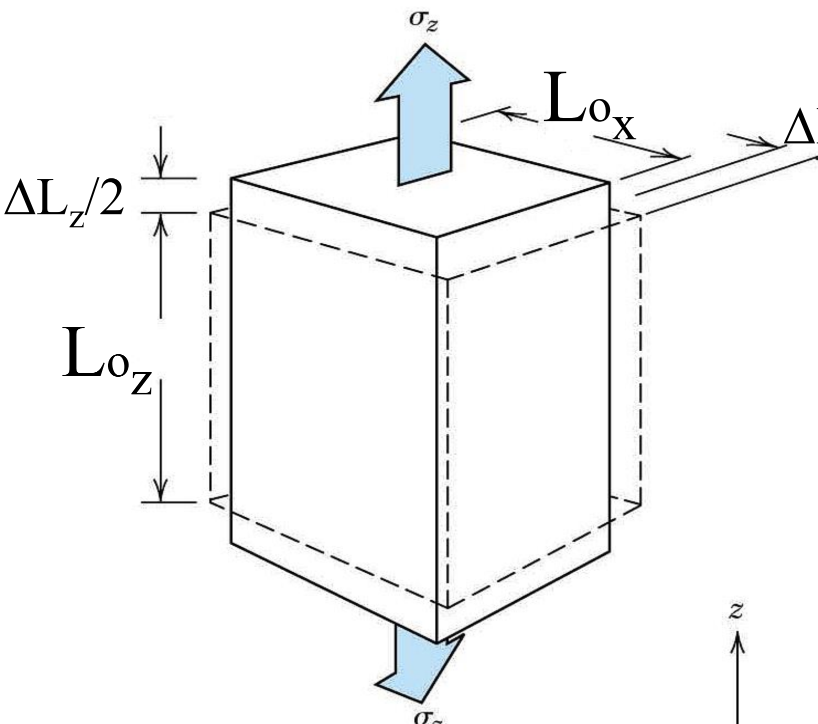
Elastic Deformation: Non-linear Behavior



For some materials (typically for polymers, also concrete) deformation is ***not linear*** even on the initial stage of S-S diagram, BUT still ***reversible***, i.e. **elastic!!**

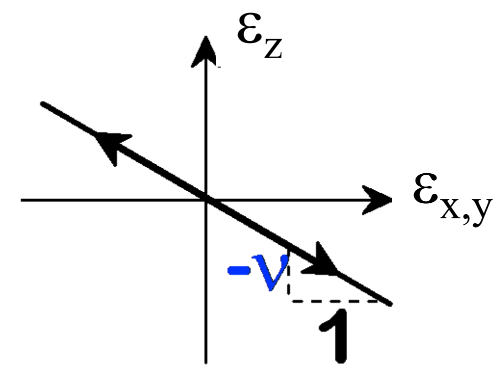
Elastic Deformation: Poisson's Ratio

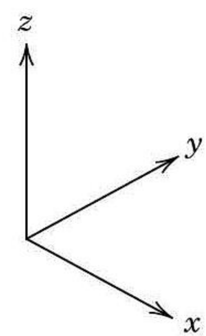
Materials subject to tension (compression) also **shrink (bulge) laterally**.



$\varepsilon_z = \Delta L_z / Lo_z$

$\varepsilon_{x,y} = \Delta L_{x,y} / Lo_{x,y}$





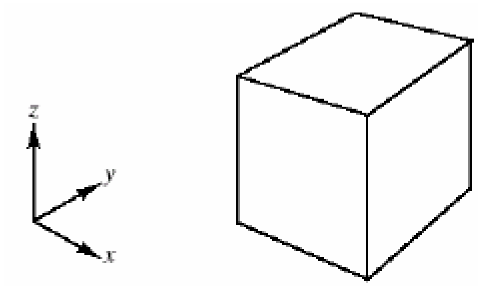
For anisotropic material
ν=**0.25**
Typical value:
Metals ~0.28-034
Ceramics ~0.25
Polymers ~0.40

Poisson's Ratio:

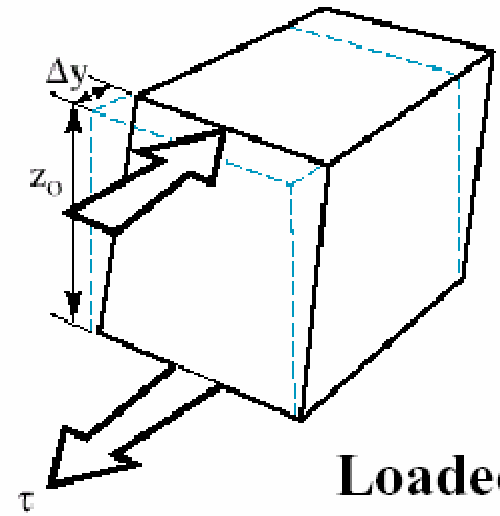
$$\nu = -\varepsilon_x/\varepsilon_z = -\varepsilon_y/\varepsilon_z$$

By definition ν is a positive dimensionless material characteristic, while $\varepsilon_{x,y}$ and ε_z are always appositive

Elastic Deformation: Shear Modulus



Unloaded



Loaded

Relationship of shear stress to shear strain:

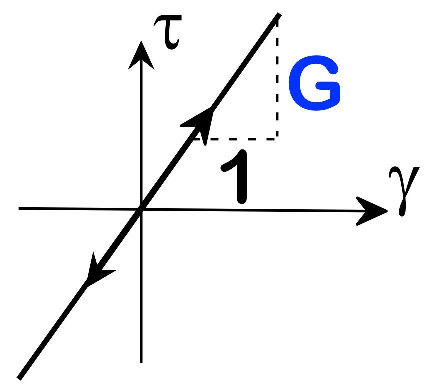
$$\tau = G\,\gamma,$$

where: $\gamma = \tan\theta = \Delta y / z_o$ and **G** is Shear Modulus (Units: N/m^2)

For isotropic material:

$$E = 2G(1+\nu) > G \sim 0.4E$$

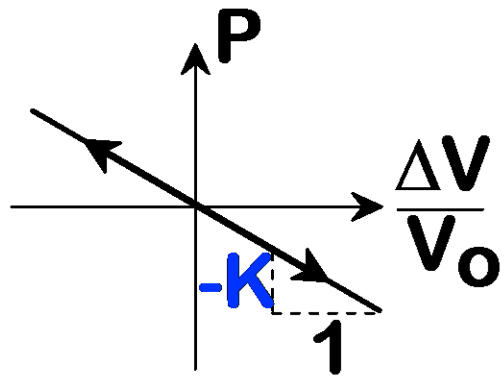
(Note: most materials are elastically anisotropic: the elastic behavior varies with crystallographic direction, see Chapter 3)

