Lecture 2 : Borda's method: A Scoring System

In the plurality and runoff methods discussed in the previous lecture, we do not take into account the voter's relative preferences for *all* of the candidates. We do not, for example take into account which candidate was ranked last by each voter. In this and the following section, we assume that voters are required to list a full set of preferences on their ballot and we look at methods that use all of the information.

Borda's Method

With **Borda's method** voters rank the entire list of candidates or choices in order of preference from the first choice to the last choice.
After all votes have been cast, they are tallied as follows:
On a particular ballot, the lowest ranking candidate is given 1 point, the second lowest is given 2 points, and so on, the top candidate receiving points equal to the number of candidates.
The number of points given to each caniddate is summed across all ballots.
This is called the **Borda Count** for the candidate. The winner is the candidate with the highest Borda count.

Example 1 A committee of 10 people needs to select a chair from among three candidates named Kelly, Holtz, Rockne. They decide to use Borda's method. The preference rankings of the ten committee members are as follows:

# Voters →	**2**	**3**	**2**	**3**
Kelly	1	3	2	3
Holtz	2	1	1	2
Rockne	3	2	3	1

Who will be the winner using Borda's method?

The Borda count for Kelly is given by:
(no. 1st place votes)3 + (no. 2nd place votes)2 + (no. 3rd place votes)1 = $2\cdot3+2\cdot2+6\cdot1 = 6+4+6 = 16$.

The Borda count for Holtz is given by:
(no. 1st place votes)3+(no. 2nd place votes)2+(no. 3rd place votes)1= $5\cdot3+5\cdot2+0\cdot1 = 15+10+0 = 25$.

The Borda count for Rockne is given by:
(no. 1st place votes)3+(no. 2nd place votes)2+(no. 3rd place votes)1= $3\cdot3+3\cdot2+4\cdot1 = 9+6+4 = 19$.
(Not that in this case the winner using the Borda method agrees with the winner using the Plurality method)

Example 2 Suppose that in a survey, squash players were asked to rank brands of squash racquets. The results are shown below:

# Players →	33	3	10	20	7	27
Dunlop	1	1	2	3	2	3
Black Knight	2	3	1	1	3	2
Prince	3	2	3	2	1	1

(a) Which brand would win using Borda's method?

(b) Which brand would win using the Plurality method?

(c) Which brand would win using the Plurality method with a runoff between the first and second place finishers?

Note: One could also apply Borda's method by just adding the rankings as they are and the person with the lowest point total wins. In some variations of Borda's method shown below this approach is no longer feasible.
Note: If there are n candidates a k th place vote adds a total of $n + 1 - k$ to that candidates Borda Count. You should be able to use this come up with an easy formula to calculate the Borda count in the example below. (This formula is also invalid for some variations of the Borda method.)

Example 3; A variation of the Borda Count The 2000 preseason rankings for the Big East college football teams are shown below, where the voters were various publications (SN = Sports News, SI= Sports Illustrated etc...)

Big East, Preseason rankings, 2000

$Team \downarrow$	AT	SN	L	PS	SS	CFN	ATS	SI	PS	SN	CNN	CPA	FA	GP	JF	CP	BR
Pittsburgh	5	6	4	4	4	4	3	5	5	4	6	4	4	5	4	4	6
Miami-FL	1	1	1	1	1	2	1	1	1	1	1	1	2	2	1	2	1
Boston College	3	3	5	6	5	6	4	3	4	5	5	5	5	6	6	5	4
Virginia Tech	2	2	2	3	2	2	2	2	2	2	2	2	1	1	2	1	2
Rutgers	8	8	8	7	8	8	8	8	7	8	8	8	8	8	7	8	7
Syracuse	4	4	3	2	3	2	5	4	3	3	3	3	3	3	3	3	3
Temple	7	7	7	8	7	7	7	7	8	6	7	7	6	7	8	7	8
West Virginia	6	5	6	5	6	5	6	6	6	7	4	6	7	4	5	7	5

Note that CFN (College Football News) has not ranked its preferences 1-8, instead it has given the top three teams a rank of 2 each. We will assign a Borda count of $9 - 2 = 7$ to each of these votes. Also College and Pro Football Newsweekly (CP) has ranked two teams as 7 instead of assigning a 6 and a 7. We will give each of these teams a Borda count of $9 - 7 = 2$.

