Performance Analysis of Coherent TCM Systems with Diversity Reception in Slow Rayleigh Fading

Saud A. Al-Semari and Thomas E. Fuja

Abstract—Coherent trellis-coded modulation (TCM) systems employing diversity combining are analyzed. Three different kinds of combining are considered: maximal ratio, equal gain, and selection combining (SC). First, the cutoff rate parameter is derived for equal gain combining (EGC) and SC assuming transmission over a fully interleaved channel with flat slow Rayleigh fading, which permits comparison with previously derived results for maximal ratio combining (MRC). Then, tight upper bounds on the pairwise error probabilities are derived for all three combining techniques. These upper bounds are expressed in product form to permit bounding of the bit error rate (BER) via the transfer function approach. In each case, it is assumed that the diversity branches are independent and that the channel state information (CSI) can be recovered perfectly.

Also included is an analysis of MRC when the diversity branches are correlated—the cutoff rate and a tight upper bound on the pairwise error probability are derived. It is shown that with double diversity a branch correlation coefficient as high as 0.5 results in only slight performance degradation.

Index Terms—Channel coding, diversity methods, fading channels, performance analysis, trellis-coded modulation.

I. INTRODUCTION

DIVERSITY combining is a well-known and effective method for improving the performance of digital communication systems over fading channels [1]–[3]. The basic principle of $M$-fold diversity is to use $M$ independent channels so that the probability of a “deep fade” on all channels is low. These independent channels can be created in a number of ways, including frequency, time, and/or polarization diversity; if multiple antennas are used to receive multiple versions of the received signal, the approach is called spatial diversity. A combining circuit is used to form a single resultant signal from the $M$ different “branch” signals. There are (at least) three different methods for combining.

- The optimal combining scheme is called maximal ratio combining (MRC) [2], [3]. In such a scheme, the matched filter output of each diversity path is weighted by the fading attenuation of that path. The resultant signal-to-noise ratio (SNR) at the output of the combiner is the sum of the SNR’s of the $M$ branches.
- In equal gain combining (EGC), the resultant signal is simply the unweighted sum of the signals from the $M$ branches.
- In selection combining (SC), the resultant signal is the one with highest SNR among the $M$-received signals; in practice, the strongest received signal, i.e., signal plus noise, is selected.

Error probability expressions for uncoded systems with different combining schemes in Rayleigh fading are presented in [4] and [5].

Bandwidth-efficient coding such as trellis-coded modulation (TCM) [6] also provides a form of diversity—time diversity. The performance of TCM schemes may be evaluated by computing the pairwise error probability and using the transfer function approach to upper bound the bit error rate (BER) [7]–[9].

Recently, the combined use of bandwidth-efficient codes with diversity reception has been investigated [10]–[14]. In [10], trellis-coded 16-QAM with MRC was proposed for use in a time-division multiple-access (TDMA) digital cellular system. Upper bounds on the bit error probability for TCM with different combining schemes were presented in [11] and [12]. These expressions use the Chernoff bound to establish an upper limit on the pairwise error probability and are loose. A tighter upper bound on the bit error probability for MRC has recently been developed by Ventura-Traveset et al. [14]; the upper bound in [14] requires numerical evaluation of the pairwise error probability and the use of a truncated transfer function.

In this paper, the use of TCM with diversity reception is investigated. Cutoff rate expressions for the Rayleigh distributed channel with diversity reception and the three different combining schemes are presented, as are tight upper bounds on the pairwise error probability. The same system configurations in [11] and [12] are used; the new bounds are shown to be tighter than those presented in [11] and [12].

The next section describes the system model and the combining metrics. In Section III, expressions for the cutoff rates for the three combining schemes are derived and compared. Tight upper bounds on the bit error probability for trellis-coded systems with the three combining schemes are derived and analyzed in Section IV. Section V analyzes the effect of branch correlation on MRC. Finally, Section VI gives conclusions.

II. SYSTEM MODEL

The underlying system can be described as follows. Suppose the complex signal $x_i$ is transmitted at time $i$ and $M$
corresponding signals \( y_i = \{y_{i,1}, y_{i,2}, \ldots, y_{i,M}\} \) are received, i.e.,
\[
\begin{align*}
y_{i,1} &= a_{i,1}x_i + n_{i,1} \\
y_{i,2} &= a_{i,2}x_i + n_{i,2} \\
&\vdots \\
y_{i,M} &= a_{i,M}x_i + n_{i,M}
\end{align*}
\]

where \( a_i = \{a_{i,1}, a_{i,2}, \ldots, a_{i,M}\} \) are the fading amplitudes assumed to be Rayleigh distributed and normalized so \( \mathbb{E}(a_i^2) = 1 \); we assume ideal interleaving and independent diversity branches, so \( a_i \) are independent and identically distributed (i.i.d.) Rayleigh. Here also, \( \{n_i\} \) are complex-valued noise samples with independent real and imaginary components, each Gaussian distributed with mean zero and variance \( \sigma_n^2 \).

The transmitter produces a sequence of signals \( z_N = \{z_1, z_2, \ldots, z_N\} \). At the receiver, the \( N \)-received \( M \)-tuples \( y_N = \{y_{1}, y_{2}, \ldots, y_{N}\} \) and the channel fade amplitudes \( a_N = \{a_1, a_2, \ldots, a_N\} \) are the inputs to a TCM decoder which performs maximum likelihood (ML) decoding assuming ideal channel state information, i.e., the assumption that \( a_N \) is available to the decoder means that the receiver can ascertain the severity of the fading during each signaling interval. Techniques such as pilot symbol insertion [17] or decision feedback coupled with adaptive linear prediction [18] can be employed to recover \( a_N \).

The decoder selects as its estimate of the transmitted sequence the one minimizing the decoding metric
\[
m(x_N, y_N; a_N) = \sum_{i=1}^{N} m(x_i, y_i; a_i).
\]

Here, the symbol metric \( m(x_i, y_i; a_i) \) depends on which form of signal combining is used:

- For MRC, the assumption of CSI means that the signal metric is given by
  \[
m(x_i, y_i; a_i) = -\sum_{k=1}^{M} |y_{i,k} - a_{i,k}x_i|^2.
\]

- For EGC
  \[
m(x_i, y_i; a_i) = \left(-\sum_{k=1}^{M} (y_{i,k} - a_{i,k}x_i)\right)^2.
\]

- For SC
  \[
m(x_i, y_i; a_i) = -|y_{i,j^*} - a_{i,j^*}x_i|^2
\]

where
\[
j^* = \arg\max\{a_{i,j}, j = 1, \ldots, M\}.
\]

A few comments about the model are appropriate. There is an implicit assumption that the fading is constant over a symbol duration and is nonfrequency selective as well, i.e., slow flat fading. Moreover, the independence of successive fading values assumes an ideal interleaver that “breaks up” the memory in the fading process due to Doppler shift. In [24], it was shown that for coded systems without diversity, practical interleavers could yield performance with less than a quarter-decibel loss, relative to ideal interleavers. Regarding the assumption of slow fading, if the fading rate becomes too great (relative to the signaling rate), then it becomes unrealistic to assume that the fading values can be accurately tracked; in this case, the fading values appearing in the above metrics (the \( a_i \)'s) would be replaced by values of unity.