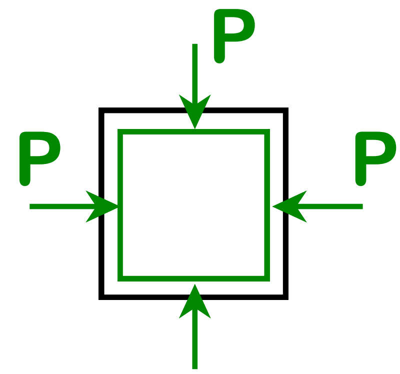


Elastic Deformation: Bulk Modulus

- Elastic **Bulk modulus, K**:

$$P = -K\frac{\Delta V}{V_o}$$



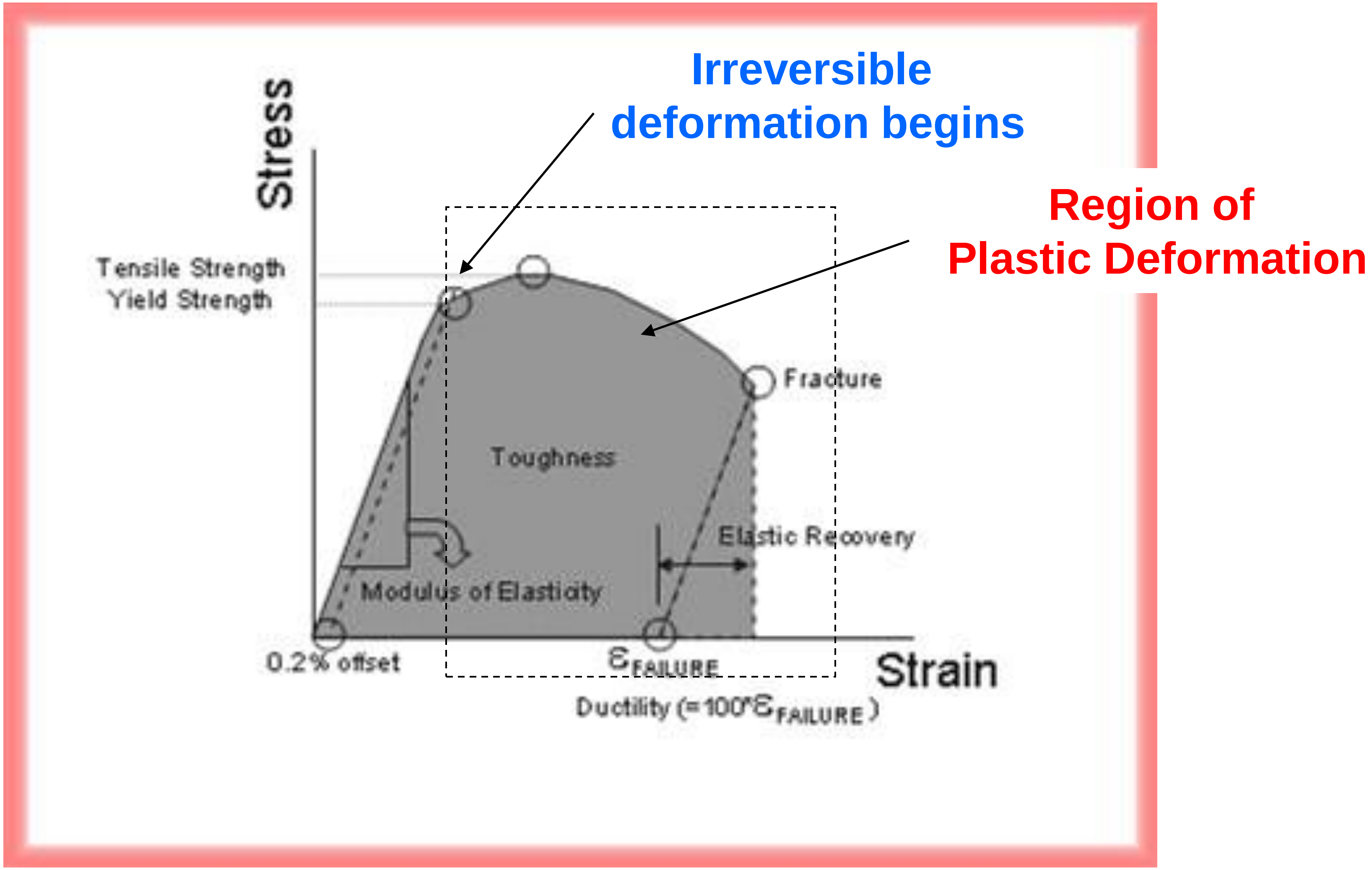


Pressure test:
V_o- initial volume
ΔV-vol. change

- **Special relations for isotropic materials:**

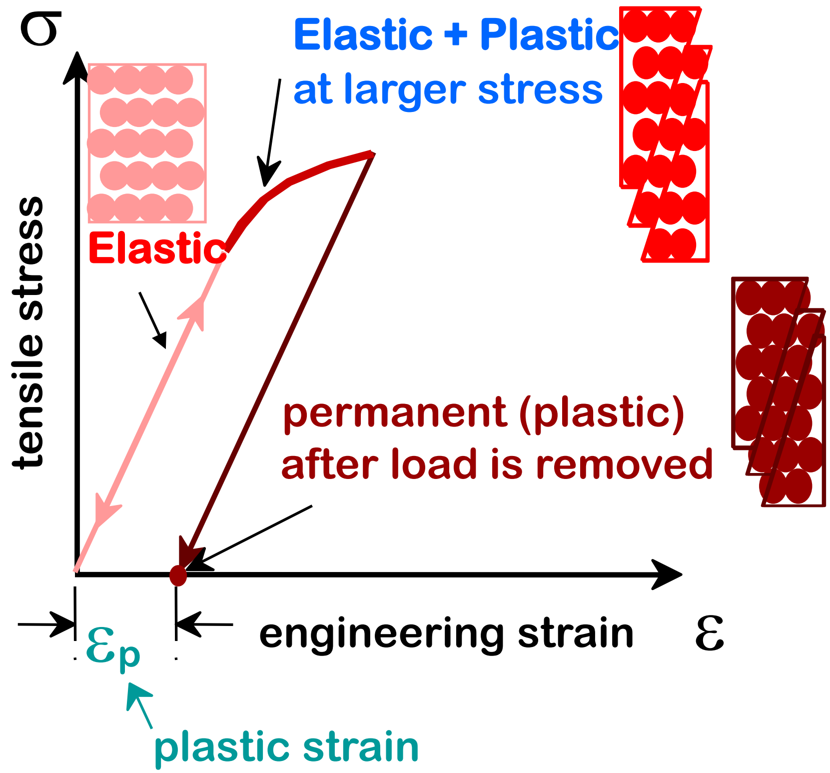
$$K = \frac{E}{3(1-2\nu)}$$

Stress Versus Strain: Plastic Deformation



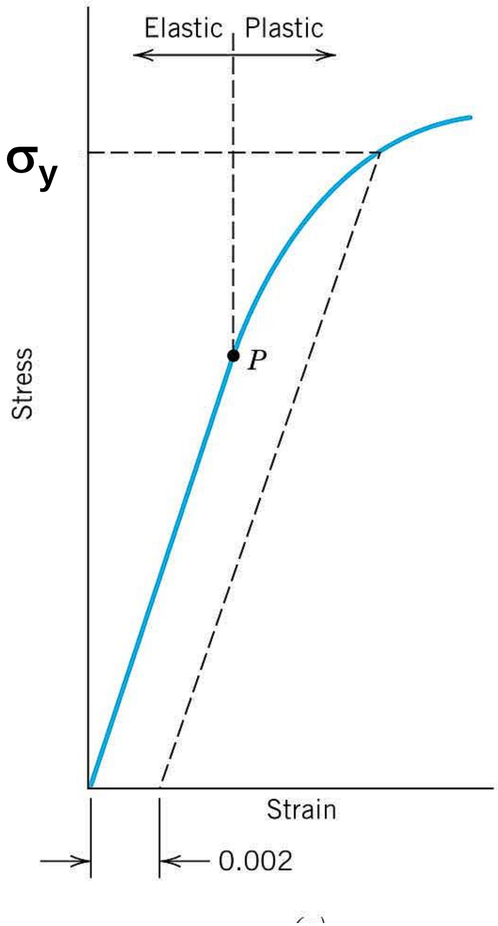
PLASTIC (PERMANENT) DEFORMATION

- For most metals elastic deformation persists to strain only of ~0.5%
- Beyond some critical point permanent non-recoverable deformation, i.e. ***yielding***, occurs
- on the atomic levels yielding means ***breaking bonds*** with original atoms
- yielding ***mechanisms*** are different for *crystalline* (Chapter 7) and *amorphous* (Chapter 12) materials



