

# Supplemental Materials to “Model-based Differentially Private Data Synthesis”

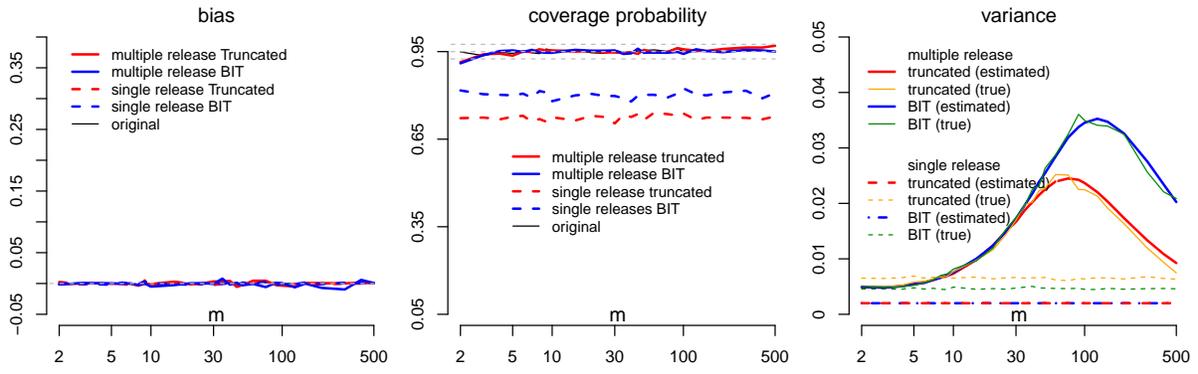
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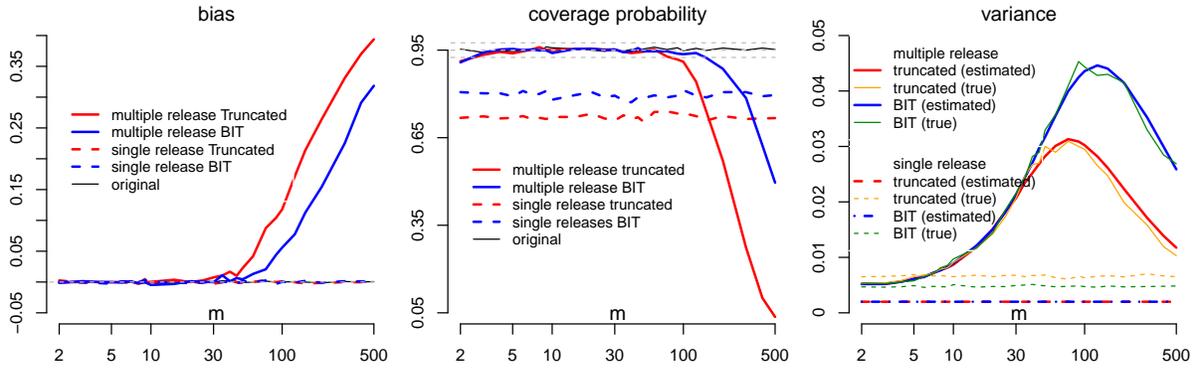
January 9, 2017

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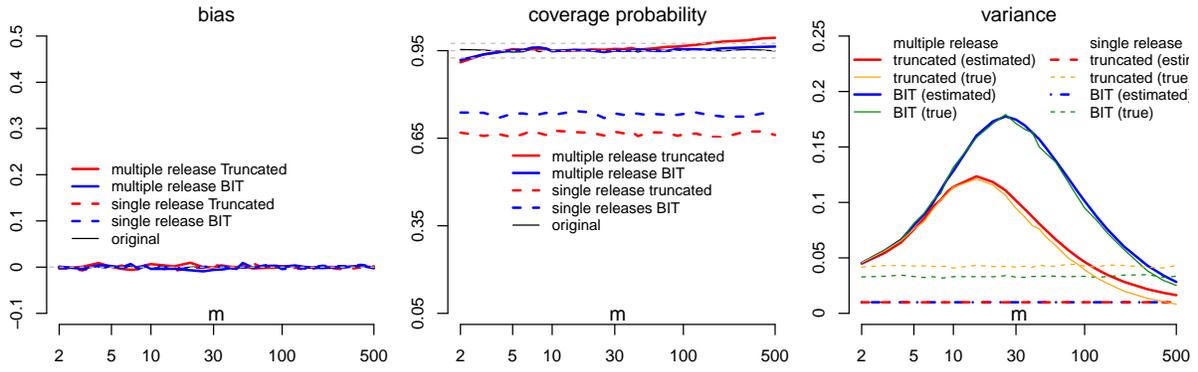
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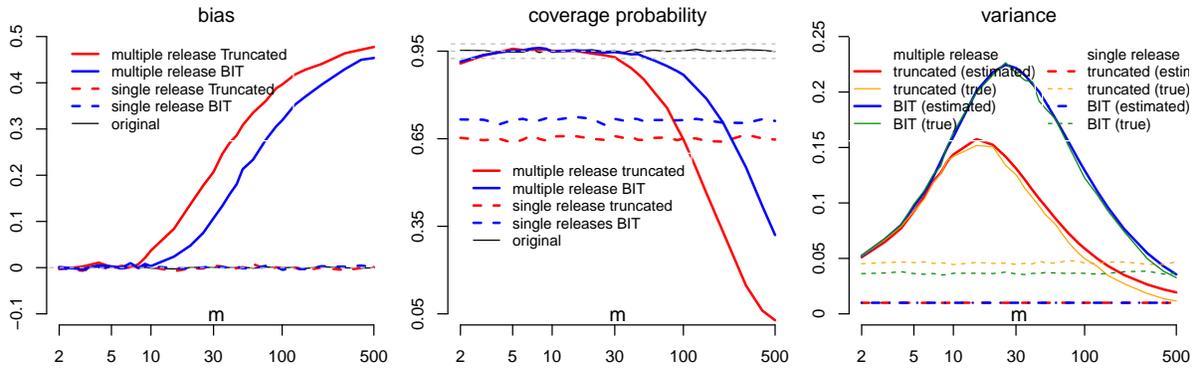
(a)  $n = 1000, (c_0, c_1) = (-4, 4)$



(b)  $n = 1000, (c_0, c_1) = (-4, 5)$



(c)  $n = 100, (c_0, c_1) = (-4, 4)$



(d)  $n = 100, (c_0, c_1) = (-4, 5)$

Figure S1: inferences based on synthetic data via modips ( $m \in [2, 500]$ )

