

Answer Key 1

Math 20580

Your Name: _____

Final Exam May 8, 2007

Instructor's name: _____

Record your answers to the multiple choice problems by placing an \times through one letter for each problem on this answer sheet. There are 24 multiple choice questions. Each problem counts 6 points and you start with 6 points. Please sign the honor statement if you agree:

"I strictly followed the Notre Dame Honor Code during this test."

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1. Let $y_1(t)$ and $y_2(t)$ are two fundamental solutions $y'' + y' + \frac{\sin t}{t}y = 0$ with initial conditions $y_1(0) = 1, y_1'(0) = 0$ and $y_2(0) = 0, y_2'(0) = 1$. Then the Wronskian $W(t) = [y_1(t)y_2'(t) - y_1'(t)y_2(t)]$ is equal to

- (a) $\frac{\sin t}{t}$. (b) e^t . (c) $\frac{1}{t}$. (d) e^{-t} . (e) $\sin t$.

2. Let $Y(t) = A_0t^2 + A_1t + A_2$ be a solution to $y'' + 4y = 4t^2$ where $\{A_0, A_1, A_2\}$ are constant numbers. Then A_2 is equal to

- (a) -1 (b) 4 (c) 1 (d) 0 (e) $-\frac{1}{2}$

3. The linear system $\begin{pmatrix} 1 & 5 & -3 \\ 1 & 4 & -1 \\ 2 & 7 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -4 \\ -3 \\ h \end{pmatrix}$ has a solution if and only if $h =$

- (a) 2 (b) 1 (c) 5 (d) 3 (e) -5

4. Suppose that $Y(t) = At^s e^{-t} + B$ is a solution to $y'' - 3y' - 4y = -5e^{-t} - 4$, where $\{A, B, s\}$ are constant numbers. Then A is equal to

- (a) $-\frac{2}{5}$ (b) 4 (c) 1 (d) -4 (e) 1

5. Find the adjoint $adj(A)$ of $A = \begin{pmatrix} 1 & 4 \\ 2 & 7 \end{pmatrix}$.

(a) $\begin{pmatrix} 7 & 4 \\ 2 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} -7 & -4 \\ -2 & -1 \end{pmatrix}$

(c) $\begin{pmatrix} 7 & -4 \\ -2 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} 7 & -2 \\ -4 & 1 \end{pmatrix}$

(e) $\begin{pmatrix} -7 & 4 \\ 2 & -1 \end{pmatrix}$

6. The reduced row echelon form of $\begin{pmatrix} 3 & -1 & 3 \\ 6 & 0 & 12 \\ 2 & 1 & 7 \end{pmatrix}$ is equal to

(a) $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & 3 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

(c) $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{pmatrix}$

(e) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

7. Find the integrating factor μ for $dx + \left(\frac{x}{y} - \sin y + y^2\right)dy = 0$.

- (a) y (b) $\sin y$ (c) 1 (d) y^2 (e) x

8. Let $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \text{proj}_V \vec{u}$ where $\vec{u} = \begin{pmatrix} 1 \\ 3 \\ 1 \\ 7 \end{pmatrix}$ and $V = \text{Span}\left\{\frac{1}{2} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \frac{1}{2} \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}\right\}$. Then x_1 is
equal to

- (a) 2 (b) 4 (c) 1 (d) 0 (e) 3

9. Which of the following sets is an orthonormal basis of R^2 ?

(a) $\left\{ \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \begin{pmatrix} -4 \\ 3 \end{pmatrix} \right\}$

(b) $\left\{ \frac{1}{5} \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \frac{1}{5} \begin{pmatrix} -4 \\ 3 \end{pmatrix} \right\}$

(c) $\left\{ \frac{1}{5} \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \frac{1}{5} \begin{pmatrix} -4 \\ 3 \end{pmatrix}, 0 \right\}$

(d) $\left\{ \frac{1}{5} \begin{pmatrix} 3 \\ 4 \end{pmatrix} \right\}$

(e) $\left\{ \frac{1}{5} \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \frac{1}{5} \begin{pmatrix} -4 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\}$

10. Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 8 & 2 \end{pmatrix}$ and $A^{-1} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$. Then b_{11} is equal to

(a) 1

(b) -2

(c) 10

(d) -6

(e) 5

11. Let $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ be a solution to $\begin{pmatrix} 1 & -2 & 1 \\ 0 & 2 & -8 \\ 4 & -5 & -9 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 8 \\ 9 \end{pmatrix}$. Then x_1 is equal to
- (a) 3 (b) 16 (c) 8 (d) 9 (e) 29

12. Use the method of reduction of order to find a second solution $y_2 = v(t)y_1(t)$ of the given differential equation $t^2y'' + 2ty' - 2y = 0$ where $y_1(t) = t$. Then $v(t)$ is equal to