YIELD STRENGTH, σ_y (1)

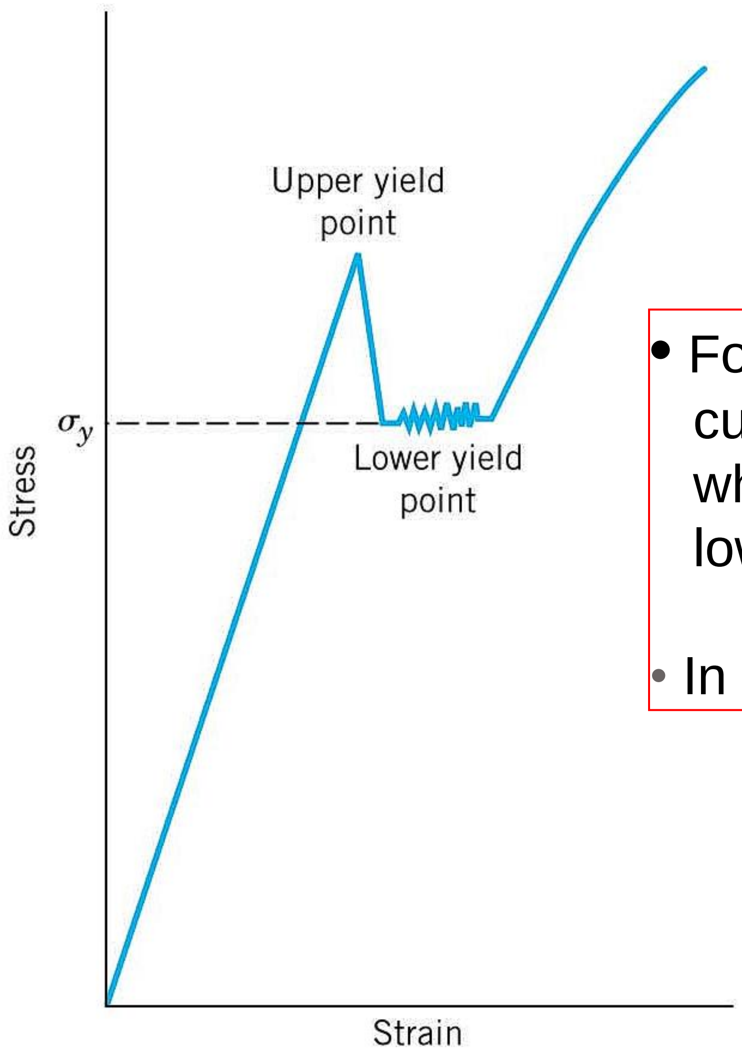
• σ_y is a stress at which ***noticeable*** (ε_p = 0.2 % ; by convention) plastic (permanent) strain has occurred.



• The yield stress is a measure of resistance to plastic deformation

• ***Proportional limit*** (**yield point P**) – S-S curve starts to deviate from linearity

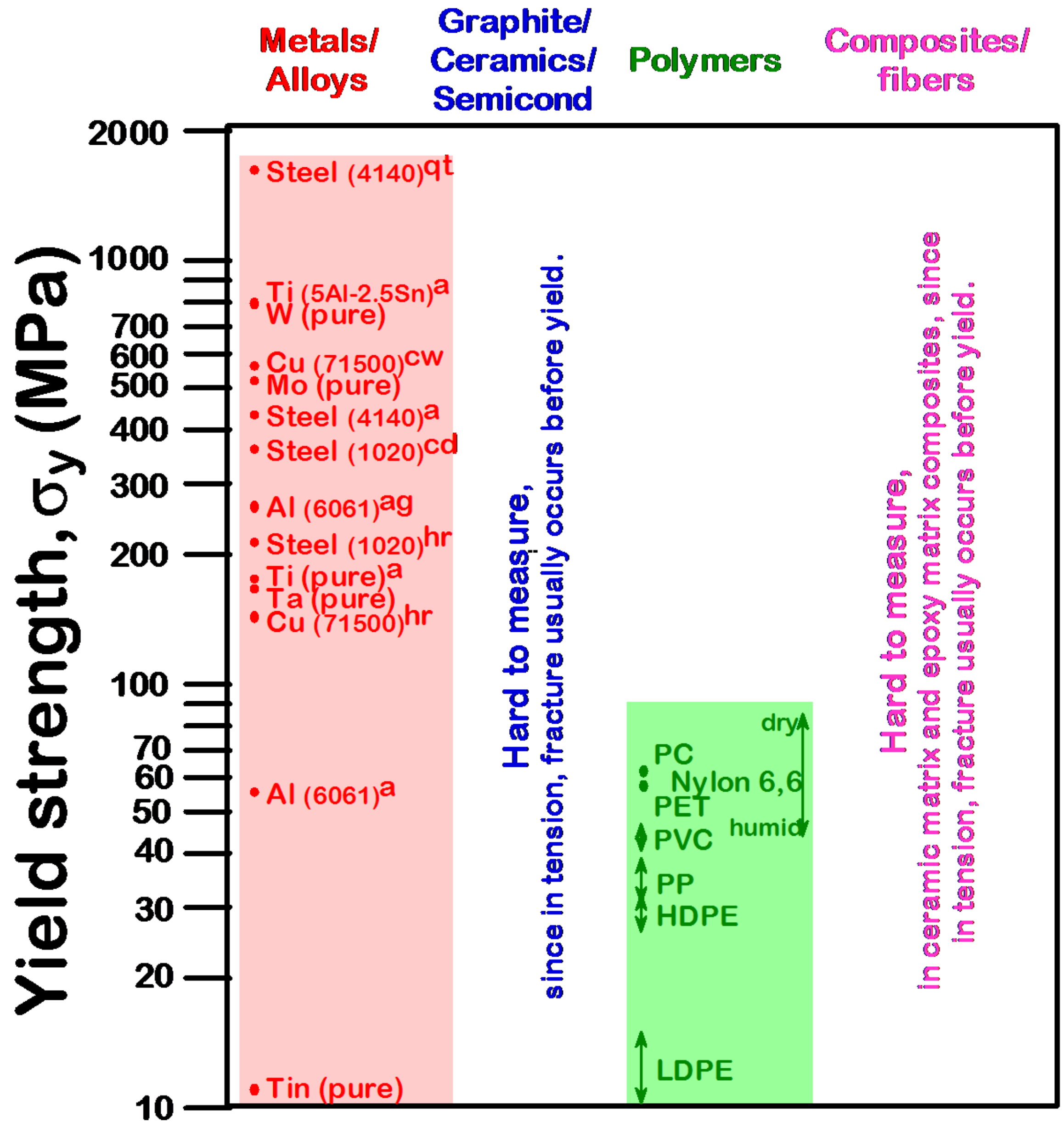
YIELD STRENGTH, σ_y (2)



- For some steels (low carbon) the S-S curve posses a ***yield point phenomenon*****,** which is characterized by an upper and lower yield points.

- In this case $\boldsymbol{\sigma_y}$ is taken at the ***lower yield*** point.

