Multiple Choice

1. (6 pts.) Find the reduced echelon form of the matrix $\begin{bmatrix} 1 & 2 & -1 & -1 \\ 2 & 4 & -1 & 0 \\ -3 & -6 & 1 & 0 \end{bmatrix}$.

(a) $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

2. (6 pts.) Determine by inspection which of the following sets is linearly independent.

(a) $\left\{ \begin{bmatrix} 2 \\ -5 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ -2 \end{bmatrix} \right\}$ (b) $\left\{ \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$

(c) $\left\{ \begin{bmatrix} 4 \\ -6 \\ 2 \\ -4 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 6 \\ -9 \\ 3 \\ -6 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 2 \\ 0 \end{bmatrix} \right\}$ (d) $\left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 2 \end{bmatrix} \right\}$

(e) all four sets are linearly dependent

3. (6 pts.) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ and $S : \mathbb{R}^3 \to \mathbb{R}^2$ be linear transformations with

$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ and

$S\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, $S\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $S\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

Which matrix below is the standard matrix of ST?

(a) $\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 2 & 0 \\ -2 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & -2 & 0 \\ 2 & 0 & -2 \\ 0 & 2 & 0 \end{bmatrix}$ (e) $\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}$

4. (6 pts.) The determinant of $\begin{bmatrix} 0 & 2 & -3 \\ -2 & 6 & -12 \\ 1 & -2 & 3 \end{bmatrix}$ is

(a) -6 (b) -12 (c) 0 (d) 6 (e) 12

5. (6 pts.) Let A be a 7×8 matrix of rank 3. Which of the following is equal to the dimension of the null space of A?

(a) 5 (b) 0 (c) 3 (d) 4 (e) 7

6. (6 pts.) Let $\mathcal{B}$ be the basis of $\mathbb{R}^3$ given by the vectors $\left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} \right\}$ and let x be the vector $x = \begin{bmatrix} 4 \\ 2 \\ 4 \end{bmatrix}$. Which of the following is the coordinate vector $[x]_{\mathcal{B}}$ of x with respect to $\mathcal{B}$?

(a) $\begin{bmatrix} -1 \\ 5 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} 14 \\ 0 \\ 6 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix}$

(d) $\begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}$ (e) $\begin{bmatrix} -2 \\ 0 \\ 5 \end{bmatrix}$

7. (6 pts.) Suppose an $n \times n$ square matrix A is such that the homogeneous linear system $Ax = 0$ has a non-trivial solution. Which of the following statements must be true?

(a) The linear system $Ax = b$ is inconsistent for some b in $\mathbb{R}^n$

(b) A has a pivot in every column.

(c) The linear map $T\colon \mathbb{R}^n \to \mathbb{R}^n$ given by $T(x) = Ax$ is onto.

(d) There is an $n \times n$-matrix B with $AB = \mathrm{I}_n$.

(e) The linear system $A^T x = 0$ has only the trivial solution.

8. (6 pts.) Which of the following is the solution for $\begin{bmatrix} x \\ y \end{bmatrix}$ of the matrix equation

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} h \\ k \end{bmatrix}?$$

(a) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5/2 & 3/2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} h \\ k \end{bmatrix}$

(b) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5/2 & -3/2 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} h \\ k \end{bmatrix}$

(c) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & 3/2 \\ 2 & -5/2 \end{bmatrix} \begin{bmatrix} h \\ k \end{bmatrix}$

(d) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & -3/2 \\ -2 & -5/2 \end{bmatrix} \begin{bmatrix} h \\ k \end{bmatrix}$

(e) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} h \\ k \end{bmatrix}$

9. (6 pts.) Let $A = \begin{bmatrix} 2 & 3 & 0 & 0 \\ 1 & 1 & 1 & 2 \\ 1 & 2 & -1 & -2 \end{bmatrix}$. What is the rank of A?

(a) 2 (b) 0 (c) 1 (d) 3 (e) 4

Partial Credit

You must show your work on the partial credit problems to receive credit!

10. (14 pts.) Express the solution set of

$$\begin{array}{rcrcrcrcr} x_1 & + & x_2 & - & x_3 & + & 2x_4 & = & -6 \\ x_1 & & & + & x_3 & + & x_4 & = & -3 \\ x_1 & - & x_2 & + & 3x_3 & & & = & 0 \end{array}$$

in *Parametric Vector Form.*

11. (14 pts.) The row-reduced echelon form of the 3×5 matrix $A = \begin{bmatrix} 2 & -4 & 1 & 1 & 5 \\ 3 & -6 & -2 & 5 & -7 \\ 5 & -10 & 3 & 2 & 4 \end{bmatrix}$ is given by $B = \begin{bmatrix} 1 & -2 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$. (You may assume this; you do not have to check it.)

(a) Determine a basis for the null space null(A).
(b) Determine a basis for the column space col(A).
(c) Determine a basis for the row space row(A).

12. (14 pts.) Compute the inverse of the matrix $A = \begin{bmatrix} 4 & 2 & 3 \\ 2 & 2 & 2 \\ 1 & 0 & 1 \end{bmatrix}$