

The 4-color problem

Math 40210, Spring 2012

February 23, 2012

A brief history of the four color problem (I)

- 1852: Francis Guthrie, while idly colorings a map of counties of England, conjectured that any map can be 4-colored
- October 23, 1852: Augustus De Morgan wrote to William Rowan Hamilton, telling him about the problem, and suggesting a proof, which really just turned out to be a proof that K_5 is not planar
- June 10, 1854: “F.G.” published the conjecture in *The Athenaeum*

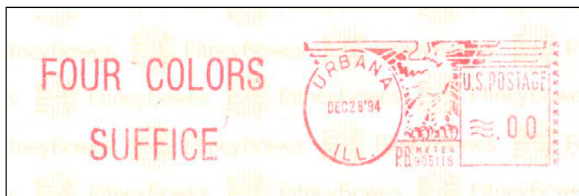
Tinting Maps.—In tinting maps, it is desirable for the sake of distinctness to use as few colours as possible, and at the same time no two conterminous divisions ought to be tinted the same. Now, I have found by experience that *four* colours are necessary and sufficient for this purpose,—but I cannot prove that this is the case, unless the whole number of divisions does not exceed five. I should like to see (or know where I can find) a general proof of this apparently simple proposition, which I am surprised never to have met with in any mathematical work. F. G.

A brief history of the four color problem (II)

- 1878: Arthur Cayley asked members of the London Mathematical Society “hasn’t anyone solved this yet?”
- 1879: Alfred Kempe proved the 4-color conjecture
- 1880: Independently, Peter Guthrie Tait gave another proof
- 1890: Percy Heawood discovered that Kempe’s proof was flawed, and proved the “5-color conjecture”
- 1891: Julius Petersen found a flaw in Tait’s proof
- 1891-1975: The 4-color conjecture was the most actively worked-on problem in graph theory

A brief history of the four color problem (III)

- 1976: Kenneth Appel and Wolfgang Haken published a 139-page, computer assisted proof, that was, to put it mildly, controversial



- 1997: Neil Robertson, Daniel Sanders, Paul Seymour and Robin Thomas published a 42-page, computer assisted proof, that is now widely accepted

Some references

There are many great books on the 4-color problem. Here are three of the best, ordered from most accessible to most mathematical:

- Robin Wilson, *Four Colors Suffice*, Princeton University Press
- Rudolf Fritsch & Gerda Fritsch, *The Four-Color Theorem*, Springer
- Robert Wilson, *Graphs, Colourings and the Four-colour Theorem*, Oxford Science Publications