YIELD STRENGTH: COMPARISON



σ_y(ceramics)

>> σ_y(metals)

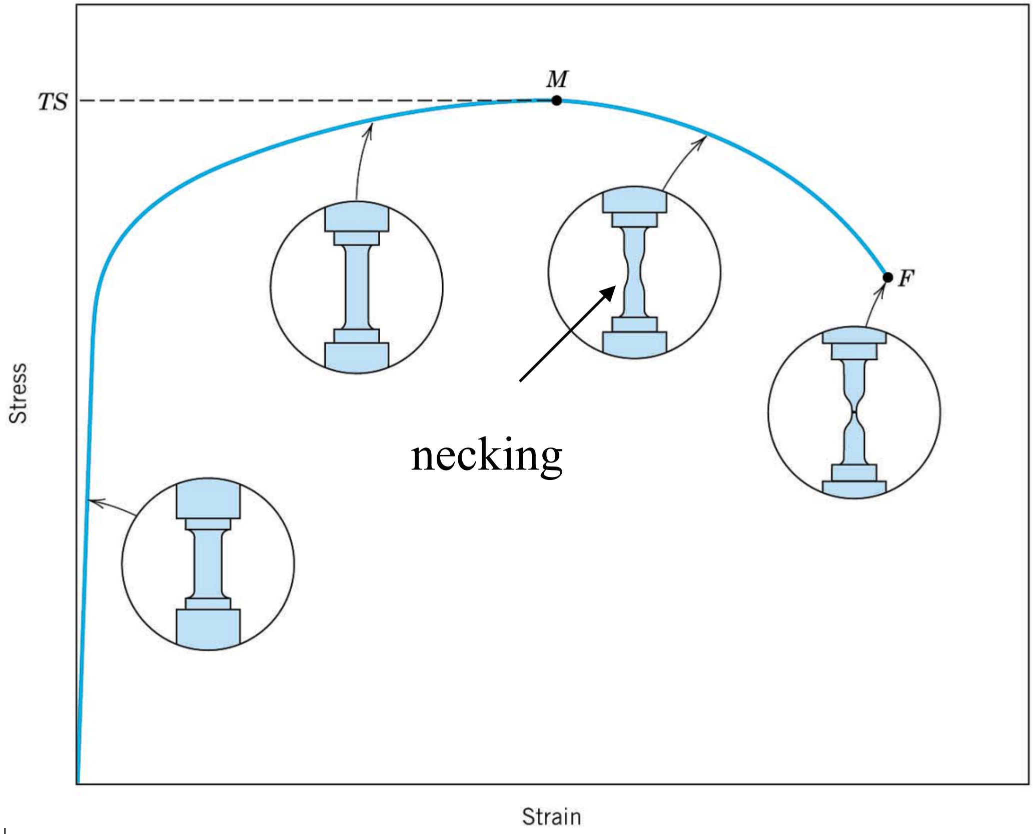
>> σ_y(polymers)

Room T values

a = annealed
hr = hot rolled
ag = aged
cd = cold drawn
cw = cold worked
qt = quenched & tempered

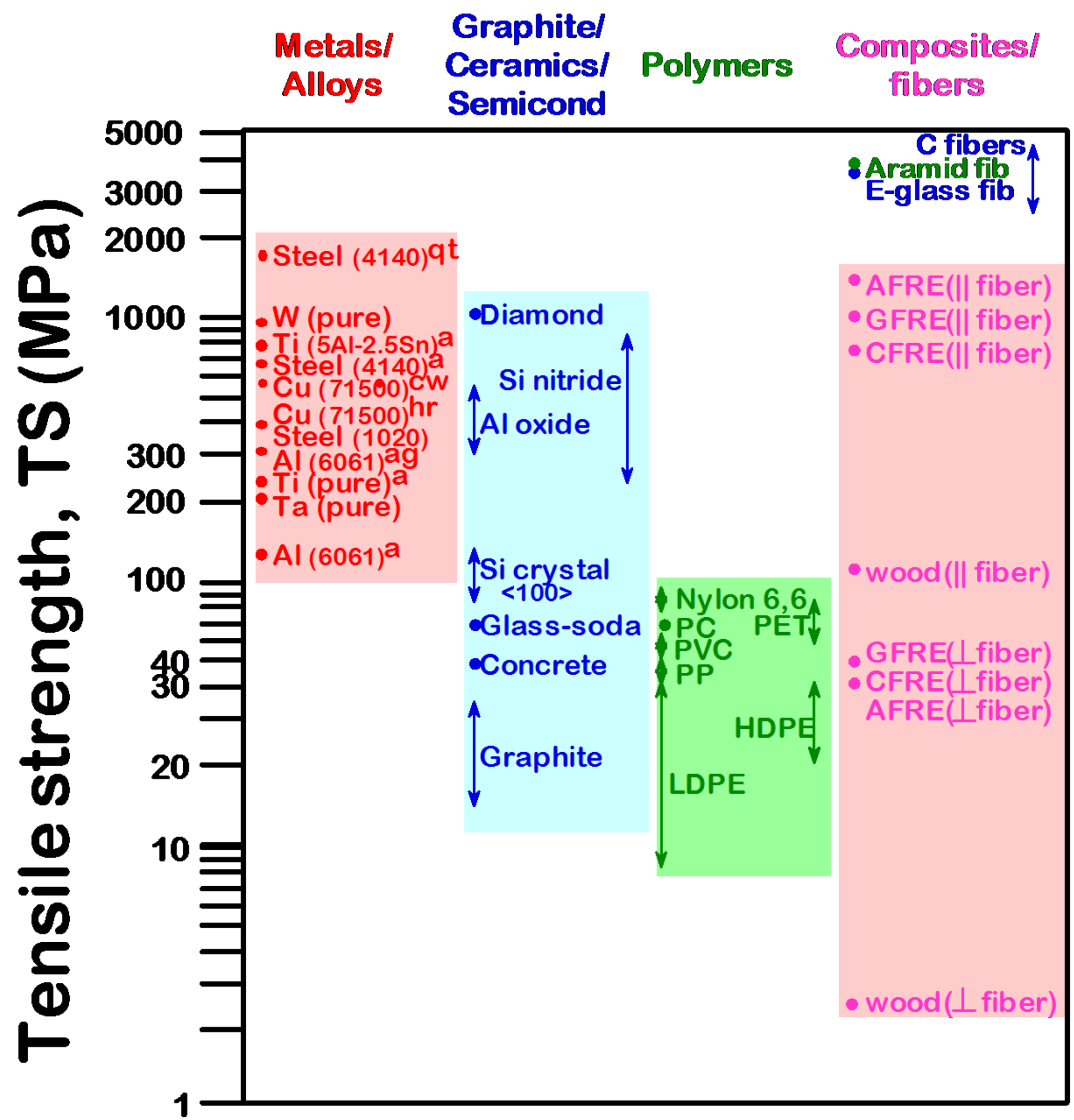
Tensile Strength, TS

- **Maximum possible engineering stress in tension (point M).**



- Metals: occurs when noticeable **necking** starts.
- Ceramics: occurs when **crack propagation** starts.
- Polymers: occurs when **polymer backbones** are aligned and about to break.

