ERRATA FOR "ON THE STRENGTH OF RAMSEY'S THEOREM FOR PAIRS"

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Several proofs given in [2] contain significant errors or gaps, although to our knowledge all results claimed there are provable. The needed corrections are described below. All references are to [2] unless otherwise stated, and we adopt the notation and terminology of that paper.

1. Lemma 7.10 asserts that the principles D^2_2 and SRT^2_2 are equivalent over RCA_0. However, the proof that D^2_2 implies SRT^2_2 has a hidden application of $\mathsf{B}\Sigma^0_2$ and thus is actually carried out in $\mathsf{RCA}_0 + \mathsf{B}\Sigma^0_2$. The problem is that, in the construction of H by adding one element at a time, each element c added to H must form a pair of the appropriate color with all previously chosen elements. To get the existence of such a c one seems to need $\mathsf{B}\Sigma^0_2$. This gap was recently closed by Chong, Lempp, and Yang, who showed in [3], Theorem 1.4, that, in RCA_0, D^2_2 implies $\mathsf{B}\Sigma^0_2$, and hence D^2_2 implies SRT^2_2.

2. Lemma 7.11 asserts that RT^2_2 is equivalent to SRT^2_2 & COH over RCA_0. However, the proof given there that RT^2_2 implies COH in RCA_0 is seriously flawed. This was pointed out by Joseph Mileti and later by Jeffrey Hirst. A proof that RT^2_2 implies COH in $\mathsf{RCA}_0 + \mathsf{I}\Sigma_2$ can easily be extracted from the proof of Theorem 12.5. Mileti, and simultaneously Lempp and Jockusch, observed that it is possible to eliminate the use of $\mathsf{I}\Sigma_2$ by effectively bounding in terms of k the number of changes changes in the characteristic function of A when it is restricted to A_k, so that proving that this number is finite requires only Σ_1-induction. Thus, it is provable in RCA_0 that RT^2_2 implies COH, and hence that RT^2_2 is equivalent to SRT^2_2 & COH.

3. Joseph Mileti pointed out a gap in the proof of the claim at the bottom of page 50 that a certain computable 2-coloring of pairs C is "jump universal" in the sense that for every C-homogeneous set A and every computable coloring $\tilde{C}$, there exists an infinite $\tilde{C}$-homogeneous set B with $B' \leq_T A'$. The proof provided works only when $\tilde{C}$ is stable. However, this assumption can be eliminated by using the density of the Turing degrees under $<<$ (see [6], Theorem 6.5) to stabilize $\tilde{C}$. Namely,

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by Theorem 12.5 let C be a computable coloring such that every infinite homogeneous set has jump of degree $>> \mathbf{0}'$, and let A be an infinite homogeneous set for C . Let $\mathbf{d}$ be the degree of A', so that $\mathbf{d} >> \mathbf{0}'$. Let $\tilde{C}$ be any computable 2-coloring of pairs. We must show that $\tilde{C}$ has an infinite homogeneous set with jump of degree at most $\mathbf{d}$. Let $\mathbf{c}$ be a degree with $\mathbf{d} >> \mathbf{c} >> \mathbf{0}'$, and let $\mathbf{a}$ be a degree with $\mathbf{a}' = \mathbf{c}$. Since $\mathbf{a}' >> \mathbf{0}'$, there is an infinite set R of degree at most $\mathbf{a}$ which is sufficiently cohesive that the restriction of $\tilde{C}$ to R is stable. (To see this, note by Theorem 2.1 of [5] there is a p-cohesive set of degree $\mathbf{a}$, and replace the primitive recursive sets by a suitable uniformly computable family which yields stability.) Relativizing the proofs of the results in Section 3 to $\mathbf{a}$ and using that $\mathbf{d} >> \mathbf{a}'$ shows that there is an infinite set H which is homogeneous for the restricted coloring (and hence for $\tilde{C}$) such that H has degree at most $\mathbf{d}$. These same remarks suffice to prove Corollary 12.6. The upshot of these remarks is that if $\mathbf{d} >> \mathbf{0}'$, then every computable 2-coloring of pairs has an infinite homogeneous set H whose jump has degree at most $\mathbf{d}$. Another proof of this may be found in [4], Theorem 3.4(ii), where it is shown that, for any uniformly Δ^0_2 family of noncomputable sets $\{C_i\}_{i\in\omega}$, H may be chosen to satisfy in addition $(\forall i)[C_i \not\leq_T H]$.

4. In the penultimate paragraph on page 52, it is claimed that it can be shown by the methods of Avigad and also by those of Hajek that every countable model of RCA_0 + $\mathrm{I}\Sigma_n$ is an ω-submodel of some countable model of RCA_0 + $\mathrm{I}\Sigma_n$ + WKL_0. However, Avigad [1] has pointed out that it seems to be necessary to combine his methods with those of Hajek to get this result. Also in [1] there are interesting comments on our paper and related matters as well as a solution to Question 13.3 and a partial solution to Question 13.4.

References

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[3] C. T. Chong, Steffen Lempp, and Yue Yang, *The collection principle for Σ_2 formulas and the partition principle PART*, to appear in Proceedings of the American Mathematical Society.

[4] Damir D. Dzhafarov and Carl G. Jockusch, Jr., *Ramsey's Theorem and cone avoidance*, this JOURNAL **74** 2009, 557–578.

[5] Carl Jockusch and Frank Stephan, *A cohesive set which is not high*, Mathematical Logic Quarterly **39** (1993), 515–530.

[6] Stephen G. Simpson, *Degrees of unsolvability: a survey of results*, Handbook of Mathematical Logic (J. Barwise, editor), North-Holland, Amsterdam, 1997, pp. 631–652.

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