

Exam II

October 30, 2014.

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Place an \times through your answer to each problem.

- | | | | | | |
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MC. _____

11. _____

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13. _____

Tot. _____

Multiple Choice

1. (6 pts.) Let $\mathbf{b}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ and $\mathbf{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$. Find $[\mathbf{x}]_{\mathbf{B}}$.

(a) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

(c) $\begin{bmatrix} 4 \\ 5 \end{bmatrix}$

(d) $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$

(e) $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

2. (6 pts.) Let $\{\lambda_1, \lambda_2\}$ be two eigenvalues of $\begin{bmatrix} 2 & 2 \\ -2 & 2 \end{bmatrix}$. Then product of the two eigenvalues, $\lambda_1 \lambda_2$, is equal to

(a) 4

(b) $4i$

(c) 8

(d) $-4i$

(e) -8

3. (6 pts.) Which of the following is a subspace of \mathbb{R}^2 ?

- (a) All the sets above.
- (b) Union of two distinct straight lines passing through 0.
- (c) Upper half plane.
- (d) Orthogonal complement of a set in \mathbb{R}^2 .
- (e) The set of all the eigenvectors of a 2×2 matrix.

4. (6 pts.) Let $A = \begin{bmatrix} -2 & 4 & 1 & 4 \\ -3 & 6 & -1 & 1 \\ 1 & -2 & 2 & 3 \\ 2 & -4 & 5 & 9 \end{bmatrix}$. Find a basis for $\text{Row}(A)$.

(a) $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

(c) All the rows of A

(d) Orthogonal complement of (a) set in \mathbb{R}^4 , $\begin{bmatrix} 1 \\ -2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

5. (6 pts.) Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, and let $\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\}$ be a basis of \mathbb{R}^2 . Find the \mathcal{B} -matrix for the transformation $\mathbf{x} \rightarrow A\mathbf{x}$.

- (a) $\begin{bmatrix} -6 & -7 \\ 5 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$ (e) $\begin{bmatrix} -6 & 5 \\ -7 & 6 \end{bmatrix}$

6. (6 pts.) Let $A = \begin{bmatrix} 1 & 3 & -2 & 5 & 3 \\ 0 & 1 & -1 & 2 & 1 \\ 2 & 1 & 1 & 2 & 3 \end{bmatrix}$. Which of the following is equal to the dimension of the column space of A ?

- (a) 4 (b) 2 (c) 1 (d) 3 (e) 5

7. (6 pts.) Which of the following statements is true of the length of vectors?

(a) $\mathbf{u} \cdot \mathbf{v} = -\mathbf{v} \cdot \mathbf{u}$

(b) $\mathbf{u} \cdot \mathbf{u} = \|\mathbf{u}\|^2$

(c) $|\mathbf{u} \cdot \mathbf{v}| = \|\mathbf{u}\| \|\mathbf{v}\|$

(d) The length of $\frac{\mathbf{u}}{\|\mathbf{u}\|}$ is $\frac{1}{\|\mathbf{u}\|}$.

(e) $\|\mathbf{u} + \mathbf{v}\|^2 = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2$

8. (6 pts.) Compute the orthogonal projection of $\begin{bmatrix} 1 \\ 7 \end{bmatrix}$ onto the line through $\begin{bmatrix} -4 \\ 2 \end{bmatrix}$ and the origin

(a) $\begin{bmatrix} -4 \\ 2 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ 7 \end{bmatrix}$

(c) $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

(d) $\frac{9}{5} \begin{bmatrix} -2 \\ 1 \end{bmatrix}$


(e) $\frac{1}{5} \begin{bmatrix} 1 \\ 7 \end{bmatrix}$

Partial Credit

You must show your work on the partial credit problems to receive credit!

11. (12 pts.) Let $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix}$.

(a) Find eigenvalues and the eigenspace corresponding to each eigenvalue.

 (b) Determine whether A is diagonalizable. If so, give the invertible matrix P and the diagonal matrix D such that $A = PDP^{-1}$. If not, give the reason.

- 12.** (12 pts.) Give a basis $\left\{ \begin{bmatrix} 0 \\ 2 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ -2 \\ 4 \end{bmatrix} \right\}$ for a subspace W , use the Gram-Schmidt process to produce an orthogonal basis for W .

13. (12 pts.) Let $\mathbf{y} = \begin{bmatrix} 6 \\ 3 \\ -2 \end{bmatrix}$, $\mathbf{u}_1 = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix}$ and $\mathbf{u}_2 = \begin{bmatrix} -4 \\ 3 \\ 0 \end{bmatrix}$.

(a) Verify that $\{\mathbf{u}_1, \mathbf{u}_2\}$ is an orthogonal set.

(b) Let \mathbf{w} be the orthogonal projection of \mathbf{y} onto $W = \text{Span}\{\mathbf{u}_1, \mathbf{u}_2\}$, find \mathbf{w} .

(c) Find the vector \mathbf{z} orthogonal to W such that $\mathbf{y} = \mathbf{w} + \mathbf{z}$.

(d) Calculate $\|\mathbf{y}\|^2$, $\|\mathbf{w}\|^2$ and $\|\mathbf{z}\|^2$ and find a numerical relations among these three numbers and briefly explain why.

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