Finally, it should be noted that in [11], Rasmussen and Wicker referred to the first metric as the “interleaved code combining” (ICC) metric. ICC is a diversity combining technique in which each of the \( M \)-received \( K \)-dimensional diversity signals is regarded as a component of a single \( MK \)-dimensional signal. Without CSI at the receiver, this technique may be regarded (as Rasmussen and Wicker did in [11]) to be a form of EGC, however, with the assumption of CSI at the receiver, this approach becomes equivalent to MRC.

### III. CUTOFF RATE FOR DIVERSITY RECEPTION

The cutoff rate of a communication channel is a real number \( R_0 \) such that, for any \( R < R_0 \), it is possible to construct for the channel a code with blocklength \( n \) and rate at least \( R \) with an average error probability \( P_e \leq 2^{-n(R_0 - R)} \). Furthermore, \( R_0 \) is the largest real number for which this property holds. Thus, \( R_0 \) provides both an achievable rate and a bound on error performance.

Cutoff rate is typically less than channel capacity; however, \( R_0 \) has been called the “practical capacity” in that communication at rates above \( R_0 \) seems to be far more complex to implement than communication at rates below \( R_0 \). (For example, at rates above \( R_0 \) the number of operations required in sequential decoding algorithms takes on an infinite variance.) While recent results regarding the near-capacity performance of “turbo codes” [19] would argue against a strict interpretation of this position, the large interleavers and resulting delay required by turbo codes suggest that, even now, a substantial price in complexity and/or delay must be paid to operate above \( R_0 \). Thus, cutoff rate remains a valid figure-of-merit in comparing coded modulation schemes, as originally suggested by Massey [20].

The pairwise error probability \( P(x_N \rightarrow \hat{x}_N) \) is the conditional probability that the metric associated with the coded sequence \( x_N \) exceeds that of \( x_N \), given \( x_N \) was in fact transmitted. It can be upper bounded using the Chernoff bound as follows:

\[
P(x_N \rightarrow \hat{x}_N) = P(m(x_N, y_N; a_N) - m(x_N, y_N; a_N) \geq 0) \\
\leq E[\exp(\lambda(m(x_N, y_N; a_N) - m(x_N, y_N; a_N)))] \\
= C(x_N, \hat{x}_N, \lambda) = \prod_{i=1}^{N} C(x_i, \hat{x}_i, \lambda)
\]
where
\[ C(x_i, \hat{x}_i; \lambda) = E\left[ \exp\left( \lambda(m(\hat{x}_i, y_i; \xi) - m(x_i, y_i; \xi)) \right) \right] \]  \hspace{1cm} (4)
and the expectation is taken with respect to the noise \( y_i \) and the fading \( \xi_i \).

The cutoff rate \( R_0 \) in bits/transmitted signal can be expressed as [21]
\[ R_0 = 2 \log_2(|A|) - \log_2 \left( \sum_{x_i \in A} \sum_{\hat{x}_i \in \mathcal{A}} C(x_i, \hat{x}_i) \right) \]  \hspace{1cm} (5)
where \( A \) is the signal set and \( C(x_i, \hat{x}_i) = \min_{\lambda} C(x_i, \hat{x}_i; \lambda) \).

For MRC, the cutoff rate was shown by Ventura-Traveset et al. [14] to be
\[ R_0 = 2 \log_2(|A|) - \log_2 \left( \sum_{x_i \in \mathcal{A}} \sum_{\hat{x}_i \in \mathcal{A}} \frac{1}{1 + \frac{|x_i - \hat{x}_i|^2}{4N_0}} \right)^M. \]  \hspace{1cm} (6)

Fig. 1 shows the cutoff rate values of the 16-QAM signal constellation and MRC with diversity orders of \( M = 1, 2, 3, 4 \). It is clear that the largest incremental gain is obtained in going from single to double diversity, with slightly less gain when the diversity is again doubled to \( M = 4 \). For example, the curves show that reliable communication at a rate of 2 b/symbol can be achieved at \( E_b/N_0 = 11 \) dB (or \( E_b/N_0 = 8 \) dB) for single-channel reception. However, the required SNR can be reduced to \( E_b/N_0 = 6.1 \) dB (or \( E_b/N_0 = 3.1 \) dB) if double diversity with MRC is used.

In EGC, the tightest conditional Chernoff bound is
\[ C(x_i, \hat{x}_i; \lambda|x_i) = \min\{C(x_i, \hat{x}_i, \lambda|x_i)\} = \exp\left(-\mu \frac{|x_i - \hat{x}_i|^2}{4MN_0}\right) \]  \hspace{1cm} (7)
where
\[ \mu = T^2 = \left( \sum_{k=1}^{M} |a_{i,k}| \right)^2. \]  \hspace{1cm} (8)

However, no closed-form expression for the sum of Rayleigh distributed random variables is available for the case of \( M > 2 \), so an approximate expression is used. This expression is based on the small argument approximation [2], [3]; Beaulieu [22] showed that this expression is very accurate for \( M \leq 8 \). The approximation to the probability density function (pdf) of \( f_T \) is given by
\[ f_T(t) = \frac{t^{(2M-1)}}{2^{2M-1}b_0^M(M-1)!} \exp(-t^2/2b_0) \]  \hspace{1cm} (9)
for \( t \geq 0 \), where
\[ b_0 = [(2M-1)!!]^{1/M} = [(2M-1) \cdot (2M-3) \cdots 1]^{1/M}. \]  \hspace{1cm} (10)

Recognizing that \( f_\mu(\mu) = f_T(\sqrt{\mu})/2\sqrt{\mu} \), we obtain the approximation
\[ f_\mu(\mu) = \frac{\mu^{(M-1)}}{b_0^M(M-1)!} \exp(-\mu/b_0). \]  \hspace{1cm} (11)
for $\mu \geq 0$, so $\mu$ has an $M$-Erlang distribution with parameter $1/b_0$. The last step is to perform the integration

$$C(x_i, \hat{x}_i) = \frac{1}{(2M-1)!} \cdot \int_0^\infty \mu^{M-1} \cdot \exp\left(-\frac{\mu}{b_0}\right) \exp\left(-\mu \frac{|x_i - \hat{x}_i|^2}{4MN_0}\right) d\mu. \quad (12)$$

Therefore, the cutoff rate for EGC receivers is expressed as

$$R_o = 2 \log_2(|A|) - \log_2\left(\sum_{x_i \in A} \sum_{\hat{x}_i \in A} \left(1 + \frac{|x_i - \hat{x}_i|^2}{M}\right)^M\right). \quad (13)$$

Comparing (13) with (6), we see that MRC always has a greater cutoff rate than EGC because $M/(2M-1)!/M$ is greater than one and monotonically increases with $M$. Fig. 2 shows the cutoff rate values of the 16-QAM signal constellation and EGC with a diversity order of $M = 1, 2, 3, 4$.

