

Topic 2 : Point Differential, Wins - Losses and Plurality and Runoff Methods for choosing a winner

Point Differential and Wins - Losses

Last day, we ran a round robin tournament with 6 competitors. Each competitor played all of the other competitors exactly once. Now we are left with the task of deciding who gets first second and third place. We will see that this may depend heavily on the method we use to decide on the winner. The full data from the tournament is contained in the following tables:

Overall Scoresheet (row player's score first, column player's score second).

Initials	E. A.	M. M.	C. R.	.Jo. D.	Ju. D.	D. S.	Point D.
E. A.		0-1	1-1	1-0	0-0	1-1	3-3=0
M. M.	1-0		3-1	2-0	1-1	2-1	9-3=6
C. R.	1-1	1-3		2-3	0-1	0-1	4-9 = -5
Jo. D.	0-1	0-2	3-2		0-2	1-2	4-9 = -5
Ju. D.	0-0	1-1	1-0	2-0		2-0	6-1 = 5
D. S.	1-1	1-2	1-0	2-1	0-2		5-6 = -1

A **Ranking** refers to a rank ordered list of the competitors and a **Rating** gives us a list of numerical scores for the competitors. Every rating gives us a ranking of the competitors.

We can assign a rating to competitors in a round robin is using **Wins minus Losses (W-L)**. The major drawbacks to this method is that it often leads to ties and one needs a method of breaking ties. In addition, it does not take into account the magnitude of a win or a loss.

Example In our tournament, we get the following results:

Counting a tie as a half win and a half loss:

Player	Name	W = Wins	L = Losses	W-L
1	Emily Aberle	2.5	2.5	0
2	Mark Miclean	4.5	0.5	4
3	Colin Rahill	0.5	4.5	-4
4	Josh Dunlap	1	4	-3
5	Jubril Dawodu	4	1	3
6	Danielle Stefania	2.5	2.5	0

Not Counting Ties

Player	Name	W = Wins	L = Losses	W-L
1	Emily Aberle	1	1	0
2	Mark Miclean	4	0	4
3	Colin Rahill	0	4	-4
4	Josh Dunlap	1	4	-3
5	Jubril Dawodu	3	0	3
6	Danielle Stefania	2	2	0

Clearly Mark should win and Jubril should get second place, but we need a method to break the tie for third place between Emily and Danielle.

If we use the Point differential to break the tie between Emily and Danielle, who gets first second and third place?

A Condorcet winner in a round robin tournament is a competitor who wins against every other competitor. If such a competitor exists, they will be the unique competitor with such a property, however, they do not always exist. A **Weak Condorcet Winner** is a competitor who wins or draws against all of the other competitors. There may be more than one weak Condorcet winner.

Example In the above example there is no Condorcet winner but Mark and Jubril are both Weak Condorcet winner's.

The other common statistic used to rate players is the point differential. The **point differential** for a player is points for minus points against. We have calculated the point differential for the competitors above. What is the resulting ranking for the players?

Initials	Point D. (rating)	Ranking
Emily Aberle	0	
Mark Miclean	6	
Colin Rahill	-5	
Josh Dunlap	-5	
Jubril Dawodu	5	
Danielle Stefania	-1	

Later with the Colley and Massey method, we will try to predict what will happen in future games between competitors based on results of a partial round robin, namely all of the games between the teams in NCAA mens basketball between the start of the season and March 15. To get an idea of how inaccurate such prediction might be, think about the following question:

“If we had a replay between Emily and Danielle who do you think would win?”

Not that the above methods of assigning ratings and corresponding rankings do not always give the same winner.

Example Suppose we had the following results for our tournament.

Initials	E. A..	M. M..	C. R.	.Jo. D.	Ju. D.	D. S.	Point D.
E. A.		0-1	1-1	1-0	0-0	1-1	3-3=0
M. M.	1-0		3-2	2-0	1-1	2-1	9-4=5
C. R.	1-1	2-3		2-3	0-1	0-1	5-9 = -4
Jo. D.	0-1	0-2	3-2		0-5	1-2	4-12 = -8
Ju. D.	0-0	1-1	1-0	5-0		5-0	12-1 = 11
D. S.	1-1	1-2	1-0	2-1	0-5		5-9 = -4

W-L are the same as before.

- Find the ranking of the competitors using W-L, breaking ties with the point differential.
- Find a ranking of the competitors using the point differential rating.