Use Borda's method to determine a ranking for the teams using the sums of the above rankings shown below:

Team	Pitt.	Miami	BC	VT	Rutgers	Syr.	Temple	WV
Sum	77	21	80	32	132	54	121	96

Formula for Borda Count When using the Borda Method with c candidates and v voters. For any given candidate, let $r_1, r_2, \ldots, r_v$ demote the ranks assigned to that candidate by each voter. Let s denote their sum $s = r_1 + r_2 + \cdots + r_v$. Then the Borda count for that candidate is given by

$$\boxed{b = v(c+1) - s}$$

Strategic Voting and Borda's Method

Example 4 There are 4 candidates for the position of President for the Notre Dame Table Tennis Club. The preferences of the 10 members of the club are shown in the following table:

Presidential preference rankings

#Voters	**1**	**1**	**1**	**3**	**1**	**1**	**1**	**1**
G. Devaney	1	1	1	2	2	3	4	4
K. Shields.	2	3	3	4	4	2	2	3
J. Gonzales	3	2	4	1	3	4	3	2
N. Li	4	4	2	3	1	1	1	1

(a) Which candidate will win if the Borda Method is used?

(b) Could the club member who voted for Li first, Devaney second, Gonzales third and Shields fourth have voted strategically to change the outcome so that Li came first when votes were counted using Borda's method (assuming the other members of the club vote as indicated in the table) ?

(c) Could the two club members who voted for Li first and Devaney fourth have influenced the outcome by voting strategically (assuming the other members of the club vote as indicated in the table) ?

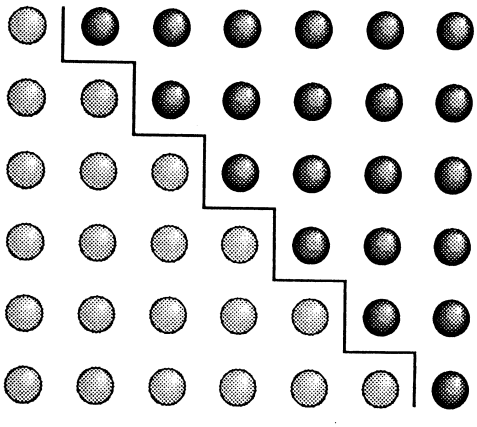
Parity Check

Since the Borda method involves a lot of calculation, it is easy to make a mistake. we can use the formula given below to run a quick check on our answers.

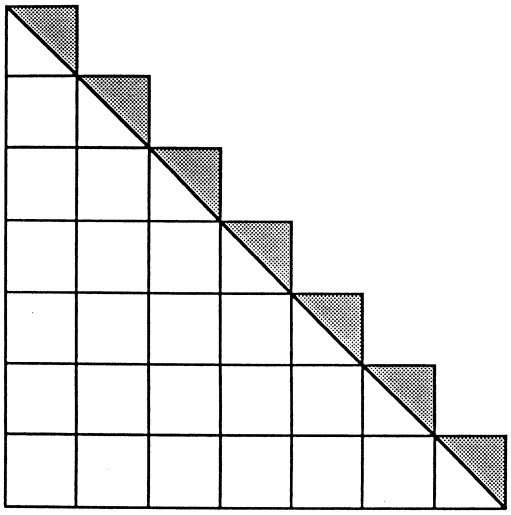
Parity Check When using Borda's method with c candidates and v voters, the sum of the Borda counts for all candidates must be

$$\boxed{\frac{vc(c+1)}{2}}.$$

Proof Since each voter contributes a total of $1+2+3+\cdots+c$ to the sum of the Borda counts, the sum of the Borda counts must be $v(1+2+3+\cdots+c)$. A visual proof of this formula is given below.