In the SC case, the Chernoff bound can be written as

$$C(x_i, \hat{x}_i, \lambda|\theta_i) = E_{\hat{x}_i} [\exp(\lambda (|x_i - \hat{x}_i|^2 - 2a_{i,j}^2| \cdot \exp(\lambda \cdot (x_i - \hat{x}_i)^2))]$$

Again, it can be simplified to

$$C(x_i, \hat{x}_i, \lambda|\theta_i) = \exp(-\lambda (a_{i,j}^2| \cdot \exp(\lambda \cdot (x_i - \hat{x}_i)^2))]$$

where

$$\nu = a_{i,j} = \max\{a_{i,1}^2, a_{i,2}^2, \cdots a_{i,M}^2\}. \quad (18)$$

Since $\{a_{i,1}, a_{i,2}, \cdots a_{i,M}\}$ are independent, $\nu$ is just the maximum of $M$ independent exponential random variables, each with mean one, so its pdf is given by

$$f_\nu(\nu) = M[1 - \exp(-\nu)]^{(M-1)} \exp(-\nu) \quad (19)$$

for $\nu \geq 0$, which can be rewritten using the binomial expansion as

$$f_\nu(\nu) = \sum_{k=1}^M (-1)^{k+1} M \left(\frac{M-1}{k-1}\right) \exp(-k\nu). \quad (20)$$
Performing the integration, the Chernoff factor simplifies to

$$C(x_i, \hat{x}_i) = \sum_{k=1}^{M} (-1)^{k+1} M \binom{M-1}{k-1} \frac{1}{k + \frac{|x_i - \hat{x}_i|^2}{4N_o}}.$$  \hfill (21)\\

Therefore, the cutoff rate for SC receivers is given by

$$R_o = 2\log_2(M)$$

$$= -\log_2 \left( \sum_{x_i \in A} \sum_{x_j \in A} \left[ \sum_{k=1}^{M} (-1)^{k+1} M \right.$$  

$$\left. \binom{M-1}{k-1} \frac{1}{k + \frac{|x_i - \hat{x}_i|^2}{4N_o}} \right] \right).$$  \hfill (22)\\

Fig. 3 shows the cutoff rate values of the 16-QAM signal constellation and SC with diversity orders of $M = 1, 2, 3, 4$.

IV. PAIRWISE ERROR PROBABILITY

In this section, tight upper bounds on the pairwise error probability are derived for the three combining schemes. Moreover, the pairwise error probability expressions are expressed in product form, i.e.,

$$P(x_N \rightarrow \hat{x}_N | a_N) = K_c \times \prod_{i=1}^{N} W(x_i, \hat{x}_i)$$  \hfill (23)\\

where $K_c$ is a constant that does not depend on the length of the error sequence and $W(x_i, \hat{x}_i)$ is the error weight profile between $x_i$ and $\hat{x}_i$. Expressing the pairwise error probability in this form allows the use of the transfer function technique of trellis codes to be used in bounding the BER.

A. Maximal Ratio Combiner

The conditional pairwise error probability for MRC can be expressed as

$$P(x_N \rightarrow \hat{x}_N | a_N) = P(m(\hat{x}_N, y_N; a_N) - m(x_N, y_N; a_N) \geq 0 | a_N)$$

$$= P \left( \sum_{i=1}^{N} \sum_{k=1}^{M} \left[ y_{i,k} - a_i x_i \right]^2 + a_i |\hat{x}_i|^2 \geq 0 | a_N \right)$$  \hfill (24)\\

which can be simplified to

$$P(x_N \rightarrow \hat{x}_N | a_N) = P \left( \sum_{i=1}^{N} \sum_{k=1}^{M} \left[ -a_i^2 |x_i - \hat{x}_i|^2 - 2a_i \hat{x}_i \right] + y_{i,k} \right)$$

$$\geq 0 | a_N \right)$$

$$= P \left( z_N \geq \sum_{i=1}^{N} \sum_{k=1}^{M} a_i |x_i - \hat{x}_i|^2 | a_N \right)$$  \hfill (25)\\

where $z_N$ is zero-mean Gaussian with variance $2N_o \sum_{i=1}^{N} \sum_{k=1}^{M} a_i^2 |x_i - \hat{x}_i|^2$. This probability can be
expressed as

$$P(\mathbf{x}_N \rightarrow \hat{\mathbf{x}}_N|\mathbf{a}_N) = \frac{1}{2} \text{erfc}\left(\sqrt{\sum_{i=1}^{N} \gamma_i d_i}\right) \tag{26}$$

where \(\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} e^{-t^2} \, dt\). Since the \(d_i\)'s are i.i.d. Rayleigh distributed random variables with \(E(d_i^2) = 1\), their squares are i.i.d. exponentially distributed random variables with a mean equal to one. Hence, \(\gamma_i\) will have an \(M\)-Erlang distribution with parameter one; its pdf is

$$f_{\gamma_i}(\gamma) = \frac{1}{(M-1)!} \gamma^{(M-1)} e^{-\gamma}, \quad \gamma \geq 0. \tag{27}$$

The unconditional pairwise error probability is thus

$$P(\mathbf{x}_N \rightarrow \hat{\mathbf{x}}_N) = \frac{1}{2} \int_{0}^{\infty} \cdots \int_{0}^{\infty} \text{erfc}\left(\sqrt{\sum_{i=1}^{N} \delta_i \omega_i}\right) \, d\omega_1 \cdots d\omega_N. \tag{28}$$

Define

$$\delta_i = \frac{d_i}{1 + d_i}, \quad \text{and} \quad \omega_i = \gamma_i(1 + d_i). \tag{29}$$

Then the unconditional pairwise error probability can be represented as

$$P(\mathbf{x}_N \rightarrow \hat{\mathbf{x}}_N) = \frac{1}{2} \prod_{i \in \eta} \frac{1}{(1 + d_i)^M} \int_{0}^{\infty} \cdots \int_{0}^{\infty} \text{erfc}\left(\sqrt{\sum_{i=1}^{N} \delta_i \omega_i}\right) \, d\omega_1 \cdots d\omega_N \tag{30}$$

where \(\eta = \{i; x_i \neq \hat{x}_i\}\) and \(L_\eta = |\Delta|\). Note that

$$\sum_{i \in \eta} \delta_i \omega_i \geq \delta_m \sum_{i \in \eta} \omega_i \tag{31}$$

where \(\delta_m = \min_{i \in \eta} \{\delta_i, i \in \eta\}\). Since \(\text{erfc}(x)e^{x^2}\) is monotonically decreasing for \(x \geq 0\), the pairwise error probability can be upper bounded by

$$P(\mathbf{x}_N \rightarrow \hat{\mathbf{x}}_N) \leq \frac{1}{2} \prod_{i \in \eta} \frac{1}{(1 + d_i)^M} \int_{0}^{\infty} \text{erfc}\left(\sqrt{\delta_m \Omega}\right) \, d\Omega \tag{32}$$

where \(\Omega = \sum_{i \in \eta} \omega_i\). Since the \(\omega_i\)'s are independent \(M\)-Erlang distributed random variables each with parameter one, \(\Omega\) will have an \((ML_\eta - 1)!\)-Erlang distribution with parameter one

$$f_{\Omega}(\Omega) = \frac{1}{(ML_\eta - 1)!} \Omega^{(ML_\eta - 1)} e^{-\Omega}, \quad \Omega \geq 0. \tag{33}$$