Name	W-L	Ranking
Emily Aberle		
Mark Miclean		
Colin Rahill		
Josh Dunlap		
Jubril Dawodu		
Danielle Stefania		

Name	P.D.	Ranking
Emily Aberle		
Mark Miclean		
Colin Rahill		
Josh Dunlap		
Jubril Dawodu		
Danielle Stefania		

Choosing a winner from a collection of rankings by voters

Our second contest was between five videos, which were ranked 1-5 by ten judges. the results were as follows:

1. Red Hot Chili Peppers - Dani California [Official Music Video] <https://www.youtube.com/watch?v=Sb5aq5HcS1A>
2. Jurassic Park Theme Song (Melodica Cover) <https://www.youtube.com/watch?v=-w-58hQ9dLk>
3. Donnie Trumpet & the Social Experiment - Sunday Candy "Short Film"<https://www.youtube.com/watch?v=i4ooH8frBWg>
4. Beyonc - Single Ladies (Put a Ring on It) <https://www.youtube.com/watch?v=4m1EFMoRFvY>
5. Drake - Hotline Bling <https://www.youtube.com/watch?v=uxpDa-c-4Mc>

Number of Voters	1	1	1	1	1	1	1	1	1	1	1	1	1	1	4	1	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1	2
Red hot Chilli Peppers	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	4	5	5	5	5		
Jurassic Park	2	2	2	2	3	3	4	5	5	5	5	1	3	3	5	5	1	1	2	2	4	5	5	5	1	3	3	5	5	5	1	2	4	4	4			
Donnie Trumpet	3	3	4	5	4	5	5	2	2	3	4	3	4	5	1	4	2	5	4	5	5	2	4	4	3	1	2	1	1	3	4	1	1	2	3			
Beyonce	4	5	3	3	5	2	2	3	4	2	3	4	1	1	3	1	4	4	1	1	1	1	1	2	2	2	1	2	3	1	3	3	3	3	1			
Drake	5	4	5	4	2	4	3	4	3	4	2	5	5	4	4	3	5	2	5	4	2	4	2	1	5	5	5	3	2	2	2	4	2	1	2			

Note that some sets of preferences have repeated voters.

Mathematicians and economists have struggled with the problem of finding a fair way to amalgamate rankings by individual voters into a single ranking (or social choice function) to produce a winner. Choosing such a method is essential in any democracy and we will see later that there is no perfect way of doing this. First we will look at several commonly used methods:

Plurality Method

One very simple method of Voting is

The Plurality Method With this method, each voter selects one candidate or choice on the ballot. The winner is the candidate or choice with the most votes.

Example 1 A committee of 10 people (with names A, B, . . . , J) must vote on a venue for their next Gymnastics competition. The choices are Indianapolis, South Bend, Fort Wayne, Terre Haute. The committee uses the plurality method of voting, and their ballots are given in the following table:

	A	B	C	D	E	F	G	H	I	J
Indianapolis		X				X	X			X
South Bend	X			X				X		
Fort Wayne									X	
Terre Haute			X		X					

Which venue did they choose?

Example It we had asked our judges to mark only their top choice, the winner of the plurality vote would have been the video with the most number 1's:

Number of Voters	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	2	1	1	2			
Red hot Chilli Peppers	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	3	3	3	3	3	3	3	4	4	4	4	4	4	4	5	5	5	5		
Jurassic Park	2	2	2	2	3	3	4	5	5	5	5	1	3	3	5	5	1	1	2	2	4	5	5	5	1	3	3	5	5	5	1	2	4	4	
Donnie Trumpet	3	3	4	5	4	5	5	2	2	3	4	3	4	5	1	4	2	5	4	5	5	2	4	4	3	1	2	1	1	3	4	1	1	2	3
Beyonce	4	5	3	3	5	2	2	3	4	2	3	4	1	1	3	1	4	4	1	1	1	1	2	2	2	1	2	3	1	3	3	3	3	1	
Drake	5	4	5	4	2	4	3	4	3	4	2	5	5	4	4	3	5	2	5	4	2	4	2	1	5	5	5	3	2	2	2	4	2	1	2

# Voters	Numberof#1votes
1. Red Hot Chili Peppers - Dani California [Official Music Video]	
2. Jurassic Park Theme Song (Melodica Cover)	
3. Donnie Trumpet & the Social Experiment - Sunday Candy "Short Film"	
4. Beyonc - Single Ladies (Put a Ring on It)	
5. Drake - Hotline Bling	

Problems With The Plurality Method

Sometimes this method leads to a **tie**. This is less likely when there are large numbers of voters.

A more serious problem is that it leads to a **Splitting of the vote on similar choices**; If there are just two choices or candidates and the plurality method is used, then the popular choice is guaranteed to win. However if there are more than two choices then it may happen that more extreme choices may win over many similar, but popular choices.

Example 2 Suppose a group of 10 people, many of whom like camping and hiking activities are deciding on where to spend fall break. The options are Camping and Hiking in Colorado, Camping and Hiking in California, Camping and Hiking in Washington, Camping and Hiking in Ireland, Disneyworld. Using the Plurality method the group may end up with a vote like this

	#Voters
Camping and Hiking in Colorado	2
Camping and Hiking in California	2
Camping and Hiking in Washington	2
Camping and Hiking in Ireland	1
Disneyworld	3

Clearly Camping and Hiking is preferred to Disneyworld, but because there are so many similar options for Camping and Hiking, the group ends up going to Disneyworld.

