TROPICAL GEOMETRY PROBLEMS, DAY 1

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(1) Find the point of intersection between the lines defined by the following two equations:
$$3 \odot x \oplus 4 \odot y \oplus 1$$
$$1 \odot x \oplus -1 \odot y \oplus 0.$$
(2) Draw the tropical curve defined by the equation:
$$1 \odot x^2 y \oplus x^2 \oplus xy^2 \oplus xy \oplus 1 \odot x \oplus 3 \odot y \oplus 2$$
(3) Find quadratic polynomials defining curves of all the types shown in lecture.
(4) Prove the Fundamental Theorem of Tropical Algebra: For any univariate tropical polynomial, there exists a factorization into linear factors which defines the same function (but is not necessarily the same polynomial).
(5) Let (a, b) and (c, d) be two points in the plane. Prove that the polynomial in x and y defined by the determinant:
$$\begin{vmatrix} a & b & 0 \\ c & d & 0 \\ x & y & 0 \end{vmatrix}$$
passes through these two points. Note that the tropical determinant is analogous to the classical determinant, except there are no signs.
(6) What happens to the defining equation when these two points line on a vertical, horizontal, or diagonal line?
(7) Prove that for any five points in the plane, there exists a tropical quadric passing through all of them.