Homework 6: Non-context-free languages

CSE 30151 Spring 2016

Due 2016/03/15

Instructions

Please note that you will **lose one point** if you don't follow these instructions.

- You can prepare your solutions however you like, but you must submit them as a single PDF file.
- Please name your PDF `netid-hw6.pdf`, where `netid` is replaced with your NetID, or `netid-hw6-1234.pdf`, where `1234` is replaced with the problems you are submitting.
- If you use the same name twice, only the most recent version will be graded!
- Submit your PDF file in Sakai. Don't forget to click the Submit (or Resubmit) button!

Problems

Each problem is worth 7 points. An additional one point is for legibility, and one point for following the submission instructions.

1. **Pumping lemma for CFLs**. Use the pumping lemma to show that the following languages are not context free:
   (a) [Problem 2.30a] $\{\mathtt{0}^n\mathtt{1}^n\mathtt{0}^n\mathtt{1}^n\}$
   (b) [Problem 2.31] $\{w \in \{\mathtt{0},\mathtt{1}\}^* \mid w = w^R$ and w has an equal number of $\mathtt{0}$s and $\mathtt{1}$s$\}$
   (c) $\{\mathtt{a}^i \mid i$ is prime$\}$

2. **The SCRAMBLE operation** [Problem 2.43]. If w and w' are strings over an alphabet Σ, define the relation $w \mathrel{\mathring{=}} w'$ to be true iff w' is a permutation of w, that is, they have the same number of each type of symbol, but possibly in a different order. If w is a string and L is a language, define

$$\begin{aligned} \mathit{SCRAMBLE}(w) &= \{w' \mid w' \mathrel{\mathring{=}} w\} \\ \mathit{SCRAMBLE}(L) &= \bigcup_{w \in L} \mathit{SCRAMBLE}(w). \end{aligned}$$

   (a) Show that if $\Sigma = \{\mathtt{0}, \mathtt{1}\}$ and L is a regular language over Σ, then $\mathit{SCRAMBLE}(L)$ is context-free.

   (b) Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$. Show that there is a regular language over Σ such that $\mathit{SCRAMBLE}(L)$ is not context-free.

3. **Non-closure properties** [Exercise 2.2]

   (a) Use the languages

$$\begin{aligned} A &= \{\mathtt{a}^m\mathtt{b}^n\mathtt{c}^n \mid m, n \geq 0\} \\ B &= \{\mathtt{a}^n\mathtt{b}^n\mathtt{c}^m \mid m, n \geq 0\} \end{aligned}$$

   to prove that context-free languages are <u>not</u> closed under intersection.

   (b) Use (a) and DeMorgan's law to prove that context-free languages are <u>not</u> closed under complementation.

4. **Intersection again.** Instructor's note: Sorry for the edits, which are my fault. You will get full credit for doing any version of this problem.

   (a) ~~[cf. Problem 2.36] Let~~ $\cancel{L = \{\mathtt{a}^m\mathtt{b}^n\mathtt{c}^n\mathtt{d}^n \mid m, n \geq 1\}}$~~. Show that~~ $\cancel{L}$ ~~satisfies the pumping lemma for context-free languages (that is, the pumping lemma fails to show that~~ $\cancel{L}$ ~~is not context-free).~~

   (b) [Problem 2.18] Let C be any context-free language, and let R be any regular language. Prove that the language $C \cap R$ is context-free.

   (c) Show that $L = \{\mathtt{a}^m\mathtt{b}^n\mathtt{c}^n\mathtt{d}^n \mid m, n \geq 1\}$ is not a ~~regular~~ context-free language.

4′. **Intersection again.** Extra credit version: You can get up to 10 points out of 7 for doing this version instead of problem 4.

   (a) Let $L = \{\mathtt{a}^m\mathtt{b}^n\mathtt{c}^n\mathtt{d}^n \mid m, n \geq 1\} \cup \{\mathtt{b}^\ell\mathtt{c}^m\mathtt{d}^n \mid \ell, m, n \geq 1\}$. Show that L acts like a context-free language in the pumping lemma. In other words, give a pumping length p and demonstrate that for any $s \in L$ such that $|s| \geq p$, s can be written as $s = uvxyz$ (where $|vy| > 0$ and $|vxy| \leq p$) such that for all $i \geq 0$, $uv^ixy^iz \in L$.

(b) [Problem 2.18] Let C be any context-free language, and let R be any regular language. Prove that the language $C \cap R$ is context-free.

(c) Show that L is not a context-free language.