TENSILE STRENGTH: COMPARISON



$TS_{(ceram)}$

$\sim TS_{(met)}$

$\sim TS_{(comp)}$

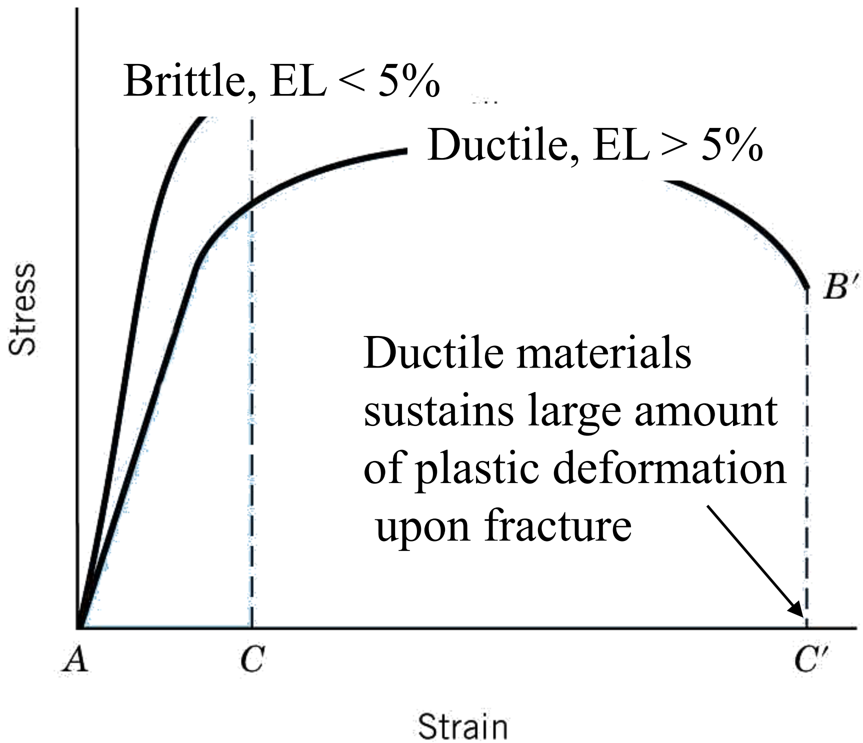
$\gg TS_{(poly)}$

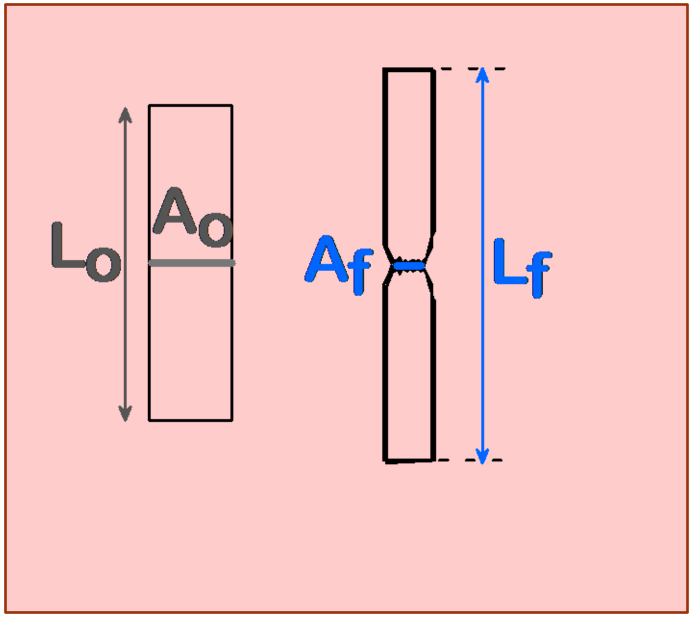
Room T values

a = annealed
hr = hot rolled
ag = aged
cd = cold drawn
cw = cold worked
qt = quenched & tempered

AFRE, GFRE, & CFRE = aramid, glass, & carbon fiber-reinforced epoxy composites, with 60 vol% fibers.

DUCTILITY





- **Plastic tensile strain at failure:**

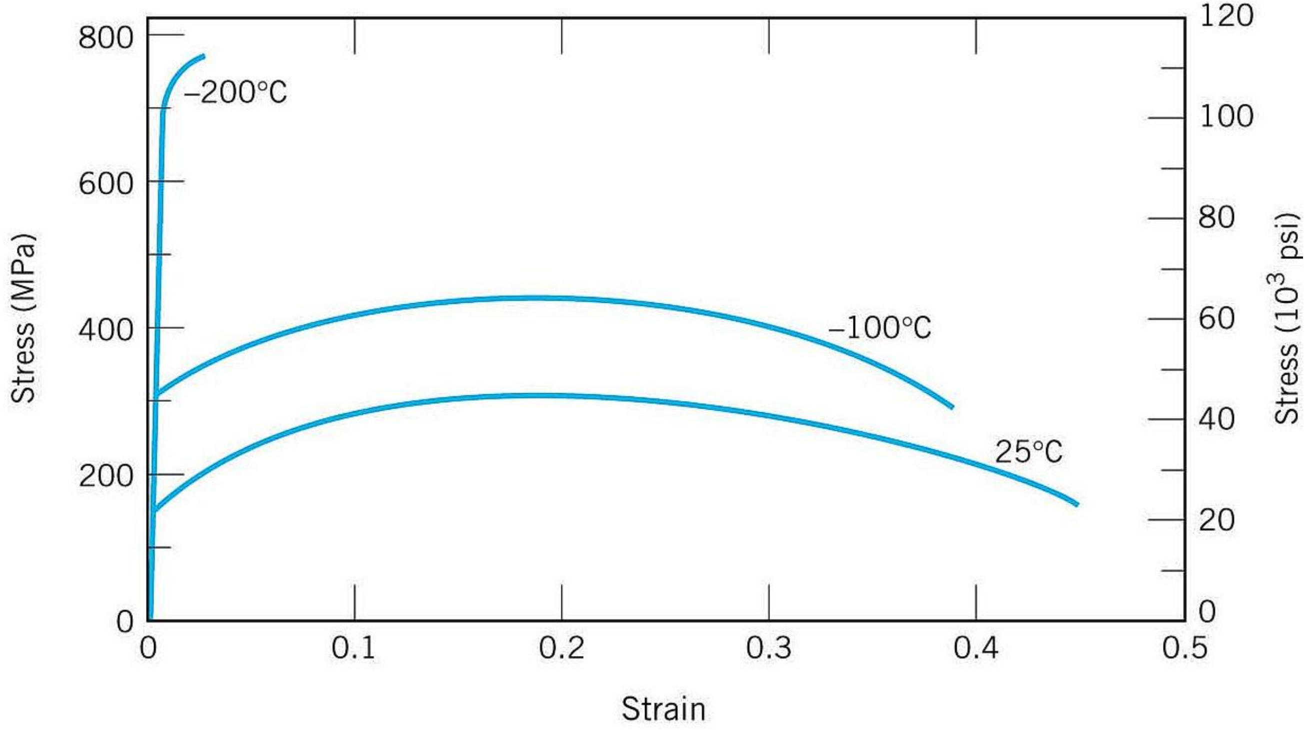
$$\%EL = \frac{L_f - L_o}{L_o} x100$$

- **Another ductility measure:**

$$\%AR = \frac{A_o - A_f}{A_o} x100$$

- Note: %AR and %EL are often comparable, because crystal slip does not change material volume. However, if internal voids form in neck. %AR > %EL possible

Temperature Effect on the S-S Diagram



σ_y, TS and **E** decrease with increasing temperature, while **ductility** increases

