Math 10250 Projects – Spring 2010

Why Projects? If this is really a mathematics class, why are projects a part of the course? Surely, math is meant to be about equations and computations, formulae and random theoretical concepts... right? Wrong. Math may be founded upon what you have learned in math classes all your life - from fractions to functions, and now calculus - but it is a lot more than that. The world of mathematics intersects with the 'real world' all the time. Modern business, science and technology, for example, simply could not take place without advanced mathematics. All the academic disciplines have something practical to contribute to the world in which we live in, and math is no exception. In Math 10250, then, it is now time for you to put your years of mathematical knowledge into action.

What Project? In your project, you will be given the opportunity to make your own connections between mathematics and society by choosing a problem that can be mitigated through mathematical means. Choose a topic that you are comfortable with, work with your classmates and have fun!

The **Topic** can be chosen from:

(a) Projects listed in Chapters 2 to 5 of your textbook.

(b) A project listed under "Sample Project Topics"

(c) Other courses you are able to establish a connection with math 10250

(d) Anything that you find interesting and is approved by your teacher

The **Rules** are:

(a) You can work in groups of size 1-4 students drawn from any section of Math 10250.

(b) Each group submits two copies of their paper. One must be submitted online via email to the course instructor, while another must be printed and submitted in person.

(c) Each member of the group receives the same score out of 20. And,10 bonus points will be awarded for great projects. If you and your team create an *exceptional* project, then you be awarded *extra credit* accordingly.

(d) Your project must clearly display the *project title*, the *names* of your team members and the *class sections* each member is from on a cover page.

(e) The *first draft* is due by March 29.

(f) Your *completed project* is **due by** April 26.

Sample Project Topics

(1) **Social Security**: Some experts project that the Social Security shortfall over the next 75 years will be about four trillion dollars. Is that true? Why or why not? How might this problem be averted or solved? Make your contribution in the national debate about saving Social Security using ideas and techniques you learned in Math 10250.

(2) **The Budget**: Visit the Webpage of the Congressional Budget Office (CBO) and try to make sense of the numbers you will find in "Current Budget Projections". How are these estimates made? Are these figures indicative of a healthy budget? Why or why not? What can be done to make things better? Note that income streams are useful in making projections.

(3) **Drilling for Oil**: "Drill, baby, drill"? A major topic for debate in the 2008 Presidential Campaign was on the benefits and pitfalls of drilling in protected areas of the United States both offshore and on line, such as in the Arctic National Wildlife Refuge (ANWR). Would such drilling be a good idea? Weigh scientific predictions about the possible economic cost and environmental risk of expanded oil drilling in these areas versus the benefits that they might bring. Do the math: how do you weigh the cost / benefit analysis?

(4) **Solar Energy**: Many think that the potential of solar energy is so great, that it will be the future solution to our energy problems. Solar 'farms' in the southwest of the United States *could* provide the energy needs for the entire country. What is stopping us? Some major problems lie in price and efficiency. Collect data about the change in price of solar energy versus that of carbon-based energy in

the U.S. since 2000 and model functions to fit the data. What needs to happen before we can switch to solar energy?

(5) **Wind Energy.** Collect data about wind energy production in the U.S. since 2000 and draw a curve that fits these data. Also, draw the oil-price curve using data from reliable sources. Furthermore, compare the shape of these curves and make sense of the current projections of wind energy production for the next 10-20 years. Finally, find out for which country in the world the percentage of the energy it uses from wind is maximum.

(6) **Oil Price.** Is the current oil price the result of world demand & supply or/and market manipulation? Draw your own conclusions by collecting data from reliable sources and analyzing them using the mathematics you learned in Math 10250.

(7) **Declining Resources**: We all know how oil is in a limited supply, but we often overlook some of the other materials that we cannot keep using at an increasing rate forever. Use what you have learned in Math 10250 to model declining resources and give some solutions to this problem.

(8) **Health Care Reform**: In 2001, the World Health Organization rated the French healthcare system as the best in the world, above the British, at 18th, and far beyond the United States' healthcare system, which came in at 37th. The French universal system costs \$3,500 per capita, whereas the British spend \$2,784, and the Americans spend \$6,714. The French have a universal healthcare insurance system, the British have a socially funded single-payer system whilst the American system is almost completely privatized. What does this data suggest? How can we determine which system is 'best', or the best value for money?

(9) **Moore's Law**: Since their invention, the performance per unit cost of computer processors have roughly doubled every two years. In 1965, the co-founder of Intel, Gordon Moore, observed this trend and predicted that it would continue. That prediction was termed "Moore's Law" after it was repeatedly confirmed. To this day, computer development has continued to follow this law. Computer components such as processing speed, memory capacity and even the number of pixels in digital cameras have also been found to follow similar logarithmic functions and exponential increases in power. Prove that such increases have been found to occur by plotting processing speed to a graph and calculating the function that fits the data best. Will this exponential growth continue indefinitely?