To evaluate the integrals, we use the following equality [8]:

$$\frac{1}{2K-1}! \int_{0}^{\infty} \text{erfc}(\sqrt{\tau y})e^{-y(1-x)y} \, dy = \frac{1}{2K} \sum_{j=1}^{K} \left(\frac{2K - j - 1}{K - 1}\right) \left(\frac{2}{1 + \sqrt{x}}\right)^{j} \tag{34}$$

which is valid for \(x < 1\). Integration yields

$$P(\mathbf{x}_N \rightarrow \hat{\mathbf{x}}_N) \leq \left[ \frac{1}{2ML_\eta} \sum_{j=1}^{ML_\eta} \left(\frac{2ML_\eta - j - 1}{ML_\eta - 1}\right) \left(\frac{2}{1 + \sqrt{\delta_m}}\right)^{j} \right] \times \prod_{i \in \eta} \left(\frac{1}{1 + d_i}\right)^{M}. \tag{35}$$

Consider the special case of uncoded BPSK modulation; in this case, the error event length is \(L_\eta = 1\), so the pairwise error probability is equal to the bit error probability \(P_b\). Also, since \(\delta_m = \delta_i\), the upper bound is satisfied with equality. Therefore

$$\delta_m = \delta_i = \frac{E_s}{N_0} \left(1 + \frac{E_s}{N_0}\right) \tag{36}$$

and

$$\frac{1}{1 + d_i} = 1 - \delta_i = (1 - \sqrt{\delta_i})(1 + \sqrt{\delta_i}). \tag{37}$$

Thus, bit error probability of uncoded BPSK can be expressed as

$$P_b = \frac{1}{2ML_\eta} \sum_{j=1}^{ML_\eta} \left(\frac{2ML_\eta - j - 1}{ML_\eta - 1}\right) \left(\frac{2}{1 + \sqrt{\delta_i}}\right)^{M-j}. \tag{38}$$

If we define \(k = M - j\), then \(P_b\) can be written as

$$P_b = \left(\frac{1 - \sqrt{\delta_i}}{2}\right)^{M-1} \sum_{k=0}^{M-1} \left(\frac{M+k-1}{2}\right) \left(\frac{1 + \sqrt{\delta_i}}{2}\right)^{k}. \tag{39}$$

This is exactly the same expression that appears in Proakis’ text [26].

Let \(L\) be the minimum time diversity of the code, i.e., the minimum Hamming distance, in signal symbols, between any two valid sequences. Then, \(L \leq L_\eta\) and we can further upper bound the pairwise error probability by

$$P(\mathbf{x}_N \rightarrow \hat{\mathbf{x}}_N) \leq \left[ \frac{1}{2ML_\eta} \sum_{j=1}^{ML_\eta} \left(\frac{2ML_\eta - j - 1}{ML_\eta - 1}\right) \left(\frac{2}{1 + \sqrt{\delta_m}}\right)^{j} \right] \times \prod_{i \in \eta} \left(\frac{1}{1 + d_i}\right)^{M}. \tag{40}$$

Not that the upper bound in [8] is a special case of this bound \((M = 1)\). The upper bound in (40) is in a product form, allowing us to use the transfer function approach to yield the following bound on the bit error probability:

$$P_b \leq \frac{1}{k} \left[ \frac{1}{2ML_\eta} \sum_{j=1}^{ML_\eta} \left(\frac{2ML_\eta - j - 1}{ML_\eta - 1}\right) \left(\frac{2}{1 + \sqrt{\delta_m}}\right)^{j} \right] . \frac{\partial T(D, 1)}{\partial I}\bigg|_{I=1, D=-E_s/4N_0} \tag{41}$$
Comparing this with the Chernoff bound, we note that the extra term on the left of the transfer function is at most one, so this bound is at least as tight as the Chernoff bound. To see the tightness of the bound, an eight-state I-Q TCM code employing 16-QAM is used as an example. Its bandwidth efficiency is 2 b/s/Hz. I-Q TCM codes are trellis codes in which the in-phase and quadrature components of the transmitted signal are encoded independently. Al-Semari and Fuja [23], [24] have shown that this approach yields better performance over Rayleigh fading channels than codes designed using the “traditional” approach. This particular code has a minimum time diversity of $L = 4$, and its performance is superior to that of the comparable eight-state conventional TCM code using 8-PSK in Rayleigh fading. (See [24] for details.) For this specific coding/modulation

$$\delta_m = \frac{0.8E_s/N_0}{1 + 0.8E_s/N_0}. \quad (43)$$

Fig. 4 compares the newly derived bound for dual diversity ($M = 2$) and MRC with the Chernoff bound for the same code; the new bound is slightly more than 1 dB tighter than the Chernoff bound at a BER of $10^{-5}$.

The performance of the 16-QAM eight-state code with MRC and different orders of diversity is shown in Fig. 5.

B. Equal Gain Combining

In the case of EGC, the conditional pairwise error probability is given by

$$P(x_N \rightarrow \hat{x}_N|a_N)$$

$$= P(m(\hat{x}_N, y_N; a_N) - m(x_N, y_N; a_N) \geq 0|a_N)$$

$$= P \left( \left| \sum_{i=1}^{N} (y_{i,t} - a_{i,t}\hat{x}_{i,t}) \right|^2 \geq 0|a_N \right). \quad (44)$$

It can be simplified to

$$P(x_N \rightarrow \hat{x}_N|a_N)$$

$$= P \left( \left| \sum_{i=1}^{N} a_{i,t} \right|^2 |x_i - \hat{x}_i|^2$$

$$- 2 \left( \sum_{i=1}^{M} a_{i,t} \right) \Re \left[ \sum_{i=1}^{M} n_{i,t}(x_i - \hat{x}_i)^* \right] \leq 0|a_N \right)$$

$$= P \left( z_N \geq \sum_{i=1}^{N} \left( \sum_{i=1}^{M} a_{i,t} \right)^2 |x_i - \hat{x}_i|^2|a_N \right) \quad (45)$$

where $z_N$ is a Gaussian random variable with variance $2N_cM \sum_{i=1}^{N} (\sum_{i=1}^{M} a_{i,t}) |x_i - \hat{x}_i|^{2}$ and zero mean. This probability can be expressed as

$$P(x_N \rightarrow \hat{x}_N|a_N) = \frac{1}{2} \text{erfc} \left( \frac{N}{\sqrt{\sum_{i=1}^{N} \mu_i d_i/b_0}} \right) \quad (46)$$
Fig. 5. The BER of the 16-QAM I-Q TCM eight-state code with MRC and different diversity orders. Solid (bound) and dashed (simulation).

