Department of Mathematics
University of Notre Dame
Math 20580 – Fall 2014.

Name: _____

Instructor: _____
& Section

Exam 3

November 18, 2014

This exam is in 2 parts on 8 pages and contains 11 problems worth a total of 96 points. An additional 4 points will be awarded for following the instructions. You have 1 hour and 15 minutes to work on it. No calculators, books, notes, or other aids are allowed. Be sure to write your name on this page and to put your initials at the top of every page in case pages become detached. Good luck!

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Signature:_____

You must record here your answers to the multiple choice problems.
Place an × through your answer to each problem.

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)

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Answers to the partial credit problems should be circled on the page with the problem.
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MC. _____
9. _____
10. _____
11. _____
Tot. _____

Multiple Choice

1. (6 pts.) Let $\phi(x)$ be a solution to $\frac{dy}{dx} = \frac{1-x^2}{y}$ that satisfying $\phi(0) = 4$. Find $\phi(3)$.

(a) 1 (b) $\sqrt{3}$ (c) $\sqrt{2}$

(d) Not a real number (e) 2

2. (6 pts.) Find an integrating factor for the differential equation

$$y' - 2xy = xe^{-2x}.$$

(a) $-x^2$ (b) $-2x$ (c) e^{-2x^2} (d) e^{-x^2} (e) e^{-2x}

3. (6 pts.) Find all the stable equilibrium solutions of the autonomous system

$$\frac{dy}{dt} = y - y^3.$$

(a) $y = 0$ (b) $y = 0, y = -1$ (c) $y = 0, y = 1$

(d) $y = 1, y = -1$ (e) $y = 0, y = 1, y = -1$

4. (6 pts.) Which of the following initial value problems is guaranteed to have a unique solution?

(a) $y' = (y-1)^{1/5}, y(0) = 1.$

(b) $y' = y^{1/5}, y(0) = 0$

(c) $y' = (y-1)^{1/5}, y(1) = 0$

(d) $y' = y^{1/5}, y(1) = 0$

(e) $y' = (y-1)^{1/5}, y(1) = 1$

5. (6 pts.) Determine a maximum interval where the solution to the initial value problem is guaranteed to exist:

$$(9-t^2)y' = \ln(2-t) + y\sqrt{t+4}, \quad y(-2) = 0.$$

(a) No such interval exists

(b) $-3 < t < 2$

(c) $t < 2$

(d) $-4 < t < -1$

(e) $-3 < t$

6. (6 pts.) Find a least squares solution of the inconsistent system $A\mathbf{x} = \mathbf{b}$ for

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 1 & 2 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}.$$

(a) $\begin{bmatrix} 6 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 216 \\ 36 \end{bmatrix}$ (c) $\begin{bmatrix} 3 \\ \frac{1}{2} \end{bmatrix}$ (d) $\begin{bmatrix} 9 \\ 12 \end{bmatrix}$ (e) $\begin{bmatrix} 27 \\ 288 \end{bmatrix}$

7. (6 pts.) Which of the following statements is NOT true of the solutions to the differential equation

$$y' + 2y = 0 \ ?$$

(a) $e^{-2t} + c$, where c is an arbitrary scalar, is the general solution

(b) It is a first order differential equation

(c) The solutions form a vector space

(d) e^{-2t} is a solution

(e) It is a linear differential equation

8. (6 pts.) Solve the differential equation $y' = 3 - \frac{1}{2}y$ subject to the condition $y(0) = 1$.

(a) $y = 6 + 5e^{-\frac{1}{2}t}$

(b) $y = e^{-\frac{1}{2}t}$

(c) $y = 6 + e^{-\frac{1}{2}t}$

(d) $y = 6 - 5e^{-\frac{1}{2}t}$

(e) $y = 6 - 5e^{\frac{1}{2}t}$

Partial Credit

You must show your work on the partial credit problems to receive credit!

9. (16 pts.) Use the method of least squares to find the coefficients a_0 and a_1 so that the line $y = a_0 + a_1 x$ is the best fit to the (x, y) data points $(-2, 2), \quad (0, 3), \quad (2, 1)$.

10. (16 pts.) A tank initially contains 120 L of pure water. A mixture containing a concentration of $(2t + 4)$ g/L of salt enters the tank at a rate of 60 L/min, and the well-stirred mixture leaves the tank at the same rate. Find an expression for the amount of salt in the tank at any time t.

11. (16 pts.) Given the following autonomous system:

$$\frac{dy}{dx} = y(2 - y).$$

(a) Find a general solution for this differential equation.
(b) Find all the stable equilibrium solutions of the autonomous system.
(c) Solve the initial value problem provided that $y(0) = 1$.

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