$1+2+\cdots+n=\frac{1}{2}n(n+1)$



$1+2+\cdots+n=\frac{n^2}{2}+\frac{n}{2}$

Example Find $1+2+3+\cdots+50$?

Check the value of the sum of all Borda counts predicted by the above parity check for the examples discussed above:

Example 1

# Voters →	2	3	2	3
Kelly	1	3	2	3
Holtz	2	1	1	2
Rockne	3	2	3	1

	Borda Count
Kelly	16
Holtz	25
Rockne	19

$v=10, \quad c=3.$

Example 2:

# Players →	**33**	**3**	**10**	**20**	**7**	**27**
Dunlop	1	1	2	3	2	3
Black Knight	2	3	1	1	3	2
Prince	3	2	3	2	1	1

	Borda Count
Dunlop	
Black Knight	
Prince	

$v = 100, \quad c = 3.$

Example 3:

Team	Pitt.	Miami	BC	VT	Rutgers	Syr.	Temple	WV
Borda Count	76	132	73	121	21	99	32	57

v = 17, c = 8.

In this case, we see that the parity check comes up incorrect; why?

Example 4:

#**Voters**	**1**	**1**	**1**	**3**	**1**	**1**	**1**	**1**
G. Devaney	1	1	1	2	2	3	4	4
K. Shields.	2	3	3	4	4	2	2	3
J. Gonzales	3	2	4	1	3	4	3	2
N. Li	4	4	2	3	1	1	1	1

	Borda Count
G. Devaney	
K. Shields.	
J. Gonzales	
N. Li	

$v = 10, \quad c = 4.$

Advantages, Disadvantages

The **advantage** of Borda's method over plurality methods is that voters are able to express their opinions about candidates other than just their first choice. This means that a candidate who is ranked highly but not necessarily first by many voters has a good chance of winning when using Borda's method. The **disadvantage** of using Borda's method is that it is more susceptible to strategic voting than either the Plurality or Runoff Plurality methods.

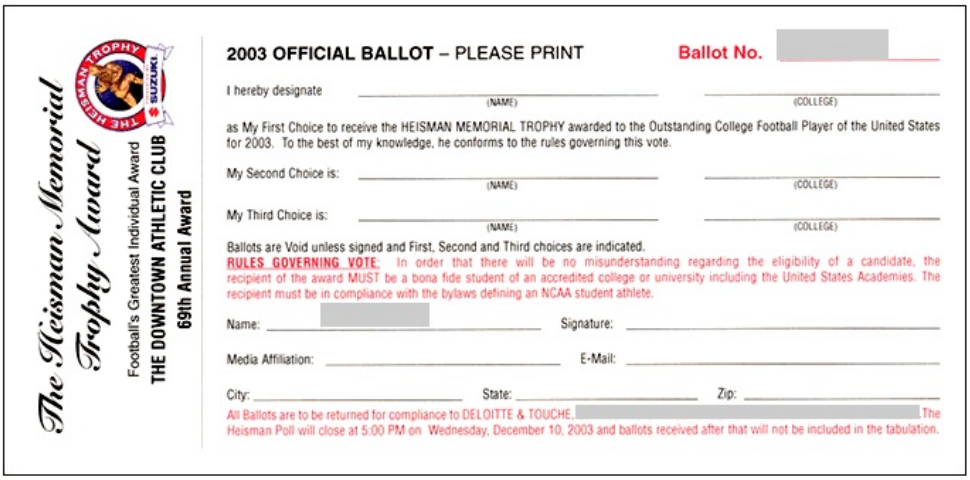
Variations of The Borda Count in Sports

In sports polls where this form of voting is commonly used, the voters may know a lot about the top teams or players and be able to rank them, but may not know enough to rank all eligible candidates, so lumping all but the top candidates together with 0 points simplifies the process for voters.