where $\tilde{d}_i = b_i|x_i - \hat{x}_i|^2/4MN_o$ and $\mu_i = (\Sigma_{t=1}^{M} a_{it})^2$. Here, $\mu_i$ has pdf

$$f_\mu(\mu_i) = \frac{\mu_i^{(M-1)} \exp\left(-\frac{\mu_i}{b_0}\right)}{b_0^M (M-1)!}, \quad \mu_i \geq 0.$$  \hspace{1cm} (47)

Define $\Gamma_i = \mu_i/b_0$. Then

$$f_{\Gamma_i}(\gamma_i) = \frac{\gamma_i^{(M-1)} \exp(-\gamma_i)}{(M-1)!}, \quad \gamma_i \geq 0.$$  \hspace{1cm} (48)

Then the unconditional pairwise error probability can be expressed as

$$P(x_N \rightarrow \hat{x}_N) = \frac{1}{2} \int_0^\infty \cdots \int_0^\infty \text{erfc}\left(\sqrt{\sum_{i=1}^{N} \gamma_i \tilde{d}_i}\right) \times f_{\Gamma_1}(\gamma_1) \cdots f_{\Gamma_N}(\gamma_N) d\gamma_1 \cdots d\gamma_N.$$  \hspace{1cm} (49)

Similarly, define

$$\tilde{\delta}_i = \frac{\tilde{d}_i}{1 + \tilde{d}_i} \quad \text{and} \quad \tilde{\omega}_i = \gamma_i (1 + \tilde{d}_i).$$  \hspace{1cm} (50)

Therefore, the unconditional pairwise error probability can be represented as

$$P(x_N \rightarrow \hat{x}_N) \leq \frac{1}{2} \prod_{i \in \eta} \frac{1}{(1 + \tilde{d}_i)^M} \int_0^\infty \text{erfc}\left(\sqrt{\sum_{j \in \eta} \delta_j \tilde{\omega}_j}\right) \times f_{\tilde{\delta}_1}(\tilde{\delta}_1) f_{\tilde{\omega}_1}(\tilde{\omega}_1) \cdots f_{\tilde{\omega}_N}(\tilde{\omega}_N) \text{d}\tilde{\delta}_1 \cdots \text{d}\tilde{\omega}_N.$$  \hspace{1cm} (51)

where $\eta = \{i: x_i \neq \hat{x}_i\}$ and $L_\eta = |\eta|$. Note that

$$\sum_{i \in \eta} \delta_j \tilde{\omega}_i \geq \sum_{i \in \eta} \delta_{ij} \tilde{\omega}_i$$  \hspace{1cm} (52)

where $\delta_{ij} = \min\{\delta_i, i \in \eta\}$. Hence, the pairwise error probability can be upper bounded as

$$P(x_N \rightarrow \hat{x}_N) \leq \frac{1}{2} \prod_{i \in \eta} \frac{1}{(1 + \tilde{d}_i)^M} \int_0^\infty \text{erfc}\left(\sqrt{\hat{\delta}_i \hat{\Omega}}\right) \times f_{\tilde{\delta}_i}(\tilde{\delta}_i) f_{\tilde{\omega}_i}(\tilde{\omega}_i) \text{d}\tilde{\delta}_i \cdots \text{d}\tilde{\omega}_i.$$  \hspace{1cm} (53)

where $\hat{\Omega} = \sum_{i \in \eta} \tilde{\omega}_i$ and $\hat{\Omega}$ is distributed as

$$f(\hat{\Omega}) = \frac{1}{(ML_\eta - 1)!} \hat{\Omega}^{(ML_\eta - 1)} e^{-\hat{\Omega}}, \quad \hat{\Omega} \geq 0.$$  \hspace{1cm} (54)

Finally, performing the integration yields

$$P(x_N \rightarrow \hat{x}_N) \leq \left[\frac{1}{2^{2ML_\eta}} \sum_{j=1}^{ML_\eta} \left(\frac{2ML_\eta - j - 1}{ML_\eta - 1}\right) \left(\frac{2}{1 + \sqrt{\hat{\delta}_i}}\right)^j\right] \times \prod_{i \in \eta} \frac{1}{(1 + \tilde{d}_i)^M}.$$  \hspace{1cm} (55)

For the uncoded BPSK system, $\tilde{d}_i = b_i E_b/MN_o$, and so

$$\tilde{\delta}_i = \frac{\tilde{d}_i}{1 + \tilde{d}_i} = 1 - \bar{\delta}_i = (1 - \sqrt{\delta}_i)(1 + \sqrt{\delta}_i).$$  \hspace{1cm} (56)
Hence, the bit error probability of uncoded BPSK can be expressed as

\[ P_b = \frac{1}{2^{2M}} \sum_{j=1}^{M} \left( \frac{2M - j - 1}{M - 1} \right) \left( 2 \cdot \frac{1}{1 + \sqrt{\delta_b}} \right)^j \]

which can be written as

\[ P_b = \left( \frac{1 - \sqrt{\delta_b}}{2} \right)^M \sum_{k=0}^{M} \left( \frac{M + k - 1}{k} \right) \left( \frac{1 + \sqrt{\delta_b}}{2} \right)^k. \]

Again, since \( L \leq L_{op} \), we can further upper bound \( P(x_N \rightarrow \hat{x}_N) \) by

\[
\begin{align*}
P(x_N \rightarrow \hat{x}_N) & \leq \left[ \frac{1}{2^{2ML}} \sum_{j=1}^{ML} \left( \frac{2ML - j - 1}{ML - 1} \right) \left( 2 \cdot \frac{1}{1 + \sqrt{\delta_e}} \right)^j \right] \\
& \times \prod_{i \in \mathcal{N}} \left( \frac{1}{1 + \delta_i} \right)^M.
\end{align*}
\]

Therefore, the bit error probability can be tightly upper bounded by

\[
P_b \leq \frac{1}{k} \left[ \frac{1}{2^{2ML}} \sum_{j=1}^{ML} \left( \frac{2ML - j - 1}{ML - 1} \right) \left( 2 \cdot \frac{1}{1 + \sqrt{\delta_e}} \right)^j \right] \cdot \frac{\partial T(D_i, I)}{\partial I} \bigg|_{I=1, D=\epsilon_s/4N_0}.
\]

where

\[
D_{D=\epsilon_s/4N_0} = \frac{1}{1 + \frac{b_0 E_s}{4MN_0}}.
\]

For the same eight-state 16-QAM I-Q TCM scheme previously considered, \( \delta_e \) is given by

\[ \delta_e = \frac{0.8b_0 E_s/MN_0}{1 + (0.8b_0 E_s/MN_0)}. \]

The performance of the previous code with EGC and different orders of diversity is shown in Fig. 6.