- (a) $\frac{4}{t}$ (b) t (c) t^{-3} (d) 1 (e) t^{-2}

13. Let r_1 and r_2 be two roots of the characteristic equation for $y'' + 100y = 0$. Then r_1 and r_2 are

(a) $\pm 10\sqrt{-1}$

(b) $0, 10$

(c) $-100, 0$

(d) ± 10

(e) $-10 \pm 10\sqrt{-1}$

14. If $\det A = 2$ where A is a 4×4 matrix, then $\det(-2A)$ is

(a) 32

(b) -4

(c) -32

(d) 16

(e) -16

15. The following two solutions form a fundamental set of solutions of linear homogeneous differential equation $2t^2y'' + 3ty' - y = 0$.

- (a) $t^{\frac{3}{2}}, t$ (b) t, t^{-1} (c) $t, 1$ (d) $t^{\frac{1}{2}}, t^{-1}$ (e) $t^{\frac{1}{2}}, 0$

16. If $\mathbf{B} = \{(1 \ 0), (1 \ 2)\}$ and $\vec{x} = (1 \ 6)$, then $[\vec{x}]_{\mathbf{B}}$ is equal to

- (a) $(1 \ 6)$ (b) $(1 \ 0)$ (c) $(3 \ 2)$ (d) $(-2 \ 3)$ (e) $(1 \ 2)$

17. The eigenvalues of $A = \begin{pmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{pmatrix}$ are

- (a) $-3, -5, -3$ (b) $1, -5, 0$ (c) $1, 3, 3$ (d) $1, 3, 5$ (e) $1, -2, -2$

18. Let $y(t)$ be the unique solution to the initial value problem $y'' - y = 0$, $y(0) = 2$, $y'(0) = 0$. Then $y(1)$ is equal to

- (a) $2e$ (b) 2 (c) $2e^{-1}$ (d) $e + e^{-1}$ (e) $2e - 2$

19. Let $y(t)$ be the unique solution to $y' + \frac{2}{t}y = 4t$ with initial condition $y(1) = 3$. Then $y(2)$ is equal to

- (a) 8 (b) $\ln 2 + 2$ (c) $4 + \frac{1}{2}$ (d) $e^4 + 2$ (e) $8 + \frac{1}{4}$

20. Let $Y(t) = v_1(t) \cos 3t + v_2(t) \sin 3t$ be a solution to $y'' + 9y = \frac{1}{\sin 3t}$. Then $v_2(t)$ is equal to

- (a) $\frac{1}{\sin 3t}$ (b) $\frac{t}{3}$ (c) $\cos 3t$
(d) $\frac{1}{9} \ln |\sin 3t|$ (e) $\frac{1}{3} \ln |\sin 3t|$

21. If $y' = 2y^{100}(3 - y)$ and $y(0) = 5$, then find $\lim_{t \rightarrow \infty} y(t)$. (Hint: This is an autonomous equation. You can find the answer by studying graphs of the solution).

- (a) 2 (b) 3 (c) 1 (d) 0 (e) 5

22. Let $y(t)$ be the unique solution to the initial value problem $y'' + 2y' + y = 0$, $y(0) = 1$, $y'(0) = 0$. Then $y(1)$ is equal to

- (a) e (b) e^{-1} (c) $e + e^{-1}$ (d) 1 (e) 0

23. Let $y(t)$ be the unique solution to the equation $y' = y^2$ with $y(0) = -1$. Then $y(1)$ is equal to

- (a) 0 (b) $\frac{-1}{2}$ (c) -4 (d) -1 (e) -3

24. The determinant of $\begin{pmatrix} 1 & 5 & 0 \\ 2 & 4 & 1 \\ 0 & -2 & 0 \end{pmatrix}$ is equal to

- (a) 1 (b) 2 (c) -2 (d) 0 (e) 5

1. By Abel's Theorem, $W = ce^{-\int p(t)dt} = ce^{-t}$. Evaluating at 0 shows $W = e^{-t}$.

2. $2A_0 + 4A_2 + 4A_1t + 4A_0t^2 = 4t^2$ so $A_2 = 1$, $A_1 = 0$ and $2A_0 + 4A_2 = 0$ so $A_2 = -1/2$.

$$3. \begin{bmatrix} 1 & 5 & -3 & | & -4 \\ 1 & 4 & -1 & | & -3 \\ 2 & 7 & 0 & | & h \end{bmatrix} \quad \begin{bmatrix} 1 & 5 & -3 & | & -4 \\ 0 & -1 & 2 & | & 1 \\ 0 & -3 & 6 & | & h+8 \end{bmatrix} \quad \begin{bmatrix} 1 & 5 & -3 & | & -4 \\ 0 & -1 & 2 & | & 1 \\ 0 & 0 & 0 & | & h+5 \end{bmatrix}$$