Heisman Trophy

A variation of this method is used to decide the winner of the Heisman Trophy. On the Heisman ballot voters are asked to **rank only their top three choices from among all college football players in the United States**. The Borda count for each player is computed by giving 3 points for each first place vote, 2 points for each second place vote and 1 point for each third place vote. The winner is declared to be the candidate with the highest Borda count.

THE 2003 OFFICIAL HEISMAN BALLOT



The Heisman Memorial Trophy Award

Football's Greatest Individual Award

THE DOWNTOWN ATHLETIC CLUB

69th Annual Award

2003 OFFICIAL BALLOT – PLEASE PRINT Ballot No. [redacted]

I hereby designate _____ (NAME) _____ (COLLEGE)

as My First Choice to receive the HEISMAN MEMORIAL TROPHY awarded to the Outstanding College Football Player of the United States for 2003. To the best of my knowledge, he conforms to the rules governing this vote.

My Second Choice is: _____ (NAME) _____ (COLLEGE)

My Third Choice is: _____ (NAME) _____ (COLLEGE)

Ballots are Void unless signed and First, Second and Third choices are indicated.

RULES GOVERNING VOTE: In order that there will be no misunderstanding regarding the eligibility of a candidate, the recipient of the award MUST be a bona fide student of an accredited college or university including the United States Academies. The recipient must be in compliance with the bylaws defining an NCAA student athlete.

Name: [redacted] Signature: _____

Media Affiliation: _____ E-Mail: _____

City: _____ State: _____ Zip: _____

All Ballots are to be returned for compliance to DELOITTE & TOUCHE, [redacted] The Heisman Poll will close at 5:00 PM on Wednesday, December 10, 2003 and ballots received after that will not be included in the tabulation.

NBA most valuable player (uses wider spread of points)

To decide on the winner of the National Basketball Association Most Valuable Player award, **116 members of the media list their first through fifth choices for the award.** Each first place vote receives 10 points, each second place vote receives 7 points, each third place vote receives 5 points, each fourth place vote receives 3 points and each fifth place vote receives 1 point.

AP Polls

The Associated Press (AP) college football poll gives a ranking for the top teams in college football. The voters in the AP poll are newspaper, radio and television sports reporters throughout the country. In 2011 there were 60 voters and the final rankings are given in the table below.

AP Top 25

RK	TEAM	RECORD	PTS	PVS
1	Alabama (55)	12-1	1495	2
2	LSU (1)	13-1	1425	1
3	Oklahoma State (4)	12-1	1399	3
4	Oregon	12-2	1250	6
5	Arkansas	11-2	1198	7
6	USC	10-2	1181	5
7	Stanford	11-2	1167	4
8	Boise State	12-1	1127	8
9	South Carolina	11-2	1013	10
10	Wisconsin	11-3	905	9
11	Michigan State	11-3	873	12
12	Michigan	11-2	839	13
13	Baylor	10-3	780	15
14	TCU	11-2	653	16
15	Kansas State	10-3	621	11
16	Oklahoma	10-3	572	19
17	West Virginia	10-3	547	23
18	Houston	13-1	518	20
19	Georgia	10-4	439	18
20	Southern Miss	12-2	411	22
21	Virginia Tech	11-3	329	17
22	Clemson	10-4	188	14
23	Florida State	9-4	154	25
24	Nebraska	9-4	143	21
25	Cincinnati	10-3	103	NR

Others receiving votes: Brigham Young 51, Auburn 40, Northern Illinois 33, Missouri 23, Texas 15, Rutgers 3, North Dakota State 2, Penn State 2, Virginia 1