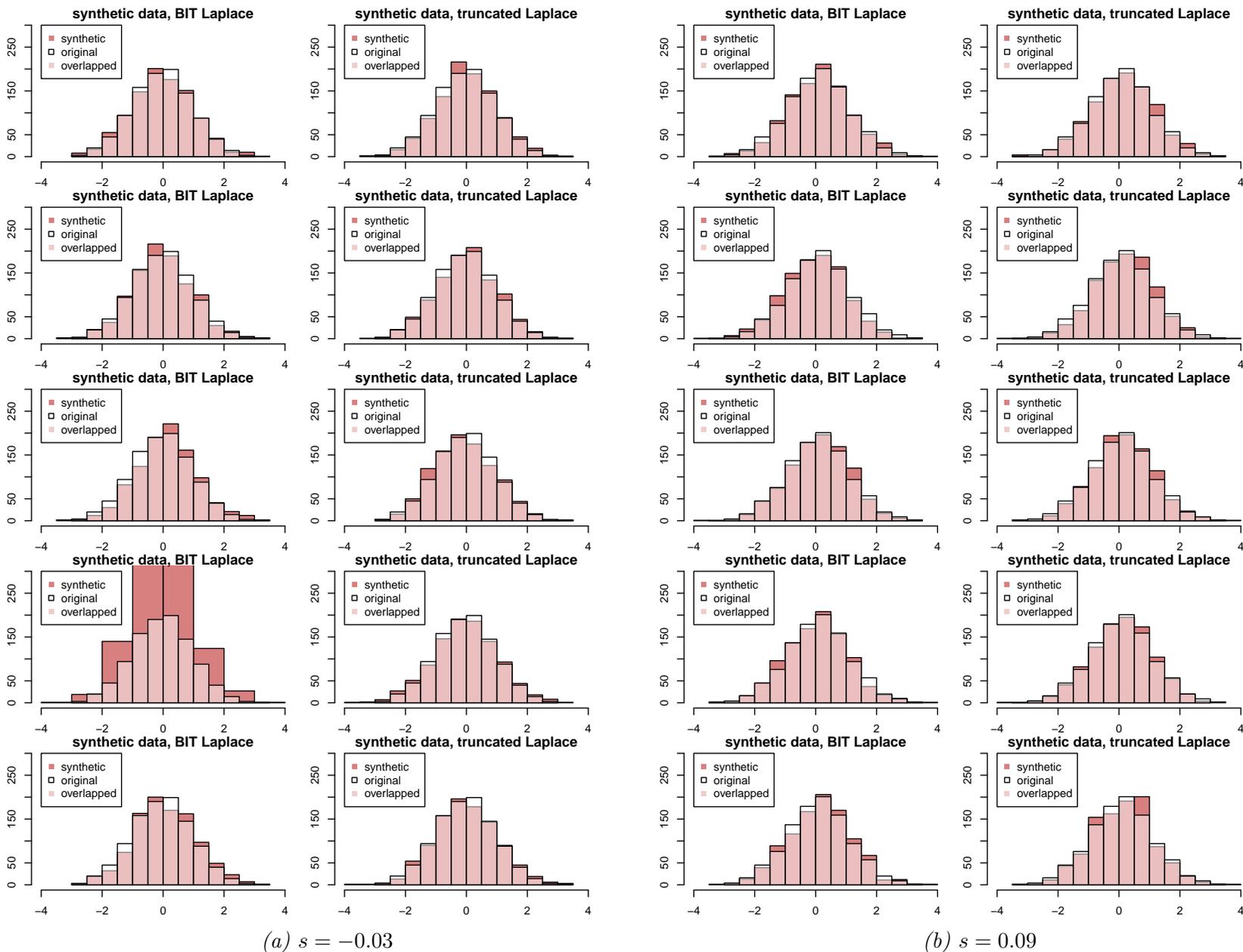


Figure S2: examples of synthetic data via modips in the simulation study ( $n = 1000, m = 5, [c_0, c_1] = [-4, 4]$ )

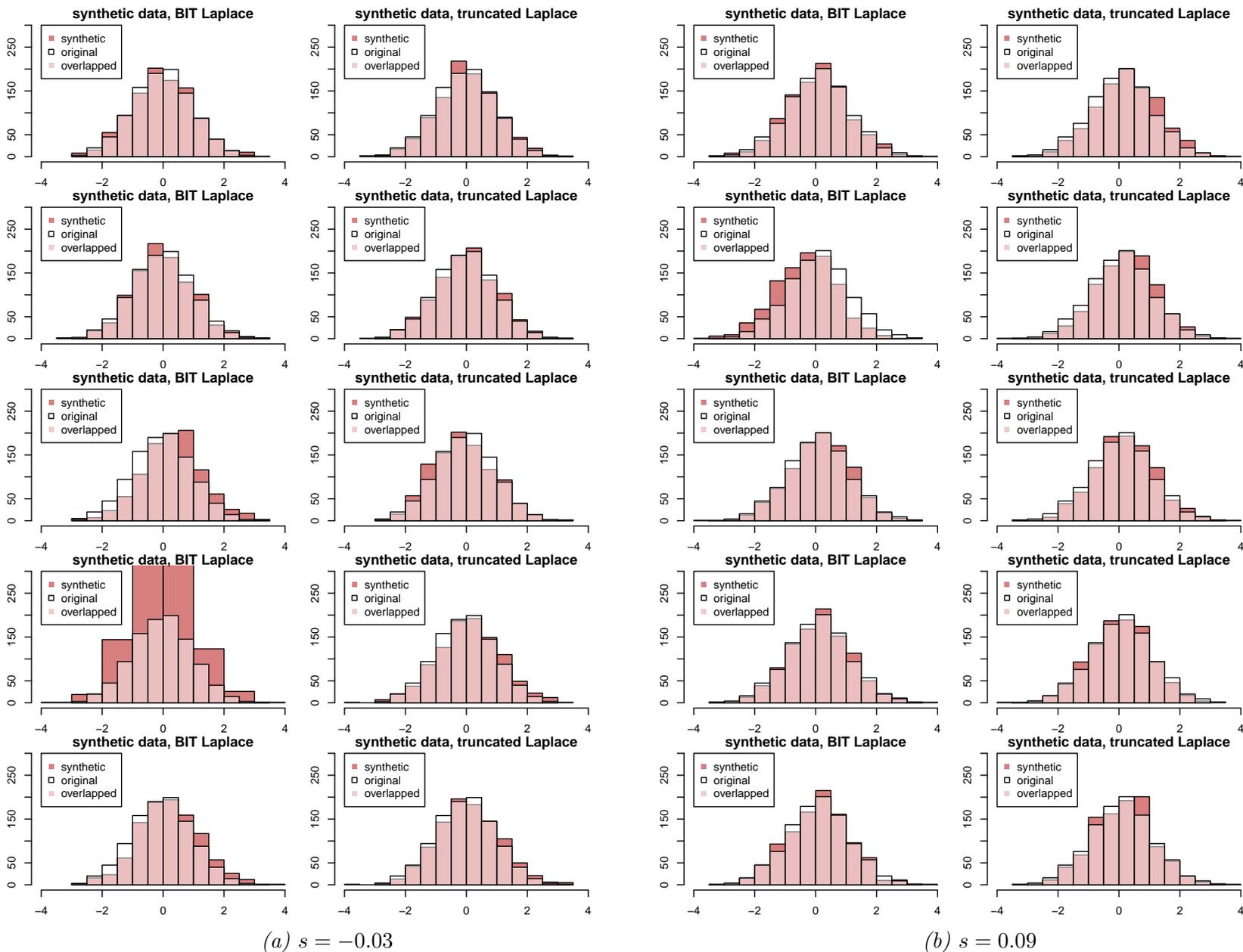


Figure S3: examples of synthetic data via modips in the simulation study ( $n = 1000, m = 15, [c_0, c_1] = [-4, 4]$ )