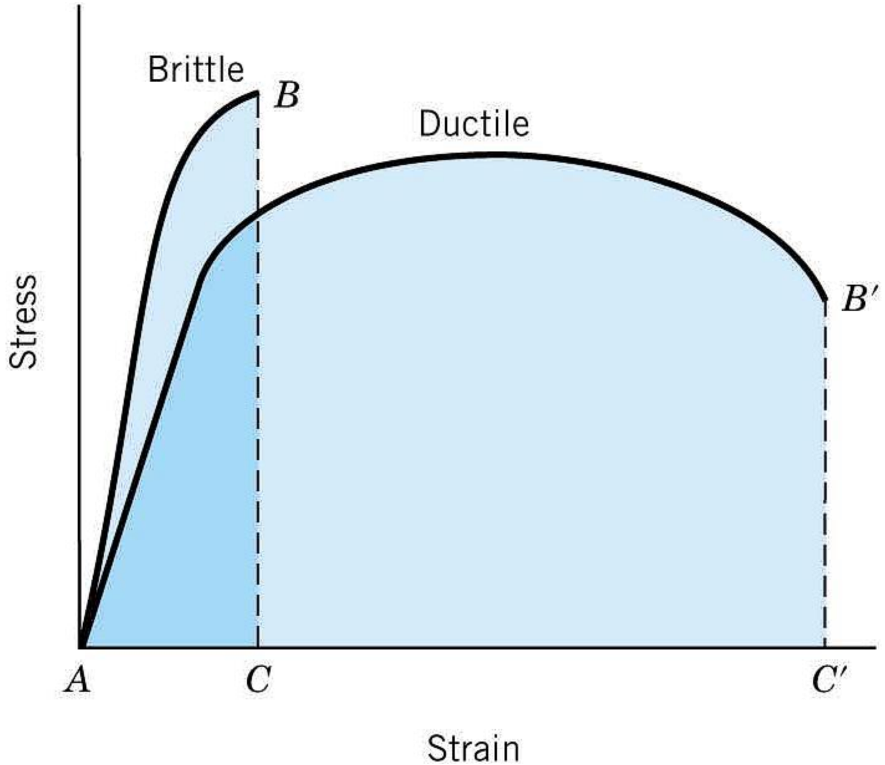
Mechanical Properties of Metals

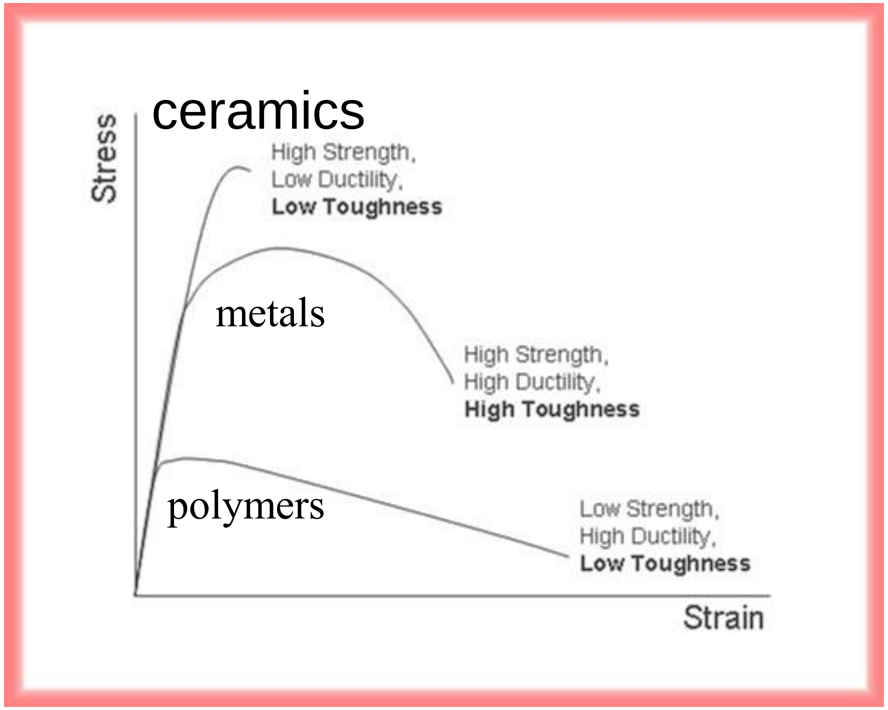
Metal Alloy	*Yield Strength MPa (ksi)*	*Tensile Strength MPa (ksi)*	*Ductility, %EL [in 50 mm (2 in.)]*
Aluminum	35 (5)	90 (13)	40
Copper	69 (10)	200 (29)	45
Brass (70Cu–30Zn)	75 (11)	300 (44)	68
Iron	130 (19)	262 (38)	45
Nickel	138 (20)	480 (70)	40
Steel (1020)	180 (26)	380 (55)	25
Titanium	450 (65)	520 (75)	25
Molybdenum	565 (82)	655 (95)	35

Note: The Yield and Tensile Strengths of material depend on its prior mechanical and thermal treatments, impurities level, etc., while elastic modulus is relatively insensitive to these effects

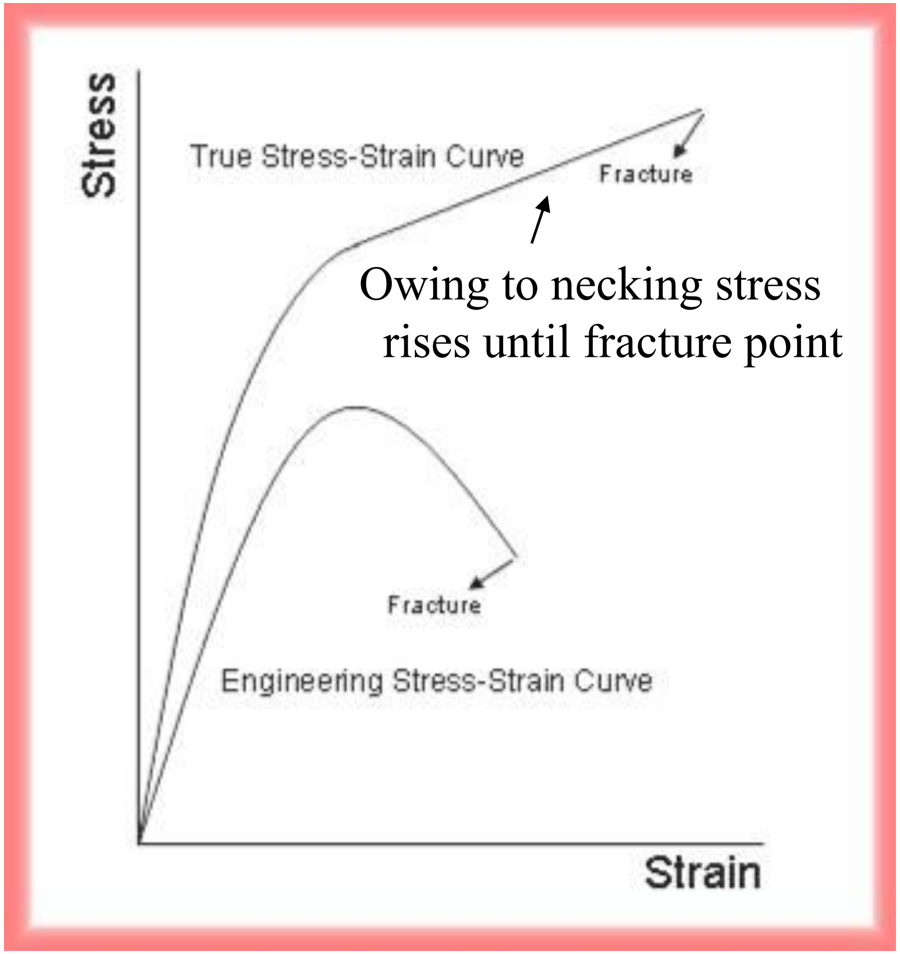
TOUGHNESS (Chapter 8)