Another problem with the plurality method is that there may be an incentive for **Strategic Voting**. Voters supporting a weak choice may be inclined to change their vote to vote strategically.

Strategic voting occurs when a voter votes in a way that does not reflect their true preferences in an attempt to improve the outcome of the election/poll.

In Example 1 above, suppose that the voters who prefer Terre Haute know that nobody else will vote for Terre Haute. Suppose also that these voters prefer South Bend to Indianapolis, how can they improve the chances that South Bend will win?

	A	B	C	D	E	F	G	H	I	J
Indianapolis		X				X	X			X
South Bend	X			X				X		
Fort Wayne									X	
Terre Haute			X		X					

Runoff Voting

Because of the problems with plurality method, a runoff election is often used.

In a **Runoff Election**, a plurality vote is taken first.

1. If one of the candidates has more than 50% of the votes, that candidate wins.
2. If no candidate has more than 50% of the votes, a second round of plurality voting occurs with a designated number of the top candidates.
3. The process continues until one of the candidates has more than 50% of the votes.

Example: Olympic Voting

The selection of the site for the Olympic Games is made by the International Olympic Committee. The voting process calls for a plurality election with a runoff between all of the candidates except the one in the last place. (This is known as the Hare Method). A number of controversial results have led to suspicions about strategic voting in the past. The results of the election for the location of the 2016 summer olympics are shown below.

Election of the Host City of the 2016 Summer Olympics — ballot results				
Candidate City	Country (NOC)	Round 1	Round 2	Round 3
Rio de Janeiro	Brazil (BRA)	26 (27.66%)	46 (48.42%)	66 (67.35%)
Madrid	Spain (ESP)	28 (29.79%)	29 (30.53%)	32 (32.65%)
Tokyo	Japan (JPN)	22 (23.40%)	20 (21.05%)	—
Chicago	United States (USA)	18 (19.15%)	—	—
121st IOC Session		Round 1	Round 2	Round 3
 Copenhagen - Denmark	Vote details			
	Eligible	95	97	99
	Participants	94	96	98
	Abstentions	0	1	0
	Valid ballots	94	95	98
Members unable to vote				
Members from countries with candidate cities			Other members	
Anita L. Defrantz · James L. Easton · Chiharu Igaya · Shun-ichiro Okano · João Havelange · Carlos Arthur Nuzman · Juan Antonio Samaranch Jr.			Jacques Rogge (IOC president) · Lee Kun-hee (suspended) · Alpha Ibrahim Diallo (absent) · Saku Koivu (absent)	

Can you find evidence of strategic voting (Hint: the number of votes for any particular city should not drop from one round to the next)?

The results of the election in other years are attached at the end of the lecture. Check to see if you can find evidence of strategic voting in the election process for other years.

Preference Ranking

In most voting situations, each voter has an order of preference of the candidates. Such an ordering is called a **Preference Ranking**. The voter may have to put some thought into making such a preference ranking and it may change over time.

The voting systems discussed below which use preference rankings make the following assumptions about them:

1. Each voter has a preference ranking that orders all candidates from most preferred to least preferred. (we assume that in the case of indifference or lack of knowledge of the candidates, the voter will choose a ranking randomly).
2. If a voter has ranked one candidate higher than another, then if the voter must choose between the two candidates, the voter would choose the higher ranked one.
3. The order of the preferences is not changed by the elimination of one or more candidates.

Note that this allows us to conduct a runoff election without revoting and it does not allow strategic voting where candidates change their preferences after the first round.

Example A fourth grade class is asked to rank their preferences for a field trip to a game of football basketball or baseball. The preference rankings of the voters are presented in a table below showing the number of voters with each preference in the top row.

# voters	1	3	3	2	4	5
football	1	1	3	2	4	3
basketball	2	4	1	4	1	4
baseball	3	2	4	3	2	1
soccer	4	3	2	1	3	2

- (a) In a plurality election, which option would win?
- (b) In a plurality election with a runoff between the top two finishers, what would the outcome be?

Instant Runoff Voting (IRV) (used in deciding winner of Oscars)

In an Instant Runoff election,

1. **each voter ranks the list of candidates in order of preference.** The candidates are ranked in ascending order with a “1” next to the most preferred candidate, a “2” next to the second most preferred candidate and so forth.
(In some implementations, the voter ranks as many or as few choices as they wish while in others they are required to rank all of the candidates or a prescribed number of them.)
2. **In the initial count, the first preference of each voter is counted and used to order the candidates.** Each first preference counts as one vote for the appropriate candidate.
3. **Once all the first preferences are counted, if one candidate holds a majority (more than 50% of votes cast), that candidate wins. Otherwise the candidate who holds the fewest first preferences is eliminated.**
(If there is an exact tie for last place in numbers of votes, tie-breaking rules determine which candidate to eliminate.)
4. **Ballots assigned to eliminated candidates are recounted and assigned to one of the remaining candidates based on the next preference on each ballot.**
5. **The process repeats until one candidate achieves a majority (more than 50%) of votes**

cast for continuing candidates. Ballots that 'exhaust' all their preferences (all its ranked candidates are eliminated) are set aside.