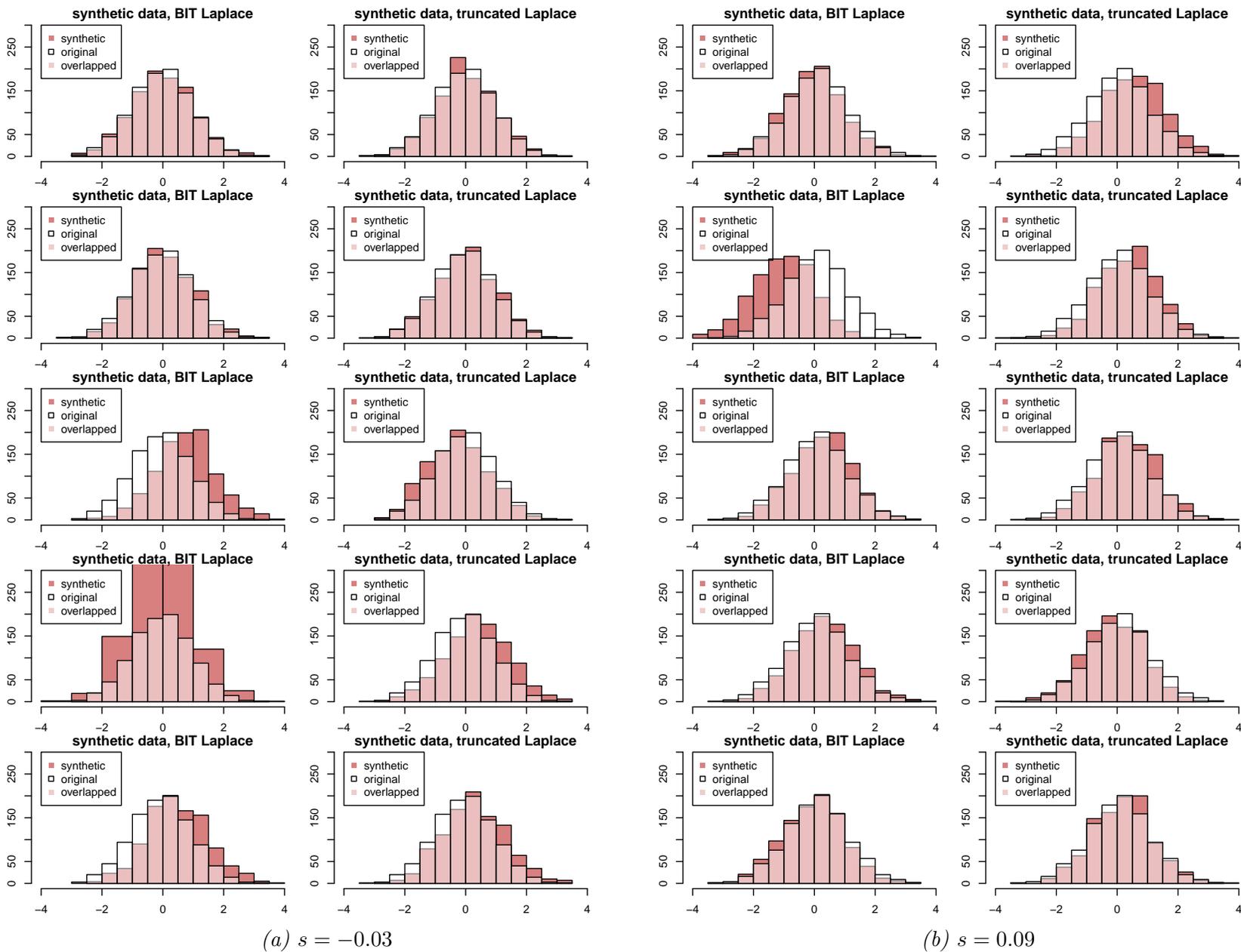


Figure S4: examples of synthetic data via modips in the simulation study ( $n = 1000, m = 40, [c_0, c_1] = [-4, 4]$ )

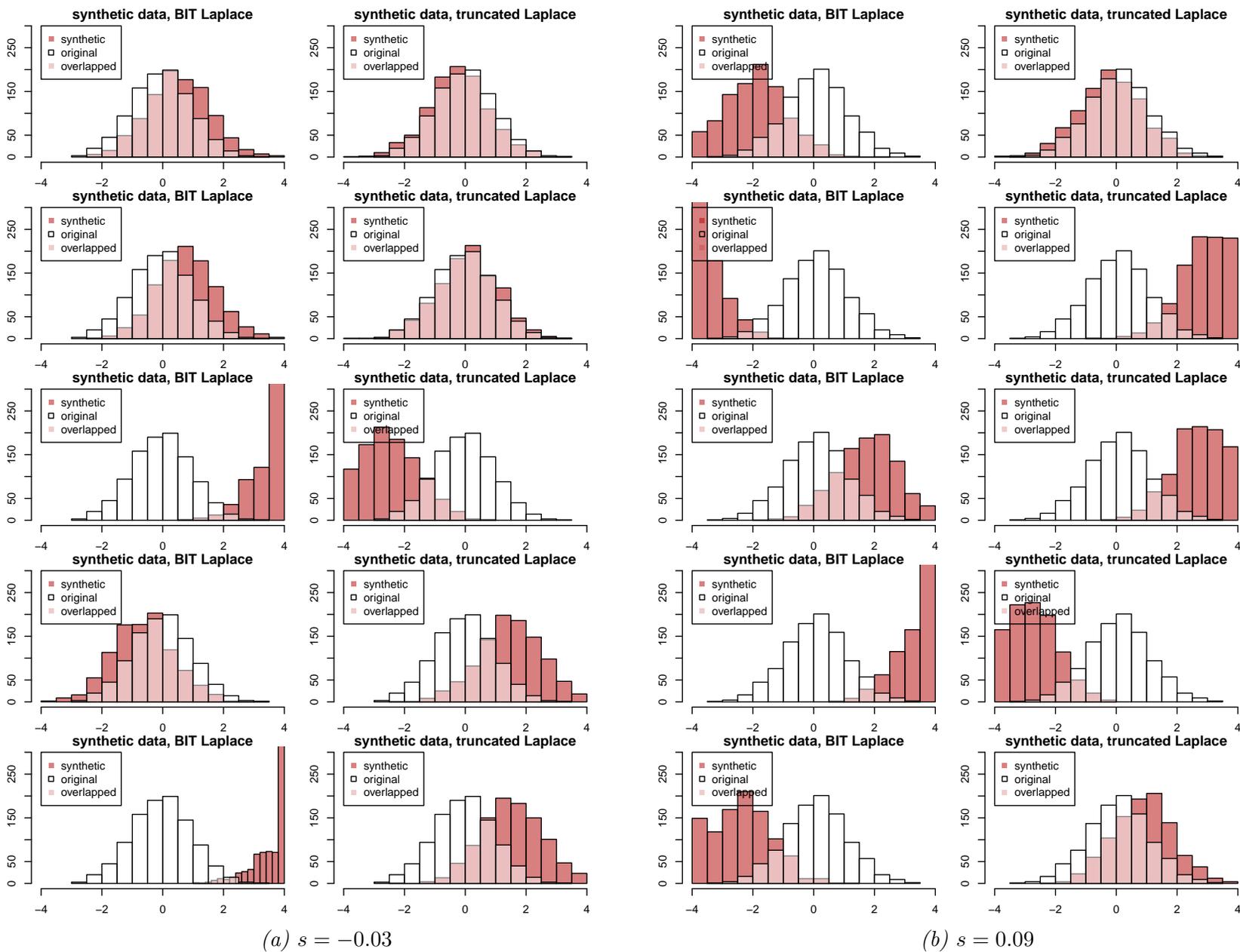


Figure S5: examples of synthetic data via modips in the simulation study ( $n = 1000, m = 500, [c_0, c_1] = [-4, 4]$ )

