Name: Date: 01/31/2019

M20580 L.A. and D.E. Tutorial

Worksheet 3

1. Describe all solutions of $A\mathbf{x} = \mathbf{0}$ in parametric form, where

$$A = \begin{bmatrix} 1 & -2 & -1 & -5 \\ 2 & -6 & 0 & -8 \\ 0 & 0 & -1 & -1 \end{bmatrix}$$

Solution: To solve the linear system, we will reduce the augmented matrix to the reduced echelon form.

$$\begin{bmatrix} 1 & -2 & -1 & -5 & 0 \\ 2 & -6 & 0 & -8 & 0 \\ 0 & 0 & -1 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -1 & -5 & 0 \\ 0 & -2 & 2 & 2 & 0 \\ 0 & 0 & -1 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -1 & -5 & 0 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & -1 & -1 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -3 & -7 & 0 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -4 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Hence we have

$$\begin{cases} x_1 = 4x_4 \\ x_2 = 0 \\ x_3 = -x_4 \\ x_4 \text{ is a free variable.} \end{cases}$$

Finally, the parametric form is given by $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = a \begin{bmatrix} 4 \\ 0 \\ -1 \\ 1 \end{bmatrix}$

2. Are the columns of B linearly independent vectors? (Show all work)

$$B = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 10 \\ -2 & -3 & -3 \end{bmatrix}$$

Solution: Let $\mathbf{b}_i$ be the i^{th} column vector of B. To see if $\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3$ are linearly independent we check if there is a nontrivial solution to $x_1\mathbf{b}_1 + x_2\mathbf{b}_2 + x_3\mathbf{b}_3 = \mathbf{0}$. Thus, we will reduce the augmented matrix $[B|\mathbf{0}]$.

$$\begin{bmatrix} 1 & 2 & 4 & 0 \\ 0 & 2 & 10 & 0 \\ -2 & -3 & -3 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 4 & 0 \\ 0 & 2 & 10 & 0 \\ 0 & 1 & 5 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 4 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 1 & 5 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -6 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Indeed this has nontrivial solutions, one of which is $x_1 = 6, x_2 = -5, x_3 = 1$. Therefore the columns of B are **NOT** linearly independent.