### C. Selection Combining

With SC, the conditional pairwise error probability can be expressed as

\[
P(x_N \rightarrow \hat{x}_N | a_N) = P(m(\hat{x}_N, y_N | a_N) - m(x_N, y_N; a_N) \geq 0 | a_N)
\]

\[ = P\left( \sum_{i=1}^{N} (|y_{i,d} - a_{i,j} \cdot \hat{x}_i|^2 - |y_{i,d} - a_{i,j} \cdot \hat{x}_i|^2) \geq 0 | a_N \right). \]

This expression can be simplified to

\[
P(x_N \rightarrow \hat{x}_N | a_N) = P\left( \sum_{i=1}^{N} (-\alpha_{i,j} |x_i - \hat{x}_i|^2 - 2\alpha_{i,j} \cdot \Re \{a_{i,j} (x_i - \hat{x}_i) \} \leq 0 | a_N \right.
\]

\[ \leq P\left( \sum_{i=1}^{N} |x_i - \hat{x}_i|^2 | a_N \right)
\]

\[
= P\left( \sum_{i=1}^{N} |x_i - \hat{x}_i|^2 | a_N \right)
\]

\[
= P\left( \sum_{i=1}^{N} |x_i - \hat{x}_i|^2 | a_N \right)
\]

\[
= P\left( \sum_{i=1}^{N} |x_i - \hat{x}_i|^2 | a_N \right)
\]
where $\nu_i = a_i^2 j_i$, and $z_N$ is a Gaussian random variable with zero mean and a variance of $2N_0 \sum_{i=1}^{N_0} \nu_i$. This probability can be expressed as

$$P(\mathbf{x}_N \to \hat{x}_N | \mathbf{a}_N) = \frac{1}{2} \text{erfc} \left( \sqrt{\sum_{i=1}^{N} \nu_i d_i} \right)$$  \hspace{1cm} (64)

where $d_i = |x_i - \hat{x}_i|^2/4N_0$. The pdf of $\nu_i$ is

$$f_{\nu i}(\nu_i) = M(1 - e^{-\nu_i})^{M-1} e^{-\nu_i} = \sum_{k_i=1}^{M} M(-1)^{k_i+1} \binom{M-1}{k_i-1} e^{-\nu_i}.$$  \hspace{1cm} (65)

The unconditional pairwise error probability can be expressed as

$$P(\mathbf{x}_N \to \hat{x}_N)$$

$$= \frac{1}{2} \sum_{k_1=1}^{M} \cdots \sum_{k_N=1}^{M} \prod_{i=1}^{N} \left\{ M(-1)^{k_i+1} \binom{M-1}{k_i-1} \right\}$$

$$\times \int_0^\infty \cdots \int_0^\infty \text{erfc} \left( \sqrt{\sum_{i=1}^{N} \nu_i d_i} \right) e^{-\nu_i} d\nu_1 \cdots d\nu_N.$$  \hspace{1cm} (66)

Define

$$\delta_{i,k_i} = \frac{d_i}{k_i + d_i} \quad \text{and} \quad \omega_{i,k_i} = \nu_i(k_i + d_i).$$  \hspace{1cm} (67)

Hence, the pairwise error probability can be expressed as

$$P(\mathbf{x}_N \to \hat{x}_N)$$

$$= \frac{1}{2} \sum_{k_1=1}^{M} \cdots \sum_{k_N=1}^{M} \prod_{i=1}^{N}$$

$$\cdot \left\{ M(-1)^{k_i+1} \binom{M-1}{k_i-1} \frac{1}{(k_i + d_i)} \right\}$$

$$\times \int_0^\infty \cdots \int_0^\infty \text{erfc} \left( \sqrt{\sum_{i=1}^{N} \delta_{i,k_i} \omega_{i,k_i}} \right)$$

$$\cdot e^{\sum_{i=1}^{N} \delta_{i,k_i} \omega_{i,k_i}} \cdots e^{-\sum_{i=1}^{N} \omega_{i,k_i}}$$

$$d\omega_1 \cdots d\omega_N.$$  \hspace{1cm} (68)

Defining $\Gamma = \sum_{i \in \eta} \omega_{i,k_i}$, we therefore obtain

$$f_{\Gamma}(\gamma) = \frac{1}{(L_{\eta} - 1)!} \gamma^{(L_{\eta} - 1)} e^{-\gamma}, \quad \gamma \geq 0.$$  \hspace{1cm} (69)

Also, observe that

$$\sum_{i \in \eta} \delta_{i,k_i} \omega_{i,k_i} \geq \sum_{i \in \eta} \delta_{i,k_i} \nu_i.$$  \hspace{1cm} (70)

where

$$\delta_s = \min\{\delta_{i,k_i}, i \in \eta, k_i \in \{1, \cdots M\} = \frac{\min\{d_i\}}{M + \min\{d_i\}}.$$  \hspace{1cm} (71)

Using the above expressions, the pairwise error probability can be expressed as

$$P(\mathbf{x}_N \to \hat{x}_N)$$

$$\leq \frac{1}{2} \prod_{i \in \eta} \left[ \sum_{k_i=1}^{M} M(-1)^{k_i+1} \binom{M-1}{k_i-1} \frac{1}{(k_i + d_i)} \right]$$

$$\times \int_0^\infty \text{erfc} \left( \sqrt{\delta_s \gamma} \right) e^{(\delta_s \gamma)} f_{\Gamma}(\gamma) d\gamma.$$  \hspace{1cm} (72)

The final step is to evaluate the integral and replace $L_{\eta}$ by $L$. Doing so yields

$$P(\mathbf{x}_N \to \hat{x}_N)$$

$$\leq \left[ \frac{1}{2^{2L}} \sum_{j=1}^{L} \left( \frac{2L - j - 1}{L - 1} \right) \frac{2^j}{(1 + \sqrt{\delta_s})^j} \right]$$

$$\times \prod_{i \in \eta} \left[ \sum_{k_i=1}^{M} M(-1)^{k_i+1} \binom{M-1}{k_i-1} \frac{1}{(k_i + d_i)} \right].$$  \hspace{1cm} (73)

For the uncoded BPSK systems

$$d_i = E_s/N_0 \quad \text{and} \quad \delta_{i,k_i} = \frac{d_i}{k_i + d_i}$$

and

$$\frac{k_i}{k_i + d_i} = (1 - \sqrt{\delta_{i,k}})(1 + \sqrt{\delta_{i,k}}).$$  \hspace{1cm} (74)

Therefore

$$P_b = \frac{1}{2} \sum_{k_i=1}^{M} M(-1)^{k_i+1} \binom{M-1}{k_i-1} \frac{1}{(k_i + d_i)} \frac{1}{(1 + \sqrt{\delta_{i,k}})}.$$  \hspace{1cm} (75)

which can be finally written as

$$P_b = \frac{1}{2} \sum_{k_i=1}^{M} (-1)^{k_i+1} \binom{M}{k_i} \left( 1 - \sqrt{\frac{d_i}{k_i + d_i}} \right).$$  \hspace{1cm} (76)