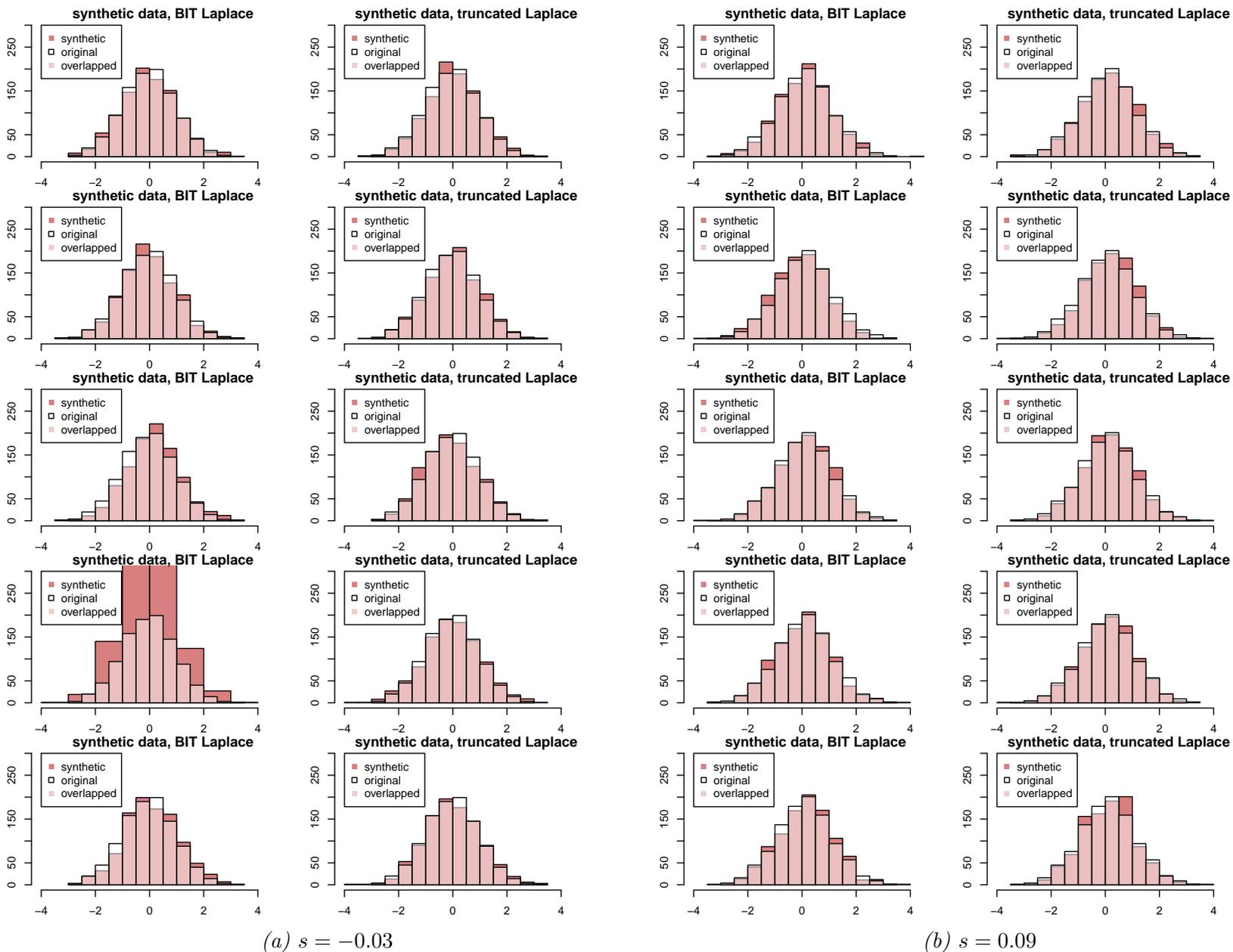


Figure S6: examples of synthetic data via modips in the simulation study ( $n = 1000, m = 5, [c_0, c_1] = [-4, 5]$ )

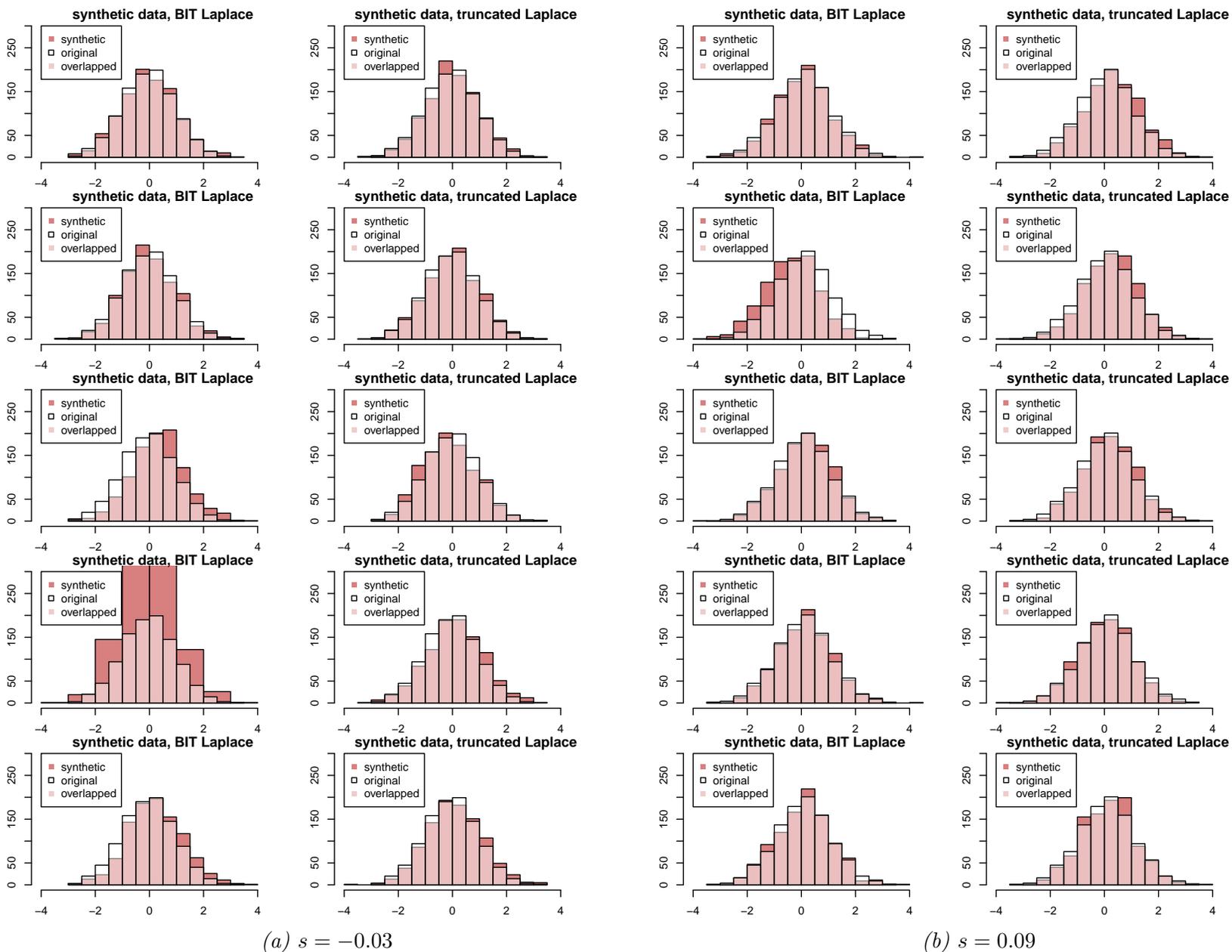


Figure S7: examples of synthetic data via modips in the simulation study ( $n = 1000, m = 15, [c_0, c_1] = [-4, 5]$ )

