Experimental Uncertainties (Errors)

Sources of Experimental Uncertainties (Experimental Errors):

All measurements are subject to some uncertainty as a wide range of errors and inaccuracies can and do happen. Measurements should be made with great care and with careful thought about what you are doing to reduce the possibility of error as much as possible.

There are three **main sources of experimental uncertainties** (experimental errors):

1. Limited accuracy of the measuring apparatus - e.g., the force sensors that we use in experiment M2 cannot determine applied force with a better accuracy than ±0.05 N.

2. Limitations and simplifications of the experimental procedure - e.g., we commonly assume that there is no air friction if objects are not moving fast. Strictly speaking, that friction is small but not equal to zero.

3. Uncontrolled changes to the environment. For example: small changes of the temperature and the humidity in the lab.

In this laboratory, we keep to a very simple form of error analysis, our purpose being more to raise your awareness of errors than to give you expertise in sophisticated methods for handling error analysis. If you ever need more information on error analysis, please check the literature[1-4].

In the *Analysis* section of the lab report, you should identify significant sources of experimental errors. Do not list all possible sources of errors there. Your goal is to identify only those significant for that experiment! For example, if the lab table is not perfectly leveled, then for the collision experiments (M6 – Impulse and Momentum) when the track is supposed to be horizontal, results will have a large, significant error. On the contrary, for the constant acceleration motion on the intentionally inclined track, the table leveling is not going to make a large contribution to the experimental error.

Do not list your mistakes as experimental errors. These mistakes should have already been detected and eliminated during the preparation of the lab report. A calculator should

[1.] Bevington, P. R., Data Reduction and Error Analysis for the Physical Sciences, New York: McGraw-Hill, 1969.
2. Taylor, J. R., An introduction to uncertainty analysis: the study of uncertainties in physical measurements, Mill Valley: University Science Books, 1982.
3. Young, H. D., Statistical Treatment of experimental data, New York: McGraw-Hill, 1962.
4. Barford N. C., Experimental Measurements: Precision, Error and Truth, Addison-Wesley, 1967.

not be listed as a source of experimental error. You may always use more significant figures (c.f. the next section) during calculations to reduce round-off error. In other words, you may always make all calculations with a better accuracy then you can do lab measurements.

Absolute and Relative Errors:

In order to learn the meaning of certain terms, consider an experiment in which we use the ultrasonic motion sensor to measure the position x of an object. The accuracy of the motion sensor has been specified by the manufacturer as ±1 mm. The **absolute error** of our measurements is thus ±1 mm. Note that the absolute error carries the same unit as the measured quantity.

Suppose the value measured in our first trial was equal to 50.6 cm. Then the **relative error** is equal to: (1 mm)/(50.6 cm) = (0.1 cm)/(50.6 cm) ≈ 0.002. The relative error is dimensionless. It is often expressed in percentage, as: 100%* (0.1 cm)/(50.6 cm) ≈ 0.2%.

We may express the above result using either absolute error or relative error, as follows:

$$x = 50.6 \pm 0.1 \text{ (cm)} \quad \text{(usually preferred)} \quad \text{or} \quad x = 50.6 \text{ cm} \pm 0.2\%$$

If we know the accepted value of the measured quantity (e.g., the temperature of melting point for water is equal to 0°C or 273K), then we can calculate the percentage error as:

$$\text{Relative error } = \left| \frac{\text{Experimental value - Accepted value}}{\text{Accepted value}} \right| \times 100\%$$

When comparing experimental data with the theoretical values, then the relative error is given by a similar formula:

$$\text{Relative error } = \left| \frac{\text{Experimental value - Theoretical value}}{\text{Theoretical value}} \right| \times 100\%$$

If we <u>do not</u> know the accepted value of the measured quantity, but the <u>measurements have been repeated</u> several times for the same conditions, one can use the <u>spread</u> of the results themselves to estimate the experimental error.

Average Values and the Standard Deviation:

Consider the following results of velocity measurements: 0.38, 0.38, 0.35, 0.44, 0.43, 0.42 m/s. The *average* value of these six velocity measurements is equal to: v = (0.38 + 0.38 + 0.35 + 0.44 + 0.43 + 0.42) / 6 = 0.40 m/s. Next, one needs to calculate the *deviations*

from the average velocity: 0.38 - 0.40 = 0.02 m/s; 0.38 - 0.40 = 0.02 m/s; 0.40 - 0.35 = 0.05 m/s; 0.40 - 0.44 = -0.04 m/s; 0.40 - 0.43 = -0.03 m/s; 0.40 - 0.42 = -0.02 m/s.

The general formula for calculation of the **average value** x_{AV} (sometimes also called *mean value*) is as follows:

$$x_{AV} = \frac{1}{n}\left(x_1 + x_2 + x_3 + ... + x_n\right)$$

where n is the number of repeated measurements (for this example $n = 6$).

The values of the deviation from the average value are used to calculate the **experimental error**. The quantity that is used to estimate these deviations is known as the **standard deviation** s_x and is defined as:

$$s_x = \sqrt{\frac{1}{n-1}\left[\left(x_1 - x_{AV}\right)^2 + \left(x_2 - x_{AV}\right)^2 + ... + \left(x_n - x_{AV}\right)^2\right]}$$

The standard deviation squared - s_x^2 is the sum of squares of deviations from the average value divided by $(n - 1)$. The subscript usually indicates the quantity that the standard deviation is calculated for, e.g., s_v stands for the standard deviation of velocity measurements, whereas s_a is the standard deviation for acceleration data.