For trellis-coded systems, the bit error probability can now be expressed as

$$P_b \leq \frac{1}{k} \left[ \frac{1}{2^{2L}} \sum_{j=1}^{L} \left( \frac{2L - j - 1}{L - 1} \right) \left( \frac{2}{1 + \sqrt{\delta_s}} \right)^j \right]$$

$$\times \frac{1}{k} \left( \frac{1}{k + d_i} \frac{1}{1 + \sqrt{\delta_{i,k}}} \right).$$  \hspace{1cm} (77)
Fig. 7. The BER of the 16-QAM I-Q TCM eight-state codes with SC and different diversity orders. Solid (bound) and dashed (simulation).

where

\[ D_{1} = -\frac{E_{s}}{4N_{0}} \sum_{k=1}^{M} M(-1)^{k+1} \frac{E_{s}}{k + \frac{1}{4N_{0}}} . \]  

(78)

For the same previous coding/modulation scheme, \( \delta_{s} \) is expressed as

\[ \delta_{s} = \frac{0.8E_{s}/N_{0}}{M + 0.8E_{s}/N_{0}} . \]  

(79)

The performance of the previous code with SC and different orders of diversity is shown in Fig. 7.

Comparing the three combining schemes shows that MRC achieves the best performance. EGC error performance is within 1 dB from MRC. As \( M \) increases, the difference between the three schemes increases. Also, it is obvious that the upper bound is very tight and gives very accurate BER values, especially at BER’s less than \( 10^{-3} \).

V. THE EFFECT OF BRANCH CORRELATION

In the previous analysis, we have assumed that the fading in the different diversity branches is independent. In some cases, this is difficult to achieve due to improper antennas positioning or receiver space limitations. Therefore, it is important to examine the possible performance degradation due to correlated branch signals. The effect of branch correlation on the distribution of the received signal was studied by Schwartz et al. in 1966 [2]. Recently, the effect of correlation on noncoherent orthogonal digital modulation was studied [25]. They derived upper bounds for binary convolutional codes and noncoherent orthogonal digital modulation.

In this section, the pairwise error probability for MRC with correlated branch signals is derived. Recall from (26) that the conditional pairwise error probability may be expressed as

\[ P(\mathbf{x}_{N} \rightarrow \hat{\mathbf{x}}_{N} | \mathbf{a}_{N}) = \frac{1}{2} \text{erfc} \left( \frac{1}{\sqrt{2}} \sum_{i=1}^{N} \sum_{l=1}^{M} \alpha_{il}^{2} d_{l} \right) \]

(80)

where \( d_{l} = |x_{i} - \hat{x}_{i}|^{2} / 4N_{0} \). Also, \( \alpha_{il}^{2} = |h_{il}|^{2} \), where \( h_{il} \) is a complex Gaussian random variable with zero mean and variance of 1/2 for both the real and imaginary parts. Observe that

\[ \sum_{l=1}^{M} \alpha_{il}^{2} = h_{i}^{*} h_{i} \]

(81)

where \( h_{i} = \{h_{i1}, \ldots, h_{iM}\} \) and \((\cdot)^{*}\) denotes the Hermitian transpose. The pdf of \( h_{i} \) is expressed as

\[ f(h_{i}) = \frac{1}{\sqrt{\pi} M \det K_{h_{i}}} \exp(-h_{i}^{*} K_{h_{i}}^{-1} h_{i}) \]

(82)

where \( K_{h_{i}} \) is an \( M \times M \) covariance matrix with entries \( (K_{h_{i}})_{jk} = E[h_{ij} h_{jk}^{*}] \). It is assumed that the real parts of \( h_{ij}^{*} \) are independent of the imaginary parts, i.e., the cross covariances are zero.

The first step is to uncorrelate the random variables using a linear transformation. We are interested in generating a new vector \( g_{i} = \{g_{i1}, \ldots, g_{iM}\} \) with a diagonal covariance matrix. Let \( U \) be the transformation, i.e.,

\[ g_{i} = U h_{i} . \]

(83)
Using this transformation, the covariance matrix of $q_i$ denoted by $K_{g_i}$ is expressed as

$$K_{g_i} = U K_{b_i} U^t$$  \hspace{1cm} (84)

where $U^t$ is the transpose of $U$. Since $K_{b_i}$ is a symmetric matrix, it can be represented as

$$K_{b_i} = Q \Lambda Q^t$$  \hspace{1cm} (85)

where $\Lambda$ is a diagonal matrix that consists of the eigenvalues of $K_{b_i}$ and $Q$ is a matrix whose columns are the orthonormal eigenvectors of $K_{b_i}$. The last equation can be rewritten as

$$K_{g_i} = \begin{cases} \lambda_j^2 & \text{if } i = j \\ 0 & \text{otherwise.} \end{cases}$$  \hspace{1cm} (86)

The second step is to make another transformation so that the covariance matrix becomes the identity matrix. This is achieved via the transformation

$$q_i = \sqrt{K_{g_i}} \tilde{q}_i.$$  \hspace{1cm} (87)

This makes the Gaussian random variables $q_i$ independent with same variance. Using this transformation, the unconditional pairwise error probability can be expressed as

$$P(x_N \rightarrow \hat{x}_N) = \frac{1}{2} \int_0^\infty \cdots \int_0^\infty \text{erfc} \left( \sqrt{\sum_{i=1}^N \sum_{j=1}^M \lambda_j d_{ij}} \right) \times f_{\tilde{q}(q_{11})} \cdots f_{\tilde{q}(q_{NM})} \, dq_{11} \cdots dq_{NM}$$  \hspace{1cm} (88)

where $q_{ij} = |p_{ij}|^2$. Define

$$\delta_{ij} = \frac{\lambda_j d_{ij}}{1 + \lambda_d d_{ij}} \quad \text{and} \quad \tilde{\omega}_{ij} = q_{ij}(1 + \lambda_d d_{ij}).$$  \hspace{1cm} (89)

Hence, the unconditional pairwise error probability can be expressed as

$$P(x_N \rightarrow \hat{x}_N) = \frac{1}{2} \prod_{i \in \eta} \prod_{l=1}^M \left( \frac{1}{1 + \lambda_d d_{il}} \right) \int_0^\infty \cdots \int_0^\infty \text{erfc} \left( \sqrt{\sum_{i=1}^N \sum_{j=1}^M \delta_{ij} \tilde{\omega}_{ij}} \right) \times f_{\tilde{\omega}(\tilde{\omega}_{11})} \cdots f_{\tilde{\omega}(\tilde{\omega}_{NM})} \, d\tilde{\omega}_{11} \cdots d\tilde{\omega}_{NM}$$  \hspace{1cm} (90)

where $\eta = \{i \in A : x_i \neq \hat{x}_i\}$ and $L_\eta$ is its cardinality. However,

$$\sum_{i \in \eta} \sum_{i=1}^M \delta_{ij} \tilde{\omega}_{ij} \geq \sum_{i \in \eta} \sum_{i=1}^M \delta_{ij} \tilde{\omega}_{ij}$$  \hspace{1cm} (91)