- **Energy to break a unit volume of material**
- Approximate by the area under the stress-strain curve.
- Units: [J/m^3]





True Stress and Strain



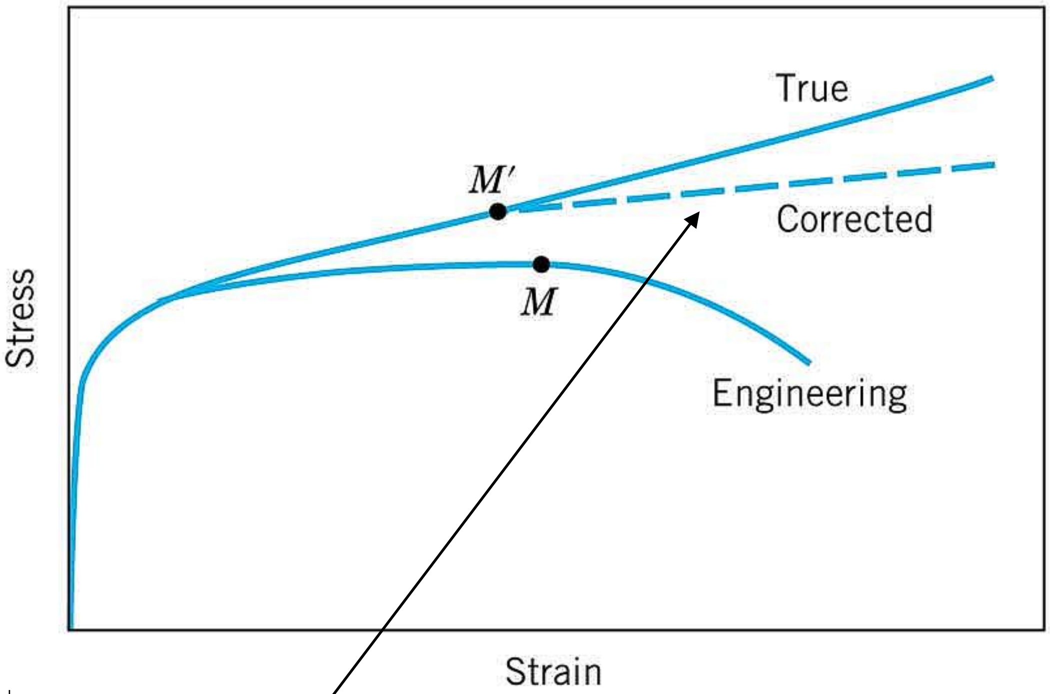
- **True stress**, σ_T , is defined by using instantaneous cross--sectional area of the sample, A_i

$$\sigma_T , = F/A_i$$

True Strain:

$$\varepsilon_T = \ln(l_i/l_o)$$

True Stress and Strain



The "corrected" true S-S curve accounts for the complex stress state within the neck region

$\sigma_T , = F/A_i$

$\varepsilon_T = \ln(l_i/l_o)$

If volume of materials remains constant (until necking point):

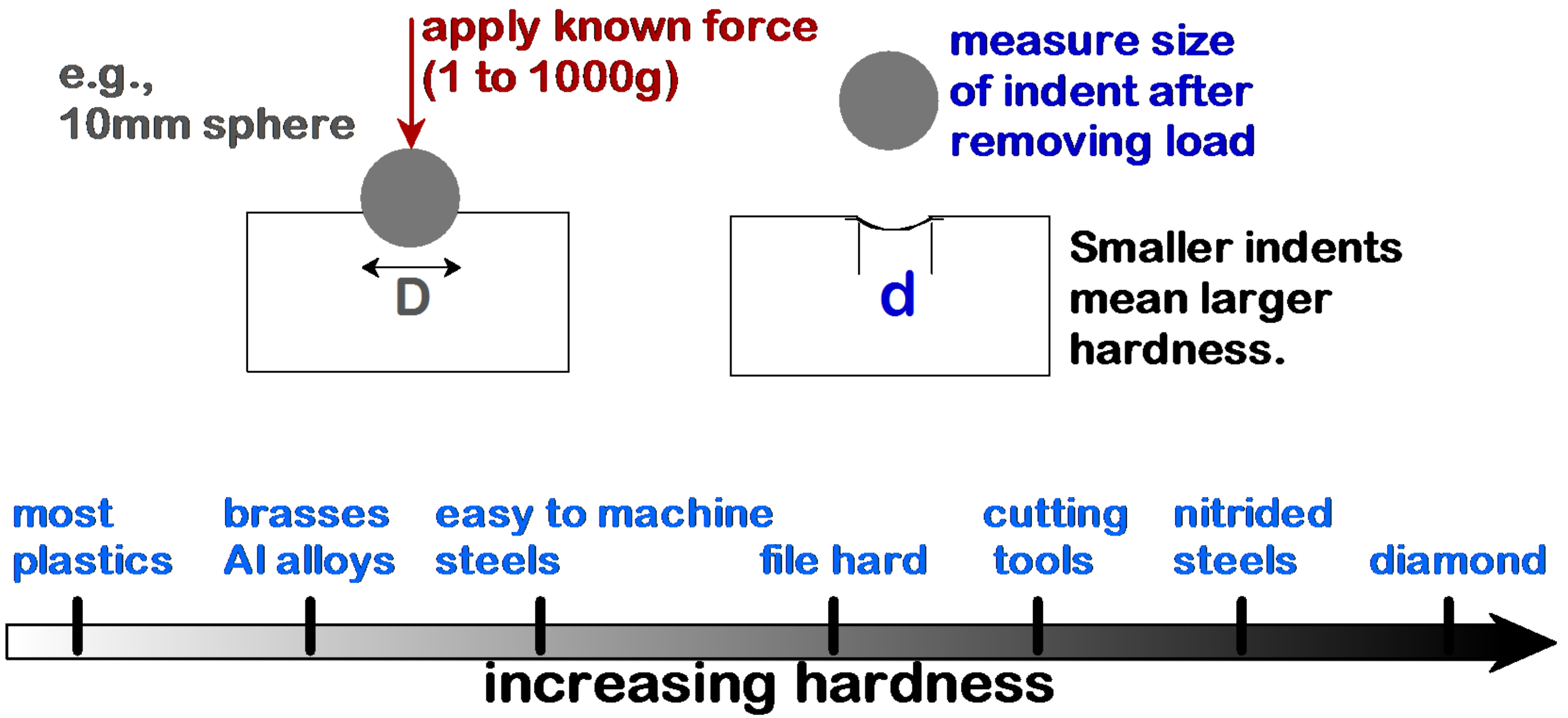
$A_i l_i = A_o l_o$

Then the following relations hold:

$\sigma_T = \sigma(1+\varepsilon)$ and $\varepsilon_T = \ln(1+\varepsilon)$

HARDNESS

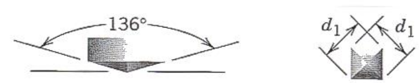
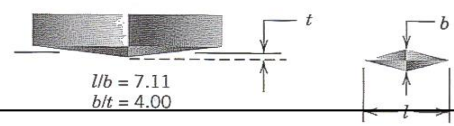
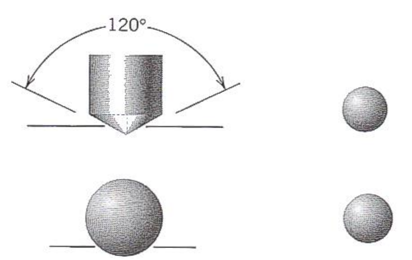
- Resistance to permanently indenting the surface.
- Large hardness means:
  --resistance to plastic deformation or cracking in compression.
  --better wear properties.



