Homework 1: Strings and languages

CSE 30151 Spring 2016

Due: 2016/01/19 11:59pm

Instructions

- You can prepare your solutions however you like (by hand, LaTeX, Jupyter/IPython notebook, etc.), but you must submit them as a single PDF file.
  - Scan written solutions in the library or using a smartphone (e.g., CamScanner).
  - Convert a Jupyter/IPython notebook using one of these commands:

    ```
    jupyter nbconvert netid-hw1.ipynb --to pdf
    ipython nbconvert netid-hw1.ipynb --to pdf
    ```

- Please give every PDF file a unique name.
  - If you're making a complete submission, name your PDF file `netid-hw1.pdf`, where `netid` is replaced with your NetID.
  - If you're submitting some problems now and want to submit other problems later, name your PDF file `netid-hw1-1234.pdf`, where `1234` is replaced with the problems you are submitting at this time.
  - If you use the same name twice, only the most recent version will be graded!
- Submit your PDF file in Sakai. Don't forget to click the Submit (or Resubmit) button!

Problems

Each problem is worth 7 points. The remaining 2 points (on this and all future homeworks) are for legibility and clarity. (Hint: One way to ensure legibility is to use LaTeX!)

1. **Languages.** How might each of the following be represented as a formal language? Write what the alphabet would be. Show how to encode an instance as a string, either in words or by giving an example.

   (a) All base-10 positive integers that are multiples of 7.

(b) All grammatical sentences of English.

(c) All legal games of chess.

2. **Language classes.** Let FINITE be the class of all finite languages, and let

$$\mathsf{coFINITE} = \{L \mid L^C \in \mathsf{FINITE}\},$$

where, for any language L over Σ, $L^C = \Sigma^* \setminus L$.

(a) If $L \in \mathsf{coFINITE}$, how might you represent L in memory, and given a string w, how would you decide whether $w \in L$?

(b) Are there any languages in $\mathsf{FINITE} \cap \mathsf{coFINITE}$? If so, give an example; if not, explain why not.

(c) Are there any languages *not* in $\mathsf{FINITE} \cup \mathsf{coFINITE}$? If so, give an example; if not, explain why not.

3. **The regular operations.** Assume that L, L_1, L_2 are languages. For each of the following statements, state whether it is true or false. If true, write a short proof; if false, give a counterexample.

(a) $L \circ (L_1 \cup L_2) = LL_1 \cup LL_2$

(b) $(L_1 \cup L_2)^* = L_1^* \cup L_2^*$

(c) $(L_1 L_2)^* = L_1^* L_2^*$

4. **The reverse operation**

(a) Write a formal definition of the reverse operation. Informally, if w is a string, w^R is the string with all the symbols of w in reverse order. The definition should have the same structure as the definition of a string (see notes).

(b) Prove that $(w^R)^R = w$.

(c) Prove that if L is a language, $(L^R)^R = L$.