If this system has a solution $h + 5 = 0$.

$$4. Y'' = As(s-1)t^{s-2}e^{-t} - Ast^{s-1}e^{-t} - Ast^{s-1}e^{-t} + At^s e^{-t}$$

$$Y' = Ast^{s-1}e^{-t} - At^s e^{-t}$$

so $L[Y] = (A+3A-4A)t^s e^{-t} + (-2As-3As+0)t^{s-1}e^{-t} + (s(s-1)A+0+0)t^{s-2}e^{-t} - 4B = -5Ast^{s-1}e^{-t} + s(s-1)At^{s-2}e^{-t} - 4B$. Since $L[Y] = -5e^{-t} - 4$, $s = 1$, $B = 1$ and $A = 1$

5. $adj(A) = [a_{ij}]$ where $a_{ij} = (-1)^{i+j}C_{ji}$ and $C_{k\ell}$ is the determinant of the $k-\ell^{\text{th}}$ minor:

$$\text{hence } \begin{bmatrix} 7 & -4 \\ -2 & 1 \end{bmatrix}$$

$$6. \begin{bmatrix} 3 & -1 & 3 \\ 6 & 0 & 12 \\ 2 & 1 & 7 \end{bmatrix} \quad \begin{bmatrix} 6 & 0 & 12 \\ 3 & -1 & 3 \\ 2 & 1 & 7 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 2 \\ 3 & -1 & 3 \\ 2 & 1 & 7 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & -3 \\ 0 & 1 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$7. \frac{\partial M}{\partial y} = 0, \quad \frac{\partial N}{\partial x} = \frac{1}{y}. \quad \text{Hence } M_y - N_x = -\frac{1}{y} = -\frac{1}{y}N. \quad \text{Hence } \mu = e^{\ln y} = y$$

8. The vectors in V are an orthonormal pair. Hence $proj_V \vec{u} = (\vec{u} \cdot \vec{v}_1)v_1 + (\vec{u} \cdot \vec{v}_2)v_2$.

$$\vec{u} \cdot \vec{v}_1 = \frac{1}{2}(1+3+1+7) = \frac{12}{2} = 6.$$

$$\vec{u} \cdot \vec{v}_2 = \frac{1}{2}(1-3-1+7) = \frac{4}{2} = 2.$$

$$\text{Hence } proj_V \vec{u} = \frac{1}{2} \begin{pmatrix} 8 \\ 4 \\ 4 \\ 8 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 2 \\ 4 \end{pmatrix}$$

9. (a) is orthogonal but not unit length; (b) is orthonormal; (c) is not linearly independent; (d) is not a spanning set; (e) is not linearly independent

$$10. \begin{bmatrix} 1 & 1 & 1 & | & 1 & 0 & 0 \\ 2 & 3 & 2 & | & 0 & 1 & 0 \\ 3 & 8 & 2 & | & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 & | & 1 & 0 & 0 \\ 0 & 1 & 0 & | & -2 & 1 & 0 \\ 0 & 5 & -1 & | & -3 & 0 & 1 \end{bmatrix} \\ \begin{bmatrix} 1 & 1 & 1 & | & 1 & 0 & 0 \\ 0 & 1 & 0 & | & -2 & 1 & 0 \\ 0 & 0 & -1 & | & 7 & -5 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 & | & 1 & 0 & 0 \\ 0 & 1 & 0 & | & -2 & 1 & 0 \\ 0 & 0 & 1 & | & -7 & 5 & -1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 10 & -6 & 1 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & -7 & 5 & -1 \end{array} \right] \text{ and so } b_{11} = 10$$

$$\mathbf{11.} \left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 0 & & \\ 0 & 2 & -8 & 8 & & \\ 4 & -5 & -9 & 9 & & \end{array} \right] \left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 0 & & \\ 0 & 2 & -8 & 8 & & \\ 0 & 3 & -13 & 9 & & \end{array} \right] \left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 0 & & \\ 0 & 1 & -4 & 4 & & \\ 0 & 3 & -13 & 9 & & \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 0 & & \\ 0 & 1 & -4 & 4 & & \\ 0 & 0 & -1 & -3 & & \end{array} \right] \left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 0 & & \\ 0 & 1 & -4 & 4 & & \\ 0 & 0 & 1 & 3 & & \end{array} \right] \left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 0 & & \\ 0 & 1 & 0 & 16 & & \\ 0 & 0 & 1 & 3 & & \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -2 & 0 & -3 & & \\ 0 & 1 & 0 & 16 & & \\ 0 & 0 & 1 & 3 & & \end{array} \right] \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 29 & & \\ 0 & 1 & 0 & 16 & & \\ 0 & 0 & 1 & 3 & & \end{array} \right]$$