For the previously discussed example of velocity measurements we have:

$$s_v = \sqrt{\frac{1}{6-1}\left[(0.02\text{ m/s})^2 + (0.02\text{ m/s})^2 + (0.05\text{ m/s})^2 + (0.04\text{ m/s})^2 + (0.01\text{ m/s})^2 + (0.02\text{ m/s})^2\right]} =$$

$$= \sqrt{\frac{1}{5}\cdot 0.0062(\text{m}^2/\text{s}^2)} = 0.0352\text{ m/s} \cong 0.04\text{ m/s}$$

We use the standard deviation as the value of the experimental error. The final result of measurements and error analysis should be written as:

$$v = v_{AV} \pm s_v = \textbf{0.40} \pm \textbf{0.04}\ (\textbf{\textit{m/s}})\qquad \text{(do not forget to write the appropriate \underline{units}!)}$$

The general format for presenting experimental results with experimental error is given by one of the following expressions:

***"final result"* = *"average value"* ± *"standard deviation"***

$$x = x_{AV} \pm s_x\ \textbf{units}\quad \text{or}\quad x = x_{AV} \pm s_x\ (\textbf{units})\quad \text{or}\quad x = (x_{AV} \pm s_x)\ \textbf{units}$$

Obviously, the average value and the standard deviation <u>must</u> have the same units.

Agreement Between Two Results:

Error estimates are necessary to be able to say whether the two independent measurements of the same thing agree *within the stated errors* or *disagree.* For example, two students measured the melting point temperature of ice and obtain results of -0.3°C and 0.8°C. Without an estimate of error, we cannot say whether these measurements agree. Suppose the results had been stated with errors: T_{melt} = -0.3 ± 1.0°C and T_{melt} = 0.8 ± 1.0°C. Since the first result admits values between -1.3°C and 0.7°C and the second between -0.2°C and 1.8°C, there is an <u>overlap</u> (between -0.2°C and 0.7°C) and **the results are in agreement within experimental errors**.
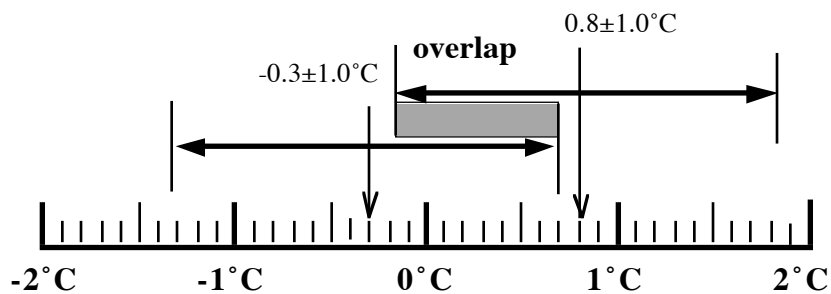
If one cannot find an overlap between the error bands, then the results do not agree with each other.

Experimental Uncertainty (Experimental Error) for a Product of Two Measurements:

Sometimes it is necessary to combine two (or even more than two) measurements to get a needed result. A good example is a determination of work done by pulling a cart on an incline that requires measuring the force and the distance independently. Then the value of work can be calculated from a simple formula: $W = F \times s = F \times \Delta x$ (more precisely: $W = F_{AV} \times \Delta x_{AV}$).

Then, the absolute error for work W is given by the following formula:

$$s_W = F_{AV} \times s_x + \Delta x_{AV} \times s_F \quad ,$$

where, s_F and s_x are standard deviations of force and distance measurements. F_{AV} and Δx_{AV} represent average values of force and traveled distance, respectively. It is quite common that one of the two measurements is more precise, i.e., has a much smaller standard deviation. In the discussed case of work measurements, we usually know the traveled distance with a

much smaller experimental error, i.e., $s_x \cong 0$. Therefore, we should be able to use the approximate formula: $$s_W \cong \Delta x_{AV} \times s_F$$

The final result of work measurements should be written as:

$$W = F_{AV} \times \Delta x_{AV} \pm s_W \ (\text{J} \ ) = F_{AV} \times \Delta x_{AV} \pm \Delta x_{AV} \times s_F \ (\text{J})$$

"work" = "average force" × "average distance" ± "average distance" × "standard deviation of force" (in Joules)

Numerical example:　　　Distance traveled　　$\Delta x = |x_1 - x_2| = \Delta x_{AV} = 1.04 \ (\text{m})$

	Measured Force F (N)	Deviations from the average force: $\Delta F = F - F_{AV}$ (N)	Squared deviations from the average force: $(\Delta F)^2$ (N^2)
1	0.170	-0.007	4.9×10^{-5}
2	0.161	-0.016	25.6×10^{-5}
3	0.152	-0.025	62.5×10^{-5}
4	0.197	0.020	40.0×10^{-5}
5	0.204	0.027	72.9×10^{-5}
Average	0.177		

Average force:　　　$F_{AV} = 0.177 \ (\text{N})$

$$s_F = \sqrt{\frac{1}{4}\left(4.9 \times 10^{-5} N^2 + 25.6 \times 10^{-5} N^2 + 62.5 \times 10^{-5} N^2 + 40.0 \times 10^{-5} N^2 + 72.9 \times 10^{-5} N^2\right)} =$$

$$= 0.0227 \ \text{N} \cong 0.023 \ \text{N}$$

Standard deviation of the force measurements:　$s_F = 0.023 \ (\text{N})$

Standard deviation of the work done:

$$s_W \cong s_F \times \Delta x = 0.023 \ \text{N} \ \times \ 1.04 \ \text{m} = 0.024 \ (\text{J})$$