BCS Unlike most college sports Division 1A football does not have a playoff to determine the national champion. Instead the Bowl Championship series is a selection system that creates five bowl match-ups involving ten of the top ranked teams in the NCAA Division I. **The BCS combines rankings from the Harris Interactive Poll, The Coaches Poll and six computer ranking systems.**

The Harris Interactive College Poll uses the Borda method with 113 voting members ranking teams from 1 through 25. The maximum Borda count for any team from this poll is $113 \times 25 = 2,825$. The Borda count for each team is scaled for use in the BCS ranking to its BCS quotient

$$\text{Harris Quotient} = \frac{\text{Harris Borda Count for team}}{2825}.$$

The USA Today Coaches Poll also uses the Borda Method has 59 voting members each season ranking teams from 1 through 25. The maximum possible Borda count for any team from this poll is $59 \times 25 = 1475$. The Borda count for each team is scaled for use in the BCS ranking to its

BCS quotient

$$\text{Coaches Quotient} = \frac{\text{Coaches Borda Count for team}}{1475}.$$

The BCS used six computer ranking systems: Jeff Sagarin, Anderson/Hester, Richard Billingsley, Colley Matrix, Kenneth Massey, and Dr. Peter Wolfe. Each ranking system assigns a number between 1 and 25 to each team, 25 to the top ranked team, 24 to the team ranked second, etc... . A team's highest and lowest computer ranking will be discarded from figuring a team's computer poll average. Points will be assigned in inverse order of ranking from 125. The four remaining computer scores will be averaged and the total will be calculated as a percentage of 100. The highest computer BCS quotient that a team can earn is therefore $(25 \times 4)/100 = 1$.

The BCS ratings at the end of each polling period are determined by adding the above 3 quotients together. Sometimes the average of the three quotients are published.

$$\text{Final BCS rating} = \frac{\text{Harris Quotient} + \text{Coaches Quotient} + \text{Computer Quotient}}{3}.$$

Controversy Because millions of dollars ride on the possibility of playing in the national championship bowl game, and because of the manipulation possible when using the Borda method, the ethical question of whether rankings should be affected by those with a vested interest in the outcome arises. The formula for the BCS ratings was rewritten after the 2004 season. Before 2004, the AP polls were used, however due to the controversial outcome in the 2004 season, the Associated Press no longer allowed its rankings to be used in the BCS formula. It was replaced by the Harris Poll. An excerpt from Wikipedia is shown below and the final two BCS rankings for 2004 are shown in subsequent pages.

> As with previous seasons, fans of successful teams left out of the BCS were disappointed. Auburn, Utah, and BSU all went unbeaten but were not offered a chance to compete for the championship. Auburn was especially the focus of national media attention on this topic; many thought that since Auburn managed to go undefeated in the traditionally tough SEC, they deserved a shot at the title. Adding to the BCS frustration was the fact that Auburn and Utah, though both in BCS bowl games, would not be able to play each other as a match-up of highly ranked unbeatens.
>
> There was also a controversy in selecting the BCS bowls' second at-large team (Utah being the first). The University of California expected to get the invite, being ranked fourth in the BCS entering the last week of the regular season; the Texas Longhorns, who had been left out of the BCS the year before, was fifth before the final BCS rankings were released. Both teams finished at 10-1, but the Longhorns ultimately received enough support from poll voters to move into the fourth slot, which ensured they would also receive the final at-large bid. Texas coach Mack Brown was criticized for publicly politicking voters to put Texas ahead of California; Cal coach Jeff Tedford called for coaches' votes to be made public. Texas went on to defeat Michigan in the Rose Bowl, while California lost to Texas Tech in the Holiday Bowl.
>
> The Associated Press, as a result of two consecutive seasons of BCS controversy, prohibited the BCS from using their poll as part of its ranking formula. The AP poll was replaced by the Harris Interactive poll, and the AP continues to award its own national championship trophy.

