5 Quiz 1 (Sept. 12) Name: SOLUTIONS

1. Formulate the following problem as a linear programming (LP) problem. Say what each of your variables represents, state the objective function and whether it is to be minimized or maximized, and state **all** constraints that must be imposed on the variables.

   A factory has two machines to make stuff. Machine one uses 80lbs of raw material per day of operation, requires 16 hours of labour per day, and produces 37lbs of stuff per day. Machine two uses 50lbs of raw material per day of operation, requires 35 hours of labour per day, and produces 43lbs of stuff per day. It is required that exactly 200lbs of stuff is produced per week (which can consists of up to seven full days of each machine running). Up to 300lbs of raw material can be purchased from supplier A at \$4 per lb, and an unlimited amount of raw material can be purchased from supplier B at \$5 per lb. 150 hours of labour is available at \$8 per hour, and an additional 30 hours of overtime labour is available at \$12 per hour. Only labour and raw materials used are paid for. It is allowable to purchase fractional lbs of raw material, use fractional hours of labour, and run each machine for a fractional number of days. What is the minimum cost required? [DON'T SOLVE!!! JUST SET UP!]

a = # days machine 1 is run for
b = # days machine 2 is run for
r_1 = lbs of raw material purchased for \$4/lb
r_2 = lbs of " " " " \$5/lb
l_1 = # hours regular labour used
l_2 = # hours overtime used

(*) These could also have been taken to be:
$\leq$
$\leq$
$\geq$
without changing the optimum

Labour cost: $16a + 35b$ hours of labour used ← this should $= l_1 + l_2$
$8l_1 + 12l_2$ labour cost

Materials cost: $80a + 50b$ lbs of raw materials used ← this should equal $r_1 + r_2$
$4r_1 + 5r_2$ materials cost

LP: Minimize total cost $8l_1 + 12l_2 + 4r_1 + 5r_2$

subject to:
$16a + 35b \overset{(*)}{=} l_1 + l_2$ (labour constraint)
$80a + 50b = r_1 + r_2$ (materials constraint)
$37a + 43b = 200$ (production constraint)
$r_1 \leq 300$ (supplier A constraint)
$l_1 \leq 150$ (regular labour constraint)
$l_2 \leq 30$ (overtime constraint)
$a, b, r_1, r_2, l_1, l_2 \geq 0$

2. Put the following LP problem in standard form: Maximize $a + b + c$ subject to

$$3a - 2b + 4c \leq 2$$

as well as $a \geq 0$, $b \geq 1$ and $c \leq 1$.

c is unconstrained ~~constrained~~, so introduce $c', c'' \geq 0$, $c = c' - c''$

Have two major constraints to deal with:

$$3a - 2b + 4c' - 4c'' \leq 2$$

and

$$c' - c'' \leq 1$$

Add two slack variables, change objective from Maximize to Minimize, to get standard form:

Minimize $-a - b - c' + c''$

subject to

$$3a - 2b + 4c' - 4c'' + d = 2$$

$$c' - c'' + e = 1$$

$$a, b, c', c'', d, e \geq 0$$