(10) **The Biggest Challenges**: What are the top major challenges for your generation? Use mathematics to show why these are such major problems. What can we do about these issues? Examples include the economic crisis, the budget deficit and climate change.

(11) **The Dollar.** What are the fundamental causes for the fall/rise of the dollar's value?

(12) **Sub-prime Loans.** What are sub-prime loans and what they have to do with the 2008 housing and banking crisis?

(13) **Mountains Beyond Mountains.** In this inspiring book Tracy Kidder describes the quest of Dr. Paul Farmer, a man who would cure the world. Curing infectious diseases and bringing the lifesaving tools of modern medicine to those who need them most is his lifes calling. Read this book and use the mathematics you have learned in Math 10250 to try to understand, analyze and propose possible solutions to the global health problem.

(14) **The End of Poverty.** In the preface of this book its author Dr. Jeffrey Sachs (Quetelet Professor of Sustainable Development at Columbia University, Director of the Earth Institute, and Director of the United Nations Millennium Project) writes:When the end of poverty arrives, as it can and should in our generation, it will be citizens in a million communities in rich and poor countries alike, rather than a handful of political leaders, who will have turned the tide. The fight for the end of poverty is a fight that all of us must join in our own way. Read this very interesting book and use the mathematics you have learned in Math 10250 to try to understand (quantify, analyze) poverty as a world problem, and propose possible solutions that our generation can realize.

(15) **A. Income distribution and Lorentz curves.** The way that income is distributed throughout a given society is an important object of study for economists. The U.S. Census Bureau collects and analyzes income data, which it makes available at its website, www.census.gov. In 2001, for instance, the poorest 20% of the U.S. population received 3.5% of the money income, while the richest 20% received 50.1%. The *cumulative* proportions of population and income are shown in the following table:

proportion of population	**proportion of income**
0	0
0.20	0.035
0.40	0.123
0.60	0.268
0.80	0.499
1.00	1.00

For instance, the table shows that the lowest 40% of the population received 12.3% of the total income. We can think of the data in this table as being given by a functional equation $y = f(x)$, where x is the cumulative proportion of the population and y is the cumulative proportion of income. For instance, $f(0.60) = 0.268$ and $f(0.80) = 0.499$. Such a function (or, more properly speaking, its graph) is called a **Lorentz curve**.

(i) Show that $f(x) = 0.1x + 0.9x^2$ is a possible Lorentz curve. Also, compute the income received by the lowest 0 %, 50%, and 100% of the population.

(ii) Show that $f(x) = 0.3x + 0.9x^2$ is **not** a Lorentz curve.

(iii) For the Lorentz curve in (i) show the following properties:

(a) $f(0) = 0, f(1) = 1,$ and $0 \leq f(x) \leq 1$ for all $0 \leq x \leq 1$,

(b) $f(x)$ is an increasing function,

(c) $f(x) \leq x$ for all x, $0 \leq x \leq 1$.

(iv) Explain why properties (a)–(c) hold for every Lorentz curve.

(v) Write many other different formulas for Lorentz curves.

(vi) Using real data produce Lorentz curves for the U.S. and Canada in 2005.

(vii) Sketch the graph of a Lorentz curve and compare it with the line $y = x$.

B. Coefficient of Inequality. If the Lorentz curve of a country is given by $f(x) = x$ then its total income is distributed equally. Otherwise there are inequalities present in the distribution of income, which are measured by the following number:

$$\textbf{coefficient of inequality} = 2\int_0^1 [x - f(x)]dx,$$

which is also called the **Gini Index**.

(i) Compute the coefficient of inequality when $f(x) = 0.1x + 0.9x^2$.

(ii) Show that the Gini Index is the **ratio** of the area of the region between $y = f(x)$ and $y = x$ to the area of the region under $y = x$, and provide an economic inerpretation of this ratio.

(iii) Using real data estimate the Gini Index of the U.S. and Canada in 2005.

(16) **The Coca-Cola can.** In this project, we will investigate whether a Coca-Cola can is designed to minimize the amount of aluminum used for the volume of soda it contains. (i) For a cylindrical can, closed at the top and bottom, with given volume V, find the ratio h/d of height to diameter that minimizes the total surface area A. (ii) Second, by measuring the height and the diameter of the base of a Coca-Cola can, determine whether it minimizes the surface for the volume it contains.

(17) **Making Connections**: Math is a vital tool for many disciplines, ranging from graphic design to computer programming and from economics to music. Investigate one of these connections and describe your findings.

(18) **What does calculus have to do with change?** The two central concepts in calculus are the derivative (instantaneous rate of change) and the integral (total change). Write in your own words the way you understand these concepts. Give examples from mathematics and its applications to demonstrate them.