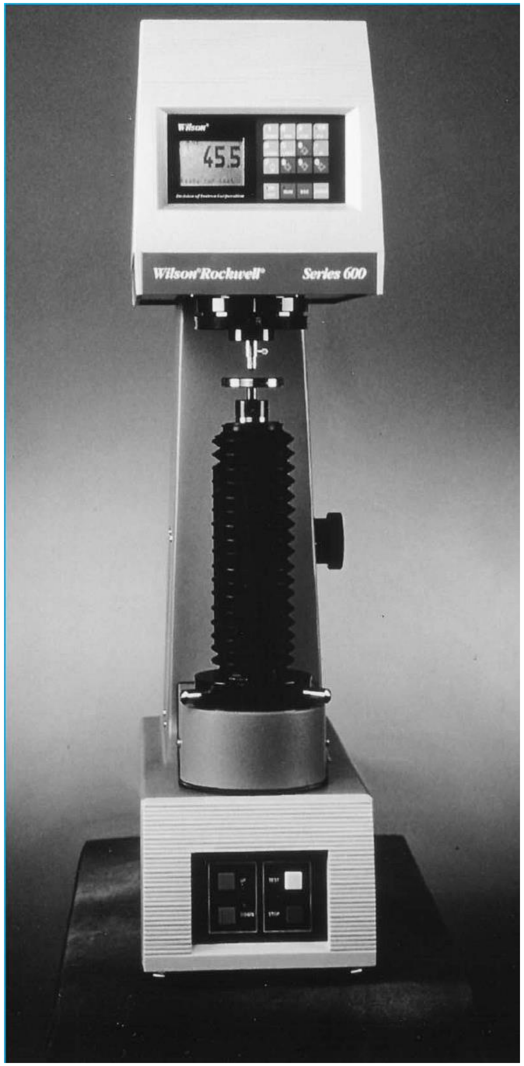
Hardness-Testing Methods

• A qualitative Mohs scale: ability of a material to scratch another material: from 1 (softest, e.g. talc) to 10 (hardest, i.e. diamond)

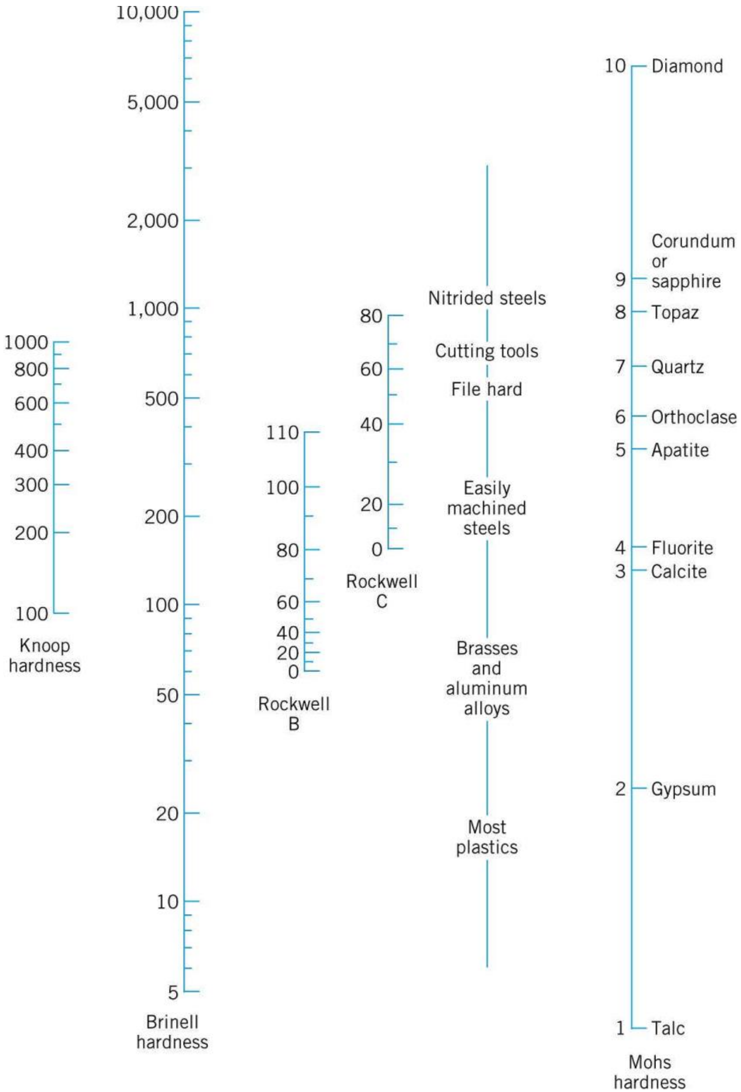
• A variety of quantitative hardness test methods:

Test	*Indenter*	*Shape of Indentation* (*Side View*, *Top View*)	*Load*	*Formula for Hardness Number*[a]
Brinell	10-mm sphere of steel or tungsten carbide		P	$HB = \frac{2P}{\pi D[D - \sqrt{D^2 - d^2}]}$
Vickers microhardness	Diamond pyramid		P	$HV = 1.854P/d_1^2$
Knoop microhardness	Diamond pyramid		P	$HK = 14.2P/l^2$
Rockwell and Superficial Rockwell	Diamond cone; $\frac{1}{16}$, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$ in. diameter steel spheres		60 kg, 100 kg, 150 kg: Rockwell; 15 kg, 30 kg, 45 kg: Superficial Rockwell	

Hardness Tests



Rockwell hardness tester



Hardness Scales

DESIGN OR SAFETY FACTORS

- **Design uncertainties mean we do not push the limit.**
- **Factor of safety, N**

$$\sigma_{working} = \frac{\sigma_y}{N}$$

Often N is between 1.2 and 4

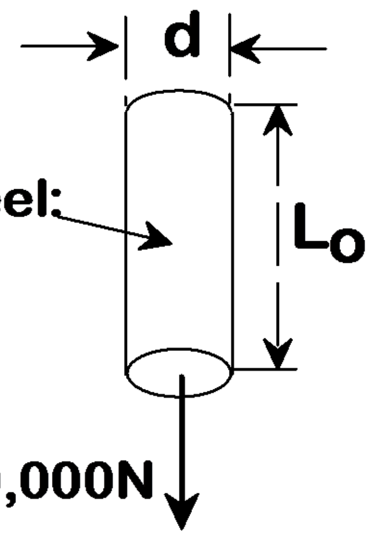
- **Ex: Calculate a diameter, d, to ensure that yield does not occur in the 1045 carbon steel rod below. Use a factor of safety of 5.**

$$\sigma_{working} = \frac{\sigma_y}{N}$$

$$\sigma_{working} = \frac{220{,}000\,N}{\pi\left(d^2/4\right)}$$

$N = 5$

1045 plain carbon steel:
σ_y=310MPa
TS=565MPa



SUMMARY

- **Stress and strain**: **These are size-independent measures of load and displacement, respectively.**
- **Elastic behavior**: **This reversible behavior often shows a linear relation between stress and strain. To minimize deformation, select a material with a large elastic modulus (E or G).**
- **Plastic behavior**: **This permanent deformation behavior occurs when the tensile (or compressive) uniaxial stress reaches σ_y.**
- **Toughness:** **The energy needed to break a unit volume of material.**
- **Ductility:** **The plastic strain at failure.**