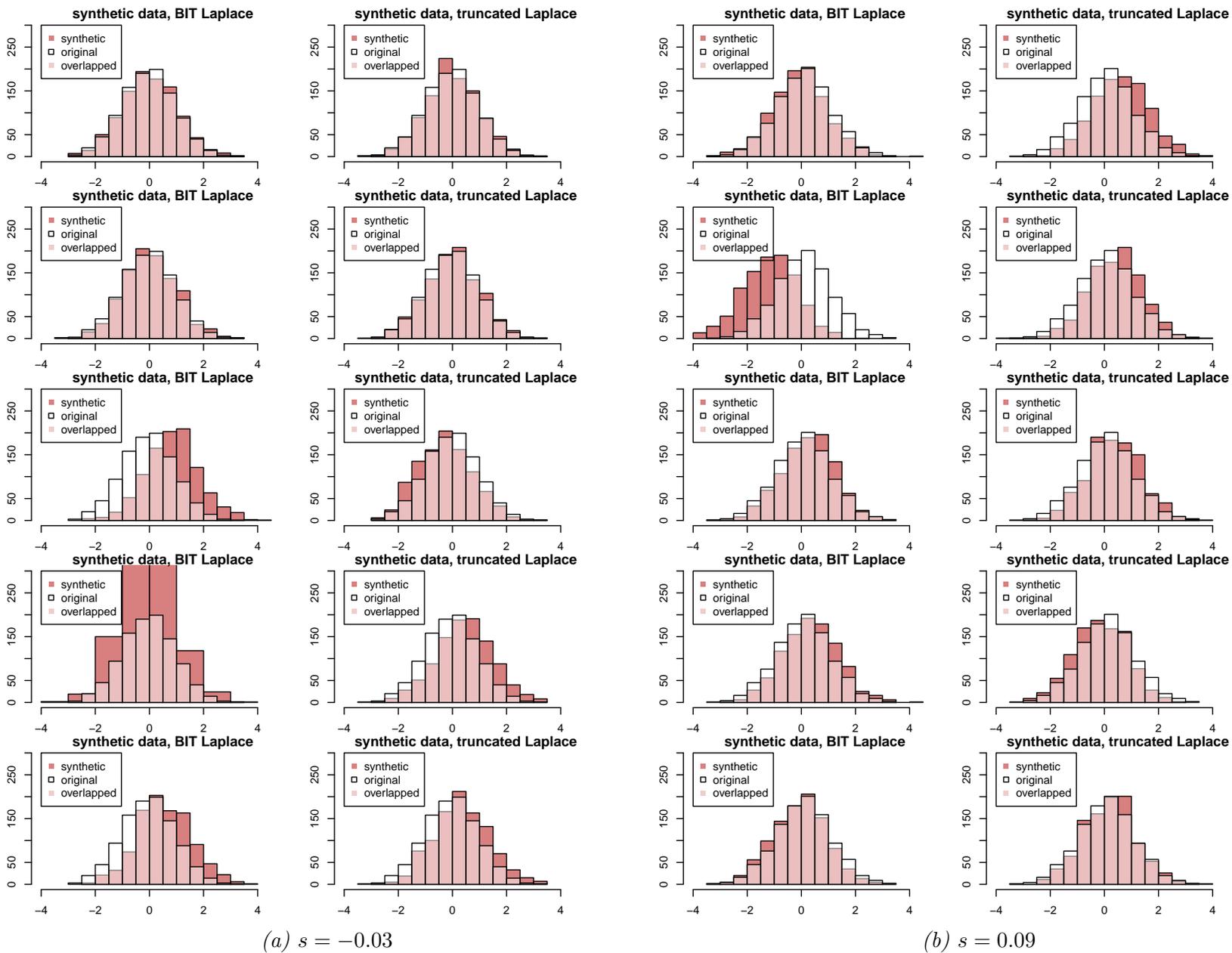


Figure S8: examples of synthetic data via modips in the simulation study ( $n = 1000, m = 40, [c_0, c_1] = [-4, 5]$ )

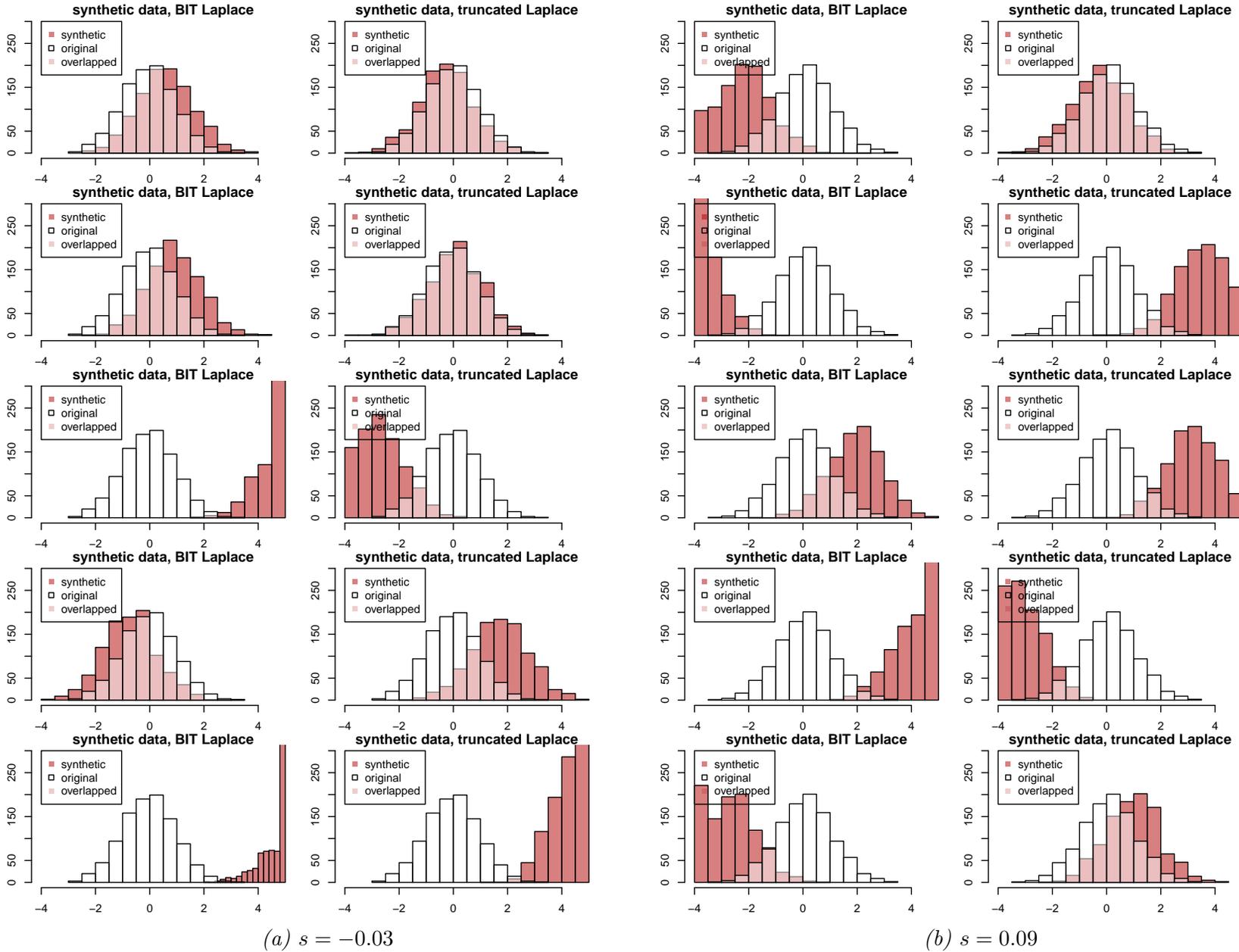


Figure S9: examples of synthetic data via modips in the simulation study ( $n = 1000, m = 500, [c_0, c_1] = [-4, 5]$ )

