

Name: _____

Instructor: _____

Math 10350, Calculus A
Fall Semester 2006
Final Exam
Monday, December 11, 2006

This Examination contains **25** problems, worth a total of **150** points, on 7 sheets of paper including the front cover. All problems are multiple choice with no partial credit, and each is worth **6** points. Record your answers to these problems by placing an \times through one letter for each problem below:

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| 1. <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e | 14. <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e |
| 2. <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e | 15. <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e |
| 3. <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e | 16. <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e |
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| 9. <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e | 22. <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e |
| 10. <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e | 23. <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e |
| 11. <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e | 24. <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e |
| 12. <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e | 25. <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e |
| 13. <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e | | | | | |

Books and notes are not allowed. You may not use your calculator.

Sign the pledge: "On my honor, I have neither given nor received unauthorized aid on this Exam":

GOOD LUCK

Part A: Multiple Choice Problems

1. (6 pts.) 1. Find the point on the parabola $y = x^2$ at which the tangent line is parallel to the line $2x + y = 0$.

- a) (1, 1) b) (0, 0) c) (-1, 1) d) (2, 4) e) (-2, 4)

2. (6 pts.) Which of the following is the value of the limit

$$\lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{x^2 - 2x - 3} \quad ?$$

- a) 4 b) 2 c) $\frac{1}{3}$ d) $\frac{1}{2}$ e) The limit does not exist.

3. (6 pts.) Which of the following is the value of the limit

$$\lim_{x \rightarrow 1} \frac{x - 1}{\sqrt{x} - 1} \quad ?$$

- a) 2 (b) ∞ (c) 0 (d) -1 (e) 1.

4. (6 pts.) Let

$$f(x) = \begin{cases} cx - 4, & \text{if } x < 2 \\ x^2 - cx, & \text{if } x \geq 2 \end{cases}$$

where c is a constant. What value of c makes $f(x)$ continuous?

- a) $c = -1$ b) $c = 0$ c) $c = 2$ d) $c = 1$
e) No value of c makes $f(x)$ continuous.

5. (6 pts.) A particle moves so that its position at time t is given by

$$s(t) = 3t^2 - \frac{32}{\sqrt{t}}.$$

What is the instantaneous velocity of the particle at time $t = 4$?

- a) 64 b) 22 c) 32 d) 16 e) 26.

6. (6 pts.) Which of the following is equal to the derivative of the function

$$f(x) = \frac{x^2 - 1}{x^2 + 1} ?$$

- a) $\frac{4x^3}{(x^2 + 1)^2}$ b) $\frac{-4x^3}{(x^2 + 1)^2}$ c) $\frac{-4x}{(x^2 + 1)^2}$
d) $\frac{4x}{(x^2 + 1)^2}$ e) $\frac{2x}{(x^2 + 1)^2}$

7. (6 pts.) Which of the following is the value of the limit

$$\lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^3 - x^3}{\Delta x} ?$$

- a) x^3 b) $3x^2$ c) x^2 d) $3x^3$ e) $2x^3$

8. (6 pts.) Find the derivative of $\frac{1}{\sin(x)}$.

- a) $\frac{-\cos x}{\sin x}$ b) $\frac{-1}{\sin^2 x}$ c) $\frac{-1}{\sin x}$
d) $\frac{\cos x}{\sin^2 x}$ e) $\frac{-\cos x}{\sin^2 x}$

9. (6 pts.) Find $\frac{dy}{dx}$ by implicit differentiation of $xy + \sin^2 y = 10350$.

- a) $\frac{-y}{x + 2 \sin y \cos y}$ b) $\frac{-(x + y)}{2 \sin y \cos y}$ c) $\frac{-(2 \sin y \cos y + y)}{x}$
d) $\frac{-y}{2 \cos y + x}$ e) $\frac{-y}{2 \sin y + x}$

10. (6 pts.) The minimum value of $f(x) = \frac{x^3}{3} - x^2 - 3x$ on $[-2, 2]$ is:

- a) $\frac{5}{3}$ b) $-\frac{22}{3}$ c) $-\frac{2}{3}$ d) 0 e) -9

11. (6 pts.) Determine the largest interval(s) on which the function $f(x) = -2x^3 + 15x^2 - 24x$ is increasing.

- a) $(-\infty, -4) \cup (-1, \infty)$ b) $(-\infty, 1) \cup (4, \infty)$ c) $(1, 4)$
d) $(2, 3)$ e) $(-\infty, \infty)$

12. (6 pts.) Determine the largest interval(s) on which the function $f(x) = -\frac{4}{x-3}$ is concave up.

