Homework 1: Strings and languages

CSE 30151 Spring 2017

Due: 2017/01/26 11:55pm

Instructions

- You can prepare your solutions however you like (handwriting, LaTeX, etc.), but you must submit them in PDF. You can scan written solutions in the library or using a smartphone (with a scanner app like CamScanner).
- Please give every PDF file a unique filename.
  - If you're making a complete submission, name your PDF file `netid-hw1.pdf`, where `netid` is replaced with your NetID.
  - If you're submitting some problems now and want to submit other problems later, name your PDF file `netid-hw1-123.pdf`, where `123` is replaced with the problems you are submitting at this time.
  - If you use the same filename twice, only the most recent version will be graded!
- Submit your PDF file in Sakai. Don't forget to click the Submit (or Resubmit) button!

Problems

Each problem is worth 10 points.

1. **Strings and languages.** How would you represent each of the following sets as a formal language? (There are many possible right answers.) For each set, write what the alphabet would be. Describe informally how to encode an element as a string. Give an example of a string belonging to the language and a string not belonging to the language. You don't need to explain how you would actually decide whether a string belongs to the language.
   (a) The set of valid telephone numbers.
   (b) The set of syntactically correct C programs.
   (c) The set of solvable Sudoku boards.

2. **String homomorphisms.** If Σ and Γ are finite alphabets, a *string homomorphism* is a mapping $\phi : \Sigma^* \to \Gamma^*$ that has the property that for any $u, v \in \Sigma^*$, $\phi(uv) = \phi(u)\phi(v)$. As you'll show below, string homomorphisms intuitively replace each symbol with a (possibly empty) string. We'll make use of them from time to time in this class.

   (a) Show that if ϕ is a string homomorphism, $\phi(\varepsilon) = \varepsilon$.

   (b) Show that if $w = w_1 \cdots w_n$ (where $w_i \in \Sigma$ for $1 \leq i \leq n$), then $\phi(w) = \phi(w_1) \cdots \phi(w_n)$.

   (c) Give an example of a string homomorphism ϕ that is not *injective*, that is, there exist u, v such that $u \neq v$ but $\phi(u) = \phi(v)$.

3. **Language classes.** The simplest example of a language class is the class of finite languages. Assume that Σ is a finite, nonempty alphabet. Let FINITE be the class of all finite languages over Σ, and let

$$\mathsf{coFINITE} = \{L \mid L^C \in \mathsf{FINITE}\},$$

   where, for any language L over Σ, L^C is the complement of L, that is, $\Sigma^* \setminus L$.

   (a) If $L \in \mathsf{FINITE}$, what data structure might you use to represent L in memory, and given a string w, how would you decide whether $w \in L$? (This is easy.)

   (b) If $L \in \mathsf{coFINITE}$, what data structure might you use to represent L, and given a string w, how would you decide whether $w \in L$?

   (c) Are there any languages in $\mathsf{FINITE} \cap \mathsf{coFINITE}$? Justify your answer with a short proof.

   (d) Are there any languages *not* in $\mathsf{FINITE} \cup \mathsf{coFINITE}$? Justify your answer with a short proof.