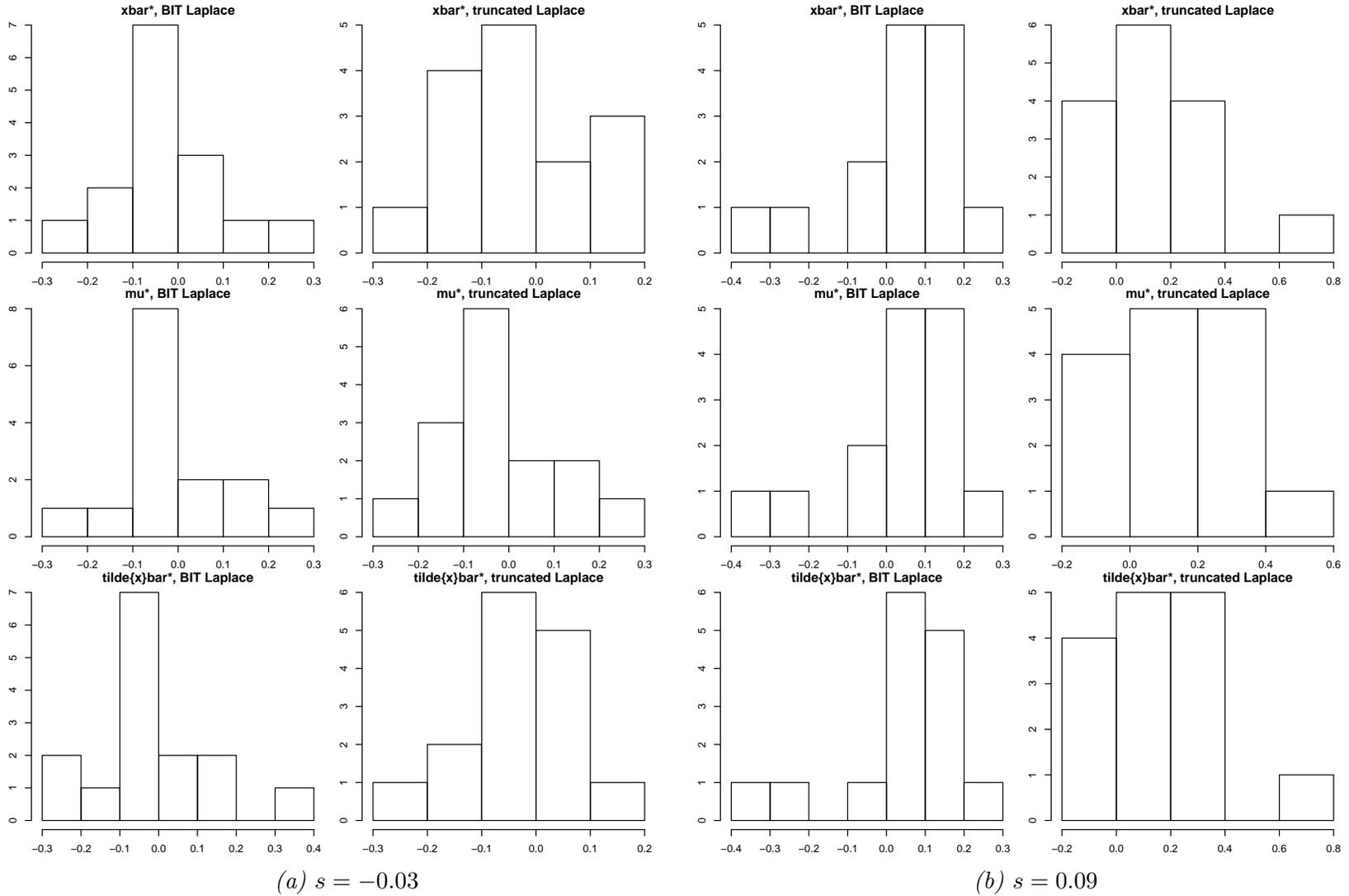


Figure S10: distribution of  $\bar{x}^{*(k)}$ ,  $\mu^{*(k)}$  and  $\tilde{x}^{*(k)}$  across synthetic sets in the simulation study ( $n = 1000$ ,  $m = 15$ ,  $[c_0, c_1] = [-4, 4]$ )

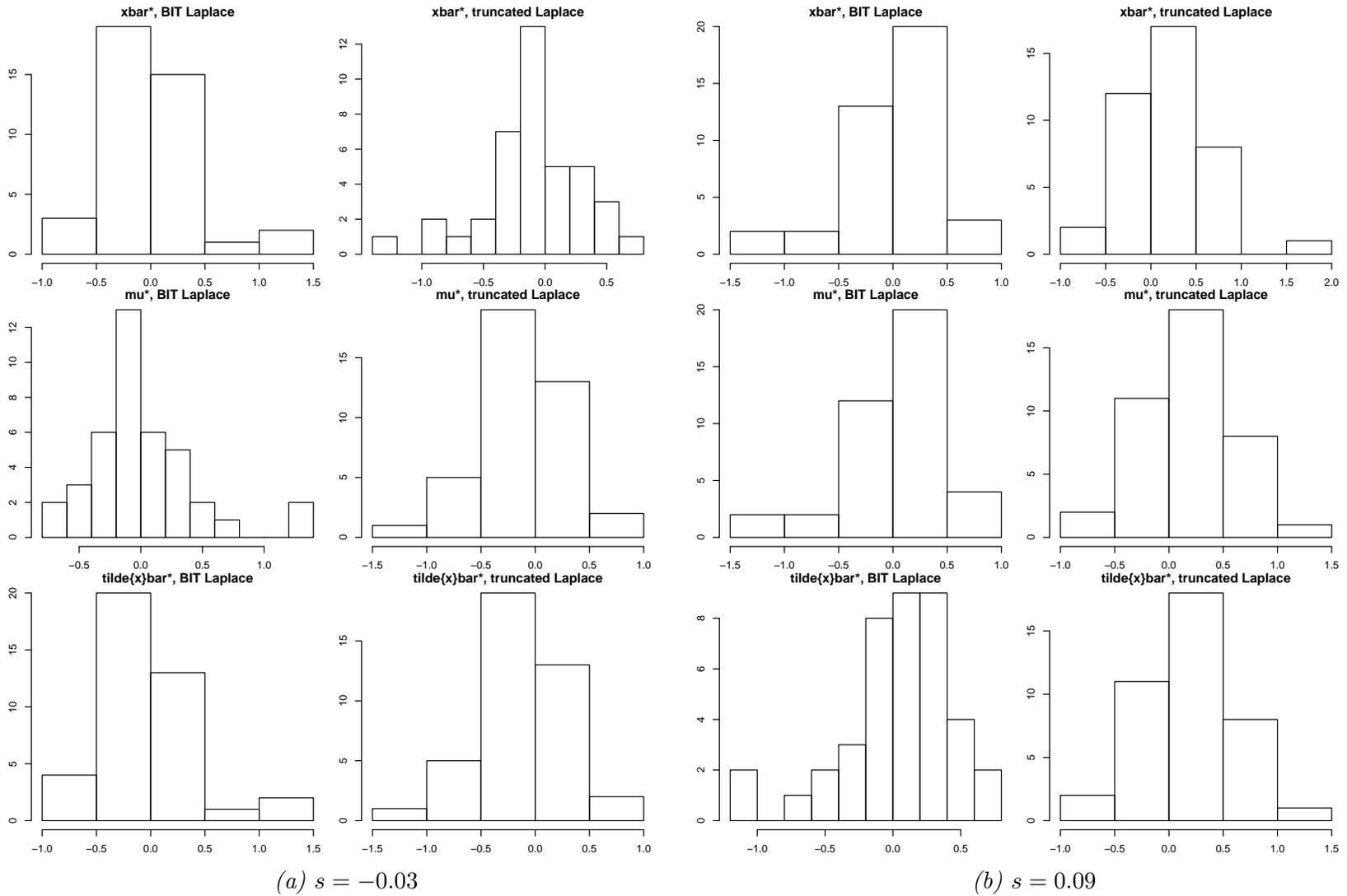


Figure S11: distribution of  $\bar{x}^{*(k)}$ ,  $\mu^{*(k)}$  and  $\tilde{x}^{*(k)}$  across synthetic sets in the simulation study ( $n = 1000$ ,  $m = 40$ ,  $[c_0, c_1] = [-4, 4]$ )