- a) $(-\infty, -3)$ b) $(3, \infty)$ c) $(-\infty, 3)$
d) $(0, 3)$ e) $(-\infty, 3) \cup (3, \infty)$

13. (6 pts.) $\lim_{x \rightarrow -\infty} \frac{4x^4 + 2x^3 + 7x - 5}{16x^5 + 3x^4 + 10350} =$

- a) 1 b) $\frac{1}{4}$ c) $-\frac{1}{4}$ d) $-\infty$ e) 0

14. (6 pts.) A duck is 6 feet north and 8 feet east of a buoy. He is swimming due east at a rate of 3 ft/sec. How quickly is his distance from the buoy changing, in ft/sec?

- a) .8 b) 2.4 c) 4.8 d) 0.5 e) $\frac{8\sqrt{45}}{45}$

15. (6 pts.) If a differentiable function $f(x)$ satisfies $f(0) = f(5)$, then which of the following must be true?

- a) There exists c in $(0, 5)$ with $f'(c) < 0$.
b) There exists c in $(0, 5)$ with $f(c) = 0$.
c) There exists c in $(0, 5)$ with $f'(c) > 0$.
d) There exists c in $(0, 5)$ with $f'(c) = 0$.
e) There exists c in $(0, 5)$ with $f''(c) = 0$.

16. (6 pts.) Find the Riemann sum for $f(x) = -x$, $-2 \leq x \leq 2$, using two subintervals of equal length, and left endpoints.

- a) 2 b) 4 c) -2 d) 3 e) 0.

17. (6 pts.) Use differentials to approximate $\sin(0.1)$ recognizing that $\sin(0) = 0$.

- a) 0.1 b) 0 c) -0.1 d) 0.2 e) 0.3.

18. (6 pts.) Compute the definite integral $\int_0^3 \sqrt{9-x^2} dx$. (Hint: interpret as an area.)

- a) 9π b) 18 c) $\frac{9}{4}\pi$ d) $\frac{9}{2}\pi$ e) $\frac{1}{3}$.

19. (6 pts.) An object is moving with velocity $v(t) = 6t + 6$ m/sec. Find the distance traveled by the object between time $t = 0$ and $t = 5$.

- a) -30 m b) 36 m c) 45 m d) 180 m e) 105 m

20. (6 pts.) An open box (that is, a box with no top) is to have a square base, and a volume of 16 cubic meters. The material for the base costs \$1 per square meter, and the material for the sides costs \$2 per square meter. Find the cost of the cheapest such box.

- a) \$16 b) \$48 c) \$32 d) \$64 e) \$24

21. (6 pts.) Determine the largest interval(s) on which the function

$$F(x) = \int_0^x \frac{1}{3+t^2} dt \text{ is concave up.}$$

- a) $(-\infty, -\sqrt{3})$ b) There is no interval on which $F(x)$ is concave up.
c) $(0, \infty)$ d) $(-\infty, 0)$ e) $(-\infty, 0) \cup (0, \infty)$

22. (6 pts.) Compute the indefinite integral $\int t^2 \left(t - \frac{2}{t} \right) dt$:

- a) $3t^2 - 2 + C$ b) $\frac{t^3}{3} \left(\frac{t^2}{2} - 2 \right) + C$ c) $t^3 - 2t + C$
d) $\frac{1}{4}t^4 - t^2 + C$ e) $\frac{2}{3}t^3 + C$.

23. (6 pts.) Suppose that $\int_0^6 f(x)dx = 8$ and $\int_0^4 f(x)dx = 2$.

Compute $\int_6^4 f(x)dx$:

- a) -10 b) 6 c) -6 d) 10 e) 12.

24. (6 pts.) Find the area of the region bounded by the graph of $f(x) = \sin(3x)$, the x -axis, and the lines $x = 0$ and $x = \pi/6$.

- a) 1 b) $\frac{1}{3}$ c) -3 d) 0 e) $-\frac{1}{3}$.

25. (6 pts.) Evaluate the integral $\int x\sqrt{x+1} dx$.

- a) $\frac{2}{5}(x+1)^{5/2} + \frac{2}{3}(x+1)^{3/2} + C$ b) $\frac{2}{3}(x^3 + x^2)^{3/2} + C$
c) $\frac{2}{3}(x+1)^{3/2} - 2(x+1)^{1/2} + C$ d) $\frac{2}{3}(x+1)^{3/2} + C$
e) $\frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + C$