The National Football Foundation and College Hall of Fame, Inc.

22 Maple Avenue, Morristown, NJ 07960

FOR RELEASE: Embargoed Until Noon ET, November 29, 2004
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Bob Burda, Assistant Commissioner, Big 12 Conference 214-753-0107

2004 BOWL CHAMPIONSHIP SERIES STANDINGS

(Games Through Nov. 27, 2004)

	Associated Press			USA Today / ESPN			Computers				
	Rank	Points	%	Rank	Points	%	Avg. Comp. Rank	Points	%	BCS Average	Previous
1. Southern California	1	1610	.9908	1	1509	.9895	2	97	.970	.9834	1
2. Oklahoma	2	1540	.9477	2	1442	.9456	1	99	.990	.9611	2
3. Auburn	3	1530	.9415	3	1435	.9410	3	92	.920	.9342	3
4. California	4	1410	.8677	4	1314	.8616	6	80	.800	.8431	4
5. Texas	6	1325	.8154	5	1266	.8302	4	88	.880	.8418	5
6. Utah	5	1342	.8258	6	1222	.8013	5	84	.840	.8224	6
7. Georgia	8	1094	.6732	7	1100	.7213	8	69	.690	.6948	8
8. Boise State	11	952	.5858	10	926	.6072	7	76	.760	.6510	7
9. Louisville	7	1175	.7231	8	1038	.6807	T-12	51	.510	.6379	10
10. Miami (FL)	9	1037	.6382	9	983	.6446	10	62	.620	.6342	9
11. LSU	13	877	.5397	12	887	.5816	9	64	.640	.5871	13
12. Virginia Tech	10	980	.6031	11	915	.6000	T-12	51	.510	.5710	14
13. Iowa	12	911	.5606	14	787	.5161	11	54	.540	.5389	11
14. Michigan	14	873	.5372	13	824	.5403	17	38	.380	.4859	12
15. Tennessee	15	802	.4935	15	706	.4630	15	49	.490	.4822	15
16. Florida State	16	628	.3865	16	606	.3974	21	24	.240	.3413	18
17. Wisconsin	17	612	.3766	17	568	.3725	19	26	.260	.3364	20
18. Virginia	18	469	.2886	18	411	.2695	18	30	.300	.2860	17
19. Arizona State	21	231	.1422	23	167	.1095	T-12	51	.510	.2539	16
20. Texas A&M	22	199	.1225	25	152	.0997	16	43	.430	.2174	19
21. Texas Tech	24	167	.1028	20	208	.1364	22	19	.190	.1431	NR
22. Florida	20	290	.1785	19	251	.1646	NR	0	.000	.1144	NR
23. Pittsburgh	19	304	.1871	21	193	.1266	NR	0	.000	.1045	NR
24. Oklahoma State	32	30	.0185	31	52	.0341	20	25	.250	.1009	22
25. Ohio State	25	143	.0880	22	172	.1128	25	5	.050	.0836	NR

EXPLANATION:

Team percentages are derived by dividing a team's actual voting points by a maximum 1625 possible points in the AP Poll and 1525 possible points in the USA Today/ESPN Coaches Poll.

Six computer rankings calculated in inverse points order (25 for #1, 24 for #2, etc.) are used to determine the overall computer component. The best and worst ranking for each team is dropped, and the remaining four are added and divided by 100 (the maximum possible points) to produce a Computer Rankings Percentage. The six computer ranking providers are Anderson & Hester, Richard Billingsley, Colley Matrix, Kenneth Massey, Jeff Sagarin, and Peter Wolfe. Each computer ranking accounts for schedule strength in its formula. Their individual weekly rankings of all teams may be found at www.BCSfootball.org.

The BCS Average is calculated by averaging the percent totals of the Associated Press, USA Today/ESPN Coaches, and Computer polls.

