Name:________________________

Instructor:________________________

MATH 20580: Introduction to Linear Algebra and Differential Equations

Exam 1 *February 17, 2011*

Record your answers to the multiple choice problems by placing an × through one letter for each problem on this page. There are 10 multiple choice questions worth 6 points each and 3 partial credit problems worth 10 points each. You start with 10 points. *On the partial credit problems you must show your work and all important steps to receive credit.*

You may not use a calculator.

1. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 3 \\ 1 \\ -2 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}$. Find the value of h such that $\mathbf{w} = \begin{bmatrix} 4 \\ 3 \\ h \\ 0 \end{bmatrix}$ is a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$.

(a) -2
(b) 1
(c) 3
(d) 2
(e) -5

2. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} -4 \\ -5 \\ 7 \end{bmatrix}$. Describe Span $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

(a) a plane
(b) $\mathbb{R}^3$
(c) a line
(d) a point
(e) three points

3. Determine by inspection which of the following sets is linearly independent.

(a) $\left\{ \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix} \right\}$

(b) $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$

(c) $\left\{ \begin{bmatrix} -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ -12 \end{bmatrix} \right\}$

(d) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

(e) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

4. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that maps $\begin{bmatrix} 1 \\ 0 \end{bmatrix} \to \begin{bmatrix} 2 \\ 5 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ 1 \end{bmatrix} \to \begin{bmatrix} -1 \\ 6 \end{bmatrix}$. Find the image of $\begin{bmatrix} 3 \\ -5 \end{bmatrix}$ under T.

(a) $\begin{bmatrix} 11 \\ -15 \end{bmatrix}$

(b) *undefined*

(c) $\begin{bmatrix} 15 \\ 7 \end{bmatrix}$

(d) $\begin{bmatrix} -13 \\ -7 \end{bmatrix}$

(e) $\begin{bmatrix} 13 \\ 11 \end{bmatrix}$

5. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that performs a horizontal shear, sending $\mathbf{e}_2$ to $\mathbf{e}_2 - 2\mathbf{e}_1$ and leaving $\mathbf{e}_1$ unchanged. Let $S : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that reflects points through the line $y = -x$, sending $\mathbf{e}_1$ to $-\mathbf{e}_2$ and $\mathbf{e}_2$ to $-\mathbf{e}_1$. Find the standard matrix for the composition $\mathbf{x} \to S(T(\mathbf{x}))$.

(a) $\begin{bmatrix} 0 & -1 \\ -1 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & -2 \\ -1 & 2 \end{bmatrix}$

(e) $\begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix}$

6. If $A = \begin{bmatrix} 3 & 4 \\ 1 & 1 \end{bmatrix}$ and $AB = \begin{bmatrix} 1 & 0 & -2 & 5 \\ 4 & -1 & 0 & 3 \end{bmatrix}$, find b_{24}, the $(2,4)$-entry of B.

(a) -4

(b) 8

(c) -2

(d) 3

(e) 11

7. Determine which of the following is *not* true for all $n \times n$ invertible matrices A, B, C.

(a) $AB = CA \Longrightarrow B = C$

(b) $(AB)^{-1} = B^{-1}A^{-1}$

(c) $(A^T)^{-1} = (A^{-1})^T$

(d) $A(B+C) = AB + AC$
(e) $AB = AC \Longrightarrow B = C$
8. Let B be an $n \times n$ matrix. Suppose the equation $B\mathbf{x} = \mathbf{c}$ is inconsistent for some $\mathbf{c}$ in $\mathbb{R}^n$. Determine which of the following statements *must* be true.
(a) The equation $B\mathbf{x} = \mathbf{0}$ has more than one solution.
(b) The columns of B span $\mathbb{R}^n$.
(c) The linear transformation $\mathbf{x} \to B\mathbf{x}$ is one-to-one.
(d) The matrix B has n pivot positions.
(e) The columns of B are linearly independent.

9. Determine the rank of the matrix $\begin{bmatrix} 0 & 0 & 1 & 2 & 1 \\ 2 & 4 & 0 & 2 & 0 \\ 3 & 6 & 2 & 7 & 2 \end{bmatrix}$.

(a) 2
(b) 1
(c) 3
(d) 4
(e) 5

10. Consider the basis $\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \right\}$ for $\mathbb{R}^3$. If $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$, find $\mathbf{x}$.

(a) $\mathbf{x} = \begin{bmatrix} 4 \\ 8 \\ 5 \end{bmatrix}$
(b) $\mathbf{x} = \begin{bmatrix} 4 \\ 6 \\ 7 \end{bmatrix}$
(c) $\mathbf{x} = \begin{bmatrix} -2 \\ -4 \\ 2 \end{bmatrix}$
(d) $\mathbf{x} = \begin{bmatrix} -2 \\ 5 \\ 0 \end{bmatrix}$
(e) $\mathbf{x} = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$

11. Find all solutions to the system of linear equations. Write your answer in parametric vector form.

$$\begin{cases} x_1 - x_2 + x_4 = 2 \\ 2x_1 - 2x_2 + x_3 = 1 \\ 5x_1 - 5x_2 + 2x_3 + x_4 = 4 \end{cases}$$

Answer:

12. Let $A = \begin{bmatrix} 1 & -2 & -5 & -4 \\ -3 & 6 & 8 & 5 \\ 2 & -4 & -9 & -7 \end{bmatrix}$. Find a basis for $\operatorname{Col} A$ and a basis for $\operatorname{Null} A$.

Answer:

13. Find the inverse of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 1 & -1 \\ 0 & -1 & 2 \end{bmatrix}$.

Answer: