Statistics for the Life Sciences

Math 20340 Section 01, Fall 2008

Homework 10 Solutions

- **10.18**:
  - **a**: $16 + 8 - 2 = 22$
- **10.19**:
  - **a**: $s^2 = \frac{9*3.4+3*4.9}{10+4-2} = 3.775$
- **10.21**:
  - **a**: $H_0 : \mu_1 = \mu_2$; $H_a : \mu_1 \neq \mu_2$
  - **b**: Test statistic > 2.771 or < -2.771 (two-tailed test; t distribution with 27 d.o.f.)
  - **c**: Pooled estimator for variance: $\frac{15*4.8+12*5.9}{27} = 5.288...$; test statistic is $(\mu_1 - \mu_2)/\sqrt{5.288(1/16 + 1/13)} = 2.794...$
  - **d**: Since $t_{.005}$ is 2.771, the approximate p-value is just under 1%
  - **e**: Results are highly significant (accept H_a at 1% significance)
- **10.28**:
  - **a**: Sample 1 has $\bar{x}_1 = .0125$; $s_1 = .001509$. Sample 2 has $\bar{x}_2 = .0138$; $s_2 = .001932$. Pooled estimator is $s = .001733$. Test statistic is -1.68. Looking at t table (18 d.o.f.) we see that data shows no significant difference between the means (p-value over 10%).
  - **b**: 95% confidence interval for difference is

    $$(\bar{x}_1 - \bar{x}_2) \pm 2.101\sqrt{\frac{s_1^2}{10} + \frac{s_1^2}{10}} = (-.000328, .002928)$$

    Since 0 is in this interval, the result of part b) agrees with the result of part a).
- **10.35**:
  - **a**: The p-value is between 2% and 5%; so the difference is significant (we reject H_0 at 5%) but not highly so (we do not reject H_0 at 1%)
  - **b**: $(0.014, .586)$

- **c**: We would need at least 62 pairs (assuming s_d^2 stays at .16)

- **10.40**: The description seems to suggests a one-tailed test, but part a) seems instead to ask for a two-tailed test; I've done both.

  - **a**: $\mu_d = -16.77$ (taking Albertsons$-$Ralphs); $s_d = 11.18$. Assuming differences are normally distributed, test statistic (which has value -2.998) is a t distribution with 3 d.o.f.

    The critical values are $t_{.05} = 2.353. t_{.025} = 3.182$. So, if we are doing the two-tailed test $H_0 : \mu_d = 0$ against $H_a : \mu_d \neq 0$, the results are not significant; but if we are doing the one-tailed test $H_0 : \mu_d = 0$ against $H_a : \mu_d < 0$, the result is significant (we would reject null at 5% but not at 1%).
  - **b**: Two-tailed test: p-value is between 5% and 10%. One-tailed test: p-value is between 2.5% and 5%
  - **c**: $(-49.43, 15.89)$. At 1% significance, can't detect a difference between the averages

- **10.41**:

- **a**: There are two populations: drivers approaching Prohibitive signs, and drivers approaching Permissive signs. A random sample of drivers has been picked, and presented with Prohibitive signs. Then that *same* random sample is presented with Permissive signs. So there is a pairing of the two random samples: first driver in first sample goes with first driver of second sample, etc.

- **b**: The p-value is $< 1\%$, so there is a significant difference

- **c**: $(80.47, 133.32)$

- **10.48**: Test statistic is $24 * 21.4/15 = 34.24$. If the variance truly was 15, this would be a χ^2 reading with 24 d.o.f., with 5% critical value 36.41. So (at 5% significance) there is not enough evidence.

- **10.50**:

  - **a**: $s^2 = .699$.
  - **b**: $(.29027, 3.3897)$
  - **c**: Test statistic: 5.2428. Not enough evidence to reject null
  - **d**: p-value is greater than 20%

- **10.56**: Want to test $H_0 : \sigma^2 = 1600$ against $H_a : \sigma^2 > 1600$ using data $s^2 = 2350$ with $n = 40$. Test statistic is $39 * 2350/1600 = 57.28127$. Since we're doing a one-tailed test, we want to compare this to the critical value $\chi^2_{.05}$ (with 39 d.o.f.). We can't see this value directly from Table 5 in the book, because it skips from d.o.f. 30 to d.o.f. 40; but notice that the 5% critical value for 40 is 55.75, and the 5% critical values increase as d.o.f. increases; so the 5% critical value for 39 will be something (a little) less than 55.75. Therefore we can safely say that the data is significant; there is reason to believe (at 5% significance) that $\sigma^2 > 1600$ and so $\sigma > 40$.