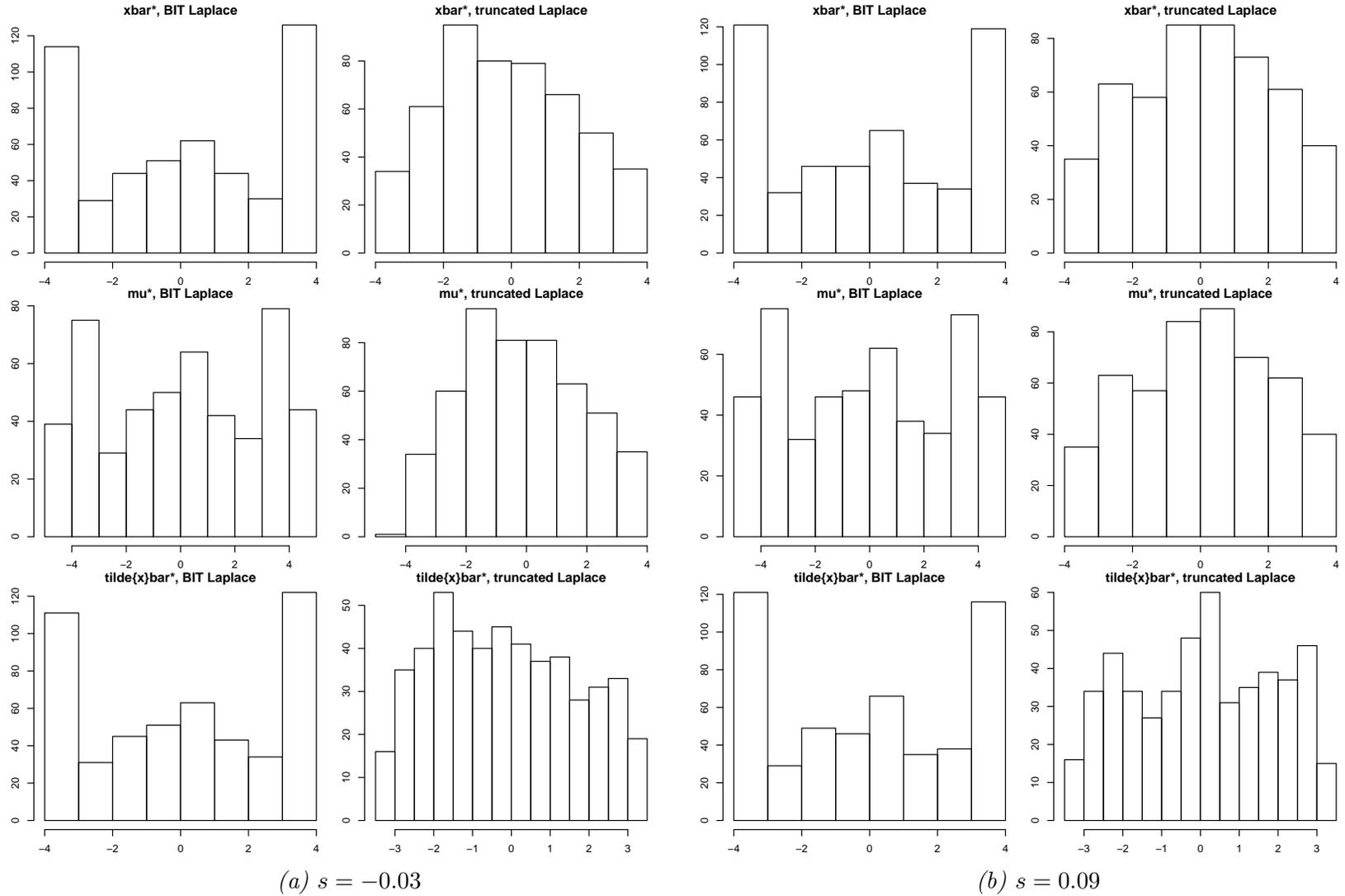


Figure S12: distribution of  $\bar{x}^{*(k)}$ ,  $\mu^{*(k)}$  and  $\tilde{\bar{x}}^{*(k)}$  across synthetic sets in the simulation study ( $n = 1000, m = 500, [c_0, c_1] = [-4, 4]$ )

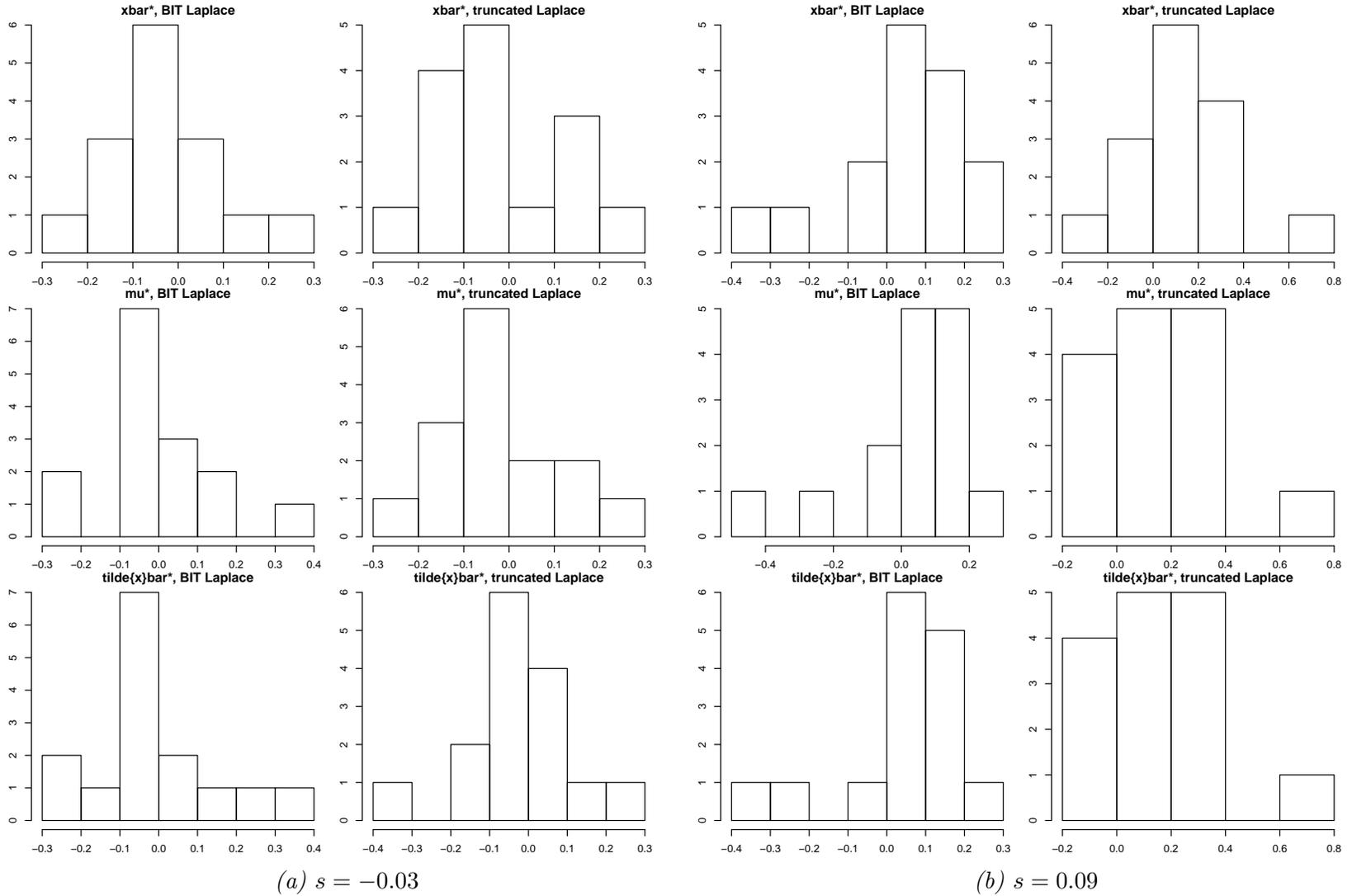


Figure S13: distribution of  $\bar{x}^{*(k)}$ ,  $\mu^{*(k)}$  and  $\tilde{x}^{*(k)}$  across synthetic sets in the simulation study ( $n = 1000$ ,  $m = 15$ ,  $[c_0, c_1] = [-4, 5]$ )