Example In an instant runoff election which of the candidates in the previous example would win?

1 votes

# voters	1	3	3	2	4	5	R1	R2	R3	R4
football	1	1	3	2	4	3				
basketball	2	4	1	4	1	4				
baseball	3	2	4	3	2	1				
soccer	4	3	2	1	3	2				

Video Competition Example Here when we eliminate the competitor with the least number ones, it puts us in a position where we have nobody with more than 50% of the votes and videos are in a tie for last place. To break the tie, one might eliminate both candidates at once in which case, our winner is [4. Beyonc - Single Ladies \(Put a Ring on It\)](#) with more than 50% of the votes. Another commonly used method is to choose one of the two (with ties in round two) at random, eliminate them and continue as above.

Number of Voters	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	4	1	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1	2	
Red hot Chilli Peppers	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	3	3	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	5	5	5	5	5
Jurassic Park	2	2	2	2	3	3	4	5	5	5	5	1	3	3	5	5	1	1	2	2	4	5	5	5	1	3	3	5	5	5	1	2	4	4	4	4	4		
Donnie Trumpet	3	3	4	5	4	5	5	2	2	3	4	3	4	5	1	4	2	5	4	5	5	2	4	4	3	1	2	1	1	3	4	1	1	2	3	3	3	1	
Beyonce	4	5	3	3	5	2	2	3	4	2	3	4	1	1	3	1	4	4	1	1	1	1	1	1	2	2	2	1	2	3	1	3	3	3	3	3	1		
Drake	5	4	5	4	2	4	3	4	3	4	2	5	5	4	4	3	5	2	5	4	2	4	2	1	5	5	5	3	2	2	2	4	2	1	2	1	2		

# Voters	R1	R2	R3	R4
1. Red Hot Chili Peppers - Dani California [Official Music Video]				
2. Jurassic Park Theme Song (Melodica Cover)				
3. Donnie Trumpet & the Social Experiment - Sunday Candy "Short Film"				
4. Beyonc - Single Ladies (Put a Ring on It)				
5. Drake - Hotline Bling				

Election of the host city for the Summer Olympics

Location for 2012 Olympics

Candidate City	Round 1	Round 2	Round 3	Round 4
London	22	27	39	54
Paris	21	32	33	50
Madrid	20	25	31	—
New York	①⑨	①⑥	—	—
Moscow	15	—	—	—
Total	97	100	103	104

Location for 2008 Olympics

Candidate City	Round 1	Round 2	Round 3	Round 4
Beijing	44	56	—	—
Toronto	20	22	—	—
Paris	15	18	—	—
Istanbul	①⑦	⑨	—	—
Osaka	6	—	—	—
Total	102	105	—	—

Location for 2004 Olympics

Candidate City	Round 1	Round 2	Round 3	Round 4
Athens	32	38	52	66
Rome	23	28	35	41
Stockholm	②⑩	①⑨	—	—
Cape Town	16	②②	②⑩	—
Buenos Aires	16	—	—	—
Total	107	107	107	107

(Buenos Aires was eliminated in round one with a tie-break vote vs. Cape Town. The result was 62-44.)

Location for 2000 Olympics

Candidate City	Round 1	Round 2	Round 3	Round 4
Sydney	30	30	37	45
Beijing	32	37	40	43
Manchester	11	①③	①①	—
Berlin	9	9	—	—
Istanbul	7	—	—	—
Total	89	89	88	88

Election of the host city for the Summer Olympics

Location for 1996 Olympics continued

Candidate City	Round 1	Round 2	Round 3	Round 4	Round 5
Athens	23	23	26	30	35
Atlanta	19	20	26	34	51
Toronto	14	17	18	22	—
Melbourne	12	(21)	(16)	—	—
Manchester	(11)	(5)	—	—	—
Belgrade	7	—	—	—	—
Total	86	86	86	86	86

Location for 1992 Olympics continued

Candidate City	Round 1	Round 2	Round 3
Barcelona	29	37	47
Paris	19	20	23
Brisbane	(11)	(9)	10
Belgrade	(13)	(11)	5
Birmingham	8	8	—
Amsterdam	5	—	—
Total	85	85	85

(Barcelona had more than 50% of the votes in the third round and thus the voting was terminated with a win for Barcelona.)