WWW.BCSFOOTBALL.ORG

On January 4, 2005, the FedEx Orange Bowl will host the BCS National Championship Game and determine which team will be presented The National Football Foundation and College Hall of Fame's MacArthur Trophy, awarded to college football's National Champion since 1959, as well as the ADT National Championship Trophy on behalf of the American Football Coaches Association.

The National Football Foundation and College Hall of Fame, Inc.

22 Maple Avenue, Morristown, NJ 07960

FOR RELEASE: Embargoed Until 5:10 PM ET, December 5, 2004
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Bob Burda, Assistant Commissioner, Big 12 Conference 214-753-0107

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(Games Through December 4, 2004)

	Associated Press			USA Today / ESPN			Computers				
	Rank	Points	%	Rank	Points	%	Avg. Comp. Rank	Points	%	BCS Average	Previous
1. Southern California	1	1599	.9840	1	1490	.9770	2	97	.970	.9770	1
2. Oklahoma	2	1556	.9575	2	1459	.9567	1	99	.990	.9681	2
3. Auburn	3	1525	.9385	3	1435	.9410	3	92	.920	.9331	3
4. Texas	6	1337	.8228	5	1281	.8400	4	88	.880	.8476	5
5. California	4	1399	.8609	4	1286	.8433	6	80	.800	.8347	4
6. Utah	5	1345	.8277	6	1215	.7967	5	83	.830	.8181	6
7. Georgia	8	1117	.6874	7	1117	.7325	8	67	.670	.6966	7
8. Virginia Tech	9	1111	.6837	9	1037	.6800	T-9	65	.650	.6712	12
9. Boise State	10	960	.5908	10	943	.6184	7	76	.760	.6564	8
10. Louisville	7	1183	.7280	8	1066	.6990	13	52	.520	.6490	9
11. LSU	12	929	.5717	11	932	.6111	T-9	65	.650	.6109	11
12. Iowa	11	948	.5834	13	812	.5325	12	55	.550	.5553	13
13. Michigan	13	917	.5643	12	874	.5731	17	38	.380	.5058	14
14. Miami (FL)	14	776	.4775	14	738	.4839	T-14	45	.450	.4705	10
15. Tennessee	15	651	.4006	17	559	.3666	T-14	45	.450	.4057	15
16. Florida State	17	647	.3982	15	643	.4216	21	22	.220	.3466	16
17. Wisconsin	16	648	.3988	16	599	.3928	20	24	.240	.3439	17
18. Virginia	18	482	.2966	18	455	.2984	18	30	.300	.2983	18
19. Arizona State	21	222	.1366	24	173	.1134	11	56	.560	.2700	19
20. Texas A&M	22	213	.1311	25	147	.0964	16	44	.440	.2225	20
21. Pittsburgh	19	415	.2554	20	318	.2085	NR	0	.000	.1546	23
22. Texas Tech	23	168	.1034	21	234	.1534	22	19	.190	.1489	21
23. Florida	20	325	.2000	19	324	.2125	NR	0	.000	.1375	22
24. Oklahoma State	32	16	.0098	28	35	.0230	19	25	.250	.0943	24
25. Ohio State	24	155	.0954	22	181	.1187	NR	4	.040	.0847	25

EXPLANATION:

Team percentages are derived by dividing a team's actual voting points by a maximum 1625 possible points in the AP Poll and 1525 possible points in the USA Today/ESPN Coaches Poll.

Six computer rankings calculated in inverse points order (25 for #1, 24 for #2, etc.) are used to determine the overall computer component. The best and worst ranking for each team is dropped, and the remaining four are added and divided by 100 (the maximum possible points) to produce a Computer Rankings Percentage. The six computer ranking providers are Anderson & Hester, Richard Billingsley, Colley Matrix, Kenneth Massey, Jeff Sagarin, and Peter Wolfe. Each computer ranking accounts for schedule strength in its formula. Their individual weekly rankings of all teams may be found at www.BCSfootball.org.

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