where $\delta_c = \min \{ \delta_{ij}, i \in \eta, l = 1, \ldots, M \}$. Hence, the pairwise error probability can be upper bounded as

$$P(x_N \rightarrow \hat{x}_N) \leq \frac{1}{2} \prod_{i \in \eta} \prod_{l=1}^M \frac{1}{1 + \lambda_d d_{il}} \int_0^\infty \text{erfc} \left( \sqrt{\delta_c \Phi} \right) \times e^{(\delta_c \Phi)} f_\Phi(\Phi) \, d\Phi$$  \hspace{1cm} (92)

where $\Phi = \sum_{i \in \eta} \sum_{l=1}^M \tilde{\omega}_{il}$. Since the $\tilde{\omega}_{il}$ are independent exponentially distributed random variables each with parameter one, $\Phi$ will have an $(ML_\eta - 1)$-Erlang distribution with parameter one, i.e.,

$$f_\Phi(\Phi) = \frac{1}{(ML_\eta - 1)!} \Phi^{(ML_\eta - 1)} e^{-\Phi}, \quad \Phi \geq 0.$$  \hspace{1cm} (93)

Finally, performing the integration yields

$$P(x_N \rightarrow \hat{x}_N) \leq \left[ \frac{1}{2^{2ML_\eta}} \sum_{j=1}^{ML_\eta} \left( \frac{2ML_\eta - j - 1}{1 + \sqrt{\delta_c}} \right)^{\frac{1}{2}} \right]$$ \hspace{1cm} (94)

where

$$D_{D = -E_d/4N_0} = \prod_{j=1}^{ML_\eta} \frac{1}{1 + \lambda_j \frac{E_d}{4N_0}}.$$  \hspace{1cm} (95)

Therefore, the bit error probability can be expressed as

$$P_b \leq \frac{1}{k} \left[ \frac{1}{2^{2ML_\eta}} \sum_{j=1}^{ML_\eta} \left( \frac{2ML_\eta - j - 1}{1 + \sqrt{\delta_c}} \right)^{\frac{1}{2}} \right] \cdot \frac{\partial T(\bar{D}, 1)}{\partial I} \bigg|_{I = \bar{D}, D = -E_d/4N_0}$$  \hspace{1cm} (96)

Similarly, the cutoff rate can be expressed as

$$R_o = 2 \log_2 (|A|) - \log_2 \left( \left( \frac{1}{1 + \lambda \frac{|A|}{4N_0}} \right)^{\frac{1}{2}} \right).$$  \hspace{1cm} (97)

As an example, dual diversity is used in many practical systems. The covariance matrix $K_{b_i}$ is represented as

$$K_{b_i} = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}.$$  \hspace{1cm} (98)

So, $\rho$ is the correlation coefficient between the two antenna elements; the eigenvalues of $K_{b_i}$ are $(1 - \rho)$ and $(1 + \rho)$, A four-state I-Q TCM 16-QAM scheme (2 b/s/Hz) is used as an example. For this configuration, $\delta_c$ will be

$$\delta_c = \left( 1 - \rho \right) (0.8 E_d/4N_0) \left( 1 + (1 - \rho) (0.8 E_d/4N_0) \right).$$  \hspace{1cm} (99)
A comparison between the bound and simulations is shown in Fig. 8 for the case of $\rho = 0.5$. Clearly, the bound is very tight. Fig. 9 shows the bit error probability upper bound curves for this code with different values of $\rho$. It is noted that values as large as $\rho = 0.5$ degrade the performance slightly. The effect of space correlation is not as severe as time correlation (which can be minimized via interleaving).

Finally, a comparison between three schemes is shown in Fig. 10. The first scheme uses a four-state code and MRC of two branches. The branch correlation $\rho$ is assumed to be 50%. The second scheme uses a 64-state code, but no diversity combining. Both schemes are 16-QAM. The third scheme employs diversity only. It uses uncoded QPSK with Gray mapping; the diversity order is $M = 3$ and independent.
branches are assumed. All systems have a bandwidth efficiency of 2 b/s/Hz. Simulation results are plotted for the first two systems and analytical values are shown for the third system. Clearly, the first scheme outperforms the other schemes at BER $<4 \times 10^{-3}$ even though moderate branch correlation (50%) exists. This suggests that a combination of simple channel coding and double diversity might yield in general better performance than using complex channel coding schemes or several diversity receivers. Moreover, increased delay and interleaving for complicated channel codes are avoided. This results in less system delay, which is favorable in mobile and personal communications.

VI. CONCLUSIONS

In this paper, cutoff rate expressions of coherent systems with maximal ratio, equal gain, and SC schemes have been evaluated using Chernoff bounds. Moreover, tight upper bounds on the pairwise error probability have been derived. These upper bounds were used to evaluate a variety of system configurations, including uncoded and coded systems. The upper bounds were expressed in product form to allow the use of the transfer function approach for evaluating the performance of trellis-coded systems. Simulations of different systems show that the derived bounds are very tight.

For the case of branch correlation, the cutoff rate and a tight upper bound on the pairwise error probability were derived for MRC. Again, the pairwise error probability was expressed in product form so that the transfer function approach could be used. Branch correlation with correlation coefficients less than 0.5 results in a slight performance loss. The results indicate that the joint use of simple coding and diversity results in a substantial improvement in Rayleigh fading over the use of a separate more complex codes (without diversity) or a higher degree of diversity (without coding).

REFERENCES


Saud A. Al-Semari was born on December 25, 1968. He received the B.Sc. and M.Sc. degrees in electrical engineering from King Fahd University of Petroleum & Minerals (KFUPM), Dhahran, Saudi Arabia, in 1991 and 1992, respectively. He received the Ph.D. degree from the University of Maryland, College Park, in 1995.

Since 1996, he has been an Assistant Professor of Electrical Engineering at KFUPM. He is also the Director of the Communications and Computer Research Center at KFUPM. His current research interests are coded modulation, error control coding, and wireless communication systems and networks.

Thomas E. Fuja received the B.S.E.E. and B.S.Comp.E. degrees in 1981 from the University of Michigan, Ann Arbor, and the M.Eng. and Ph.D. degrees in electrical engineering in 1983 and 1987, respectively, from Cornell University, Ithaca, NY.

In 1998, he joined the faculty of the University of Notre Dame, South Bend, IN, where he is an Associate Professor of Electrical Engineering. Prior to 1998, he was on the faculty of the University of Maryland, College Park. He also recently served as Program Director for Communications Research at the U.S. National Science Foundation. His research interests are in the area of digital communication systems—specifically coding theory and applications and information theory. Most of his recent research has focused on coding for wireless applications and on the interface between source coding and channel coding.

Dr. Fuja was the recipient of a Presidential Young Investigator Award from the National Science Foundation in 1989. In 1991, he received the George Corcoran Memorial Award for his significant contributions to electrical engineering education at the University of Maryland. He has been active in the IEEE Information Theory Society and is currently serving as a Member of its Board of Governors.