Homework 3: Regular expressions and non-regular languages

CSE 30151 Spring 2017

Due 2017/02/23 at 11:55pm

Instructions

- Create a PDF file (or files) containing your solutions.

- Please name your PDF file(s) as follows:

    - If you're making a complete submission, please name your PDF file `netid-hw3.pdf`, where `netid` is replaced with your NetID.
    - If you're submitting some problems now and want to submit other problems later, name your PDF file `netid-hw3-123.pdf`, where `123` is replaced with the problems you are submitting at this time.

- Submit your PDF file in Sakai. Don't forget to click Submit!

Problems

Each problem is worth 10 points.

1. **Regular expressions vs. Unix regular expressions.** Regular expressions and Unix regular expressions have some superficial differences, but also some deeper ones that affect the class of languages recognized.

    (a) Unix regular expressions do not have $\emptyset$. Prove that a regular expression without $\emptyset$ cannot describe the empty language, and conversely, if L is a nonempty regular language, it can be described by a regular expression without $\emptyset$.

    (b) Unix regular expressions have *backreferences*.[1] Give an example of a Unix regular expression[2] that uses backreferences to describe a nonregular language, and prove that this language is not regular. You may

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[1] `http://www.regular-expressions.info/backref.html`

[2] Let's define this to mean POSIX Basic Regular Expressions, but we aren't picky about this. `http://www.regular-expressions.info/posix.html`

not simply cite one of the non-regularity results proved in the book's Examples 1.73–77; please write out a full proof.

2. **Binary addition.**

   (a) [Problem 1.32] Let

   $$\Sigma_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\},$$

   that is, an alphabet of eight symbols, each of which is a 3-tuple of binary digits. Thus, a string over Σ_3 gives three rows of binary digits. Show that the following is regular:

   $B = \{ w \in \Sigma_3^* \mid$ the bottom row of w is the sum of the top two rows$\}$.

   Hint: Since it's easier to think about addition from right to left, design an automaton for B^R first, then convert it into an automaton for B.

   (b) [Problem 1.53] Let $\Sigma = \{0, 1, +, =\}$, and prove that the following is not regular:

   $ADD = \{x\text{=}y\text{+}z \mid x,y,z$ are binary natural numbers, and $x = y + z$ is true$\}$.

3. **Two similar but different languages** [Problem 1.49].

   (a) Let $B = \{1^k y \mid y \in \{0, 1\}^*$ and y contains at least k 1s, for $k \geq 1\}$. Show that B is a regular language. Hint: Try out some strings to see what does and doesn't belong to B, in order to find another simpler way of thinking about B.

   (b) Let $C = \{1^k y \mid y \in \{0, 1\}^*$ and y contains at most k 1s, for $k \geq 1\}$. Prove that C is not a regular language.