12. $y = tv$, $y' = v + tv'$, $y'' = v' + v' + tv'' = 2v' + tv''$ so $L[ty] = t^3v'' + t^2(2v' + 2v') + t(2v - 2v) = 0$ or $t^3v'' + t^24v' = 0$ or $tv'' = -4v'$ or $\frac{d}{dt} \ln|v'| = \frac{-4}{t}$. Then $\ln|v'| = -4 \ln|t| + C$ or $v' = At^{-4}$ and $v = Bt^{-3} + K$. Hence t^{-3} is the answer.

13. r_1 and r_2 are roots of $r^2 + 100r = 0$ so $r = \pm\sqrt{-100} = \pm 10\sqrt{-1}$.

14. $\det(cA) = c^n \det A$ if A is $n \times n$ so $\det(-2A) = (-2)^4 \det A = 32$.

15. We are looking for solutions of the form t^s so $2s(s-1) + 3s - 1 = 0$ or $2s^2 + s - 1 = 0$. $(2s-1)(s+1) = 0$ so $s = -1$ and $s = 1/2$.

16. We are being asked to solve $\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$. By inspection, second entry is 3 and then first entry is -2 so $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$. Or use Cramer's rule or row reduction.

17. To set up and solve the cubic equation is hard and takes a long time. The problem is easy because you have a short list of possible answers. The trace of the matrix is the sum of the eigenvalues with multiplicity. The trace is $1 + (-5) + 1 = -3$ and $1, -2, -2$ is the only one of the answers which sums to -3 .

18. The characteristic equation is $r^2 - 1 = 0$ so $y = ae^t + be^{-t}$ and $y' = ae^t - be^{-t}$. $y(0) = a + b = 2$ and $y'(0) = a - b = 0$. Hence $a = b = 1$ so $y = e^t + e^{-t}$ and $y(1) = e + e^{-1}$.

19. $L[t^r] = rt^{r-1} + 2t^{r-1} = (r+2)t^{r-1}$. Hence one particular solution is t^2 and a solution to the homogeneous system is t^{-2} so the general solution is $y = t^2 + \frac{C}{t^2}$. Hence $y(1) = 1 + C = 3$ so $C = 2$ and $y(2) = 2^2 + \frac{2}{2^2} = 4.5$ or $4 + \frac{1}{2}$.

20. Use Variation of Parameters. The Wronskian is

$$W(t) = \det \begin{bmatrix} \cos(3t) & \sin(3t) \\ -3\sin(3t) & 3\cos(3t) \end{bmatrix} = 3$$

One easy way to remember the formulas in the book is the following. A particular solution is given by

$$Y = \det \begin{vmatrix} u_2 & -u_1 \\ y_1 & y_2 \end{vmatrix} = \det \begin{vmatrix} \int \frac{g(t) y_1(t)}{W(t)} dt & \int \frac{g(t) y_2(t)}{W(t)} dt \\ y_1 & y_2 \end{vmatrix}$$

In this problem we want

$$\int \frac{g(t) y_1(t)}{W(t)} dt = \frac{1}{3} \int \frac{\cos(3t)}{\sin(3t)} dt = \frac{1}{3} \cdot \frac{1}{3} \cdot \ln |\sin(3t)|$$

21. The solution starts out in the strip above $y = 3$ since $y(0) = 5$ and hence it stays there. In this strip, y is decreasing since $y' < 0$. Hence the limit is 3.

22. The characteristic equation is $r^2 + 2r + 1 = 0$ so $r = -1$ is a double root and hence the general solution to the homogeneous equation is $y = ae^{-t} + bte^{-t}$, $y' = -ae^{-t} + be^{-t} - bte^{-t} = (b - a)e^{-t} - bte^{-t}$. Hence $y(0) = a + b = 1$ and $y'(0) = b - a = 0$ so $a = b = \frac{1}{2}$.

Hence $y = e^{-t} \frac{1+t}{2}$ so $y(1) = \frac{2}{2e} = \frac{1}{e}$.

23. The equation is separable so $\int y^{-2} dy = \int dt$ or $-y^{-1} = t + C$ or $y^{-1} = A - t$ or $y = \frac{1}{A-t}$. $Y(0) = \frac{1}{A} = -1$ so $y = \frac{-1}{1+t}$. Hence $y(1) = \frac{-1}{2}$.

24. Expand along the last column $\det A = 0 - \det \begin{vmatrix} 1 & 5 \\ 0 & -2 \end{vmatrix} + 0 = -(-2) = 2$.