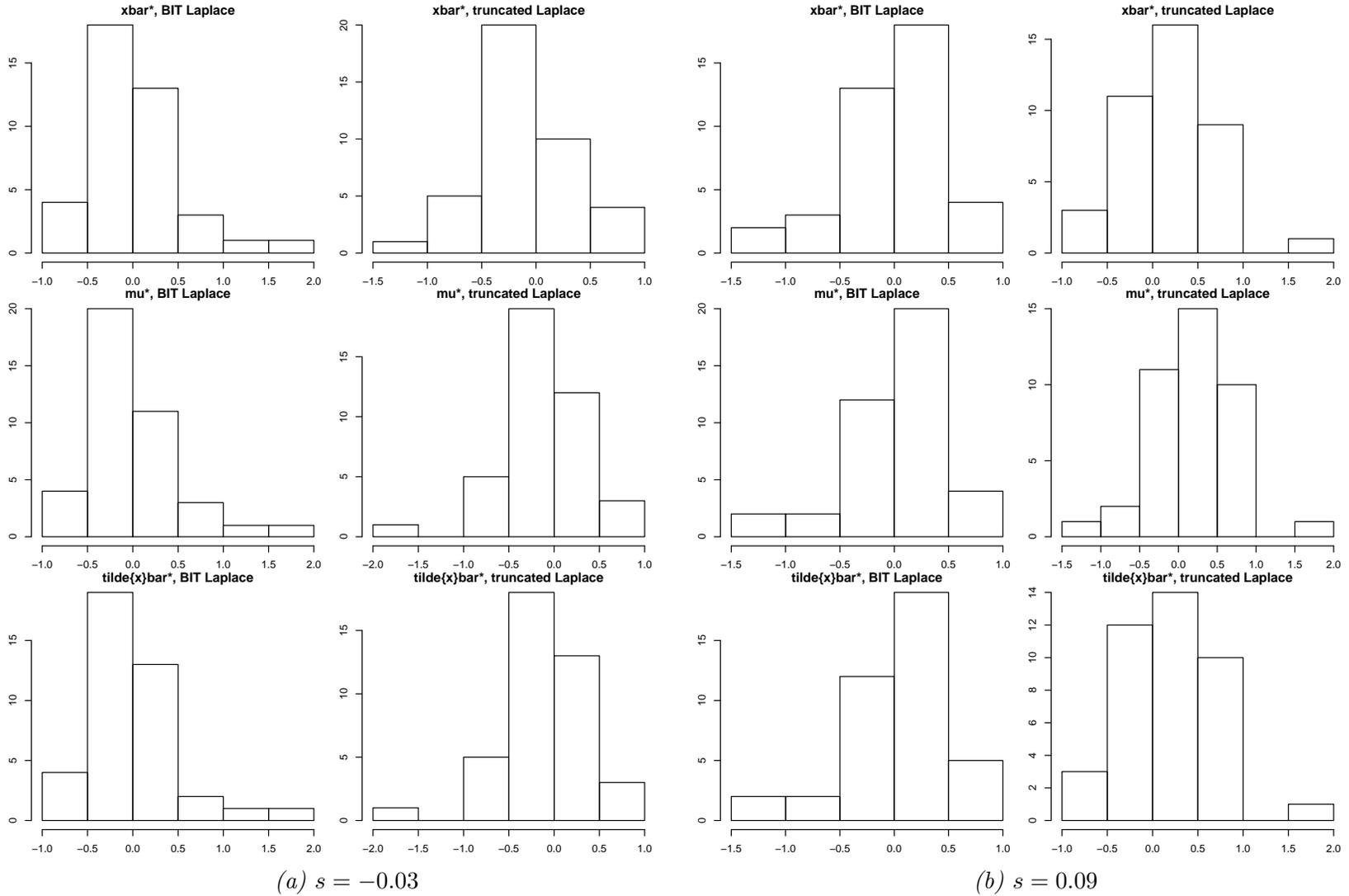


Figure S14: distribution of  $\bar{x}^{*(k)}$ ,  $\mu^{*(k)}$  and  $\tilde{x}^{*(k)}$  across the  $m$  sets in the simulation study ( $n = 1000, m = 40, [c_0, c_1] = [-4, 5]$ )

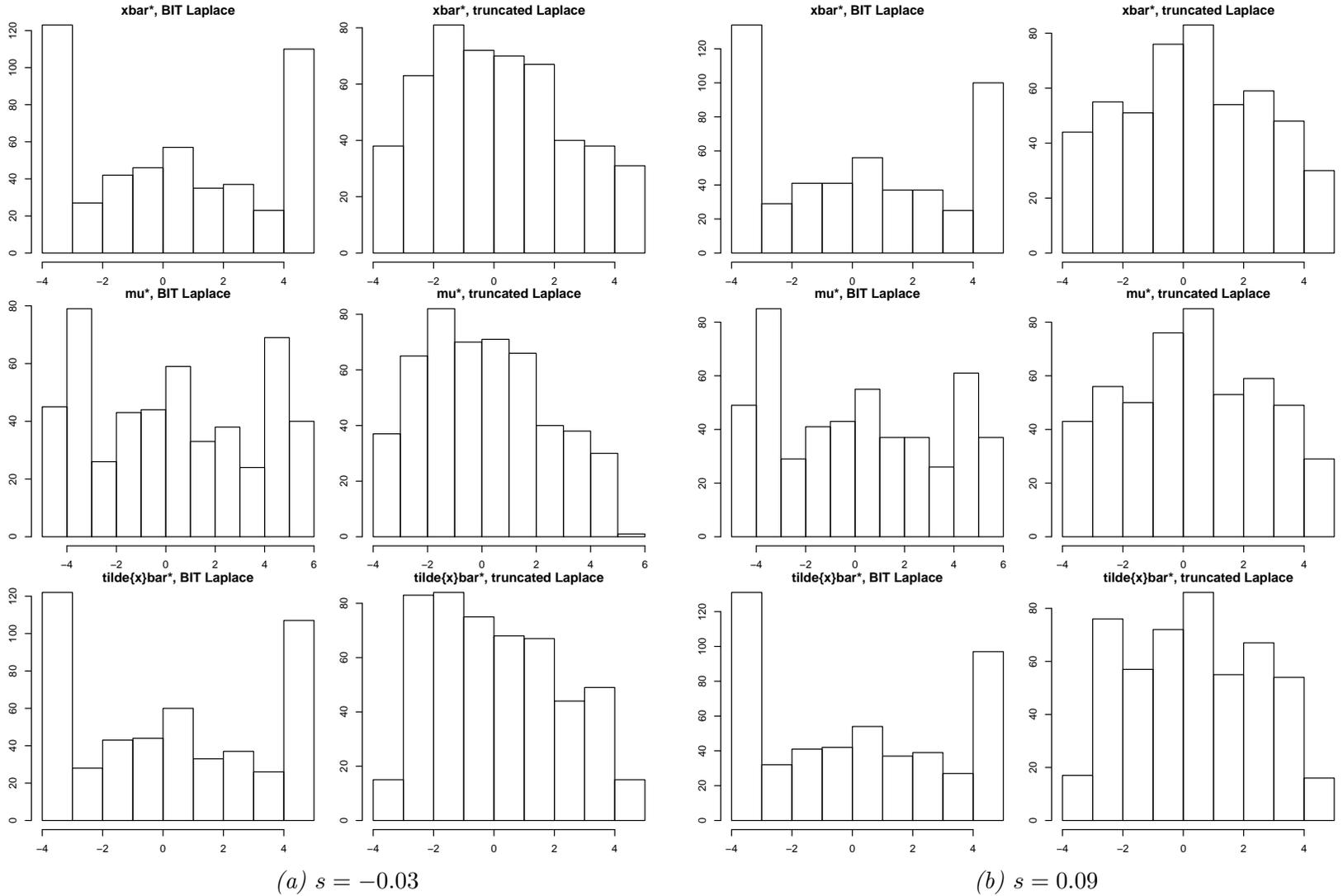


Figure S15: distribution of  $\bar{x}^{*(k)}$ ,  $\mu^{*(k)}$  and  $\tilde{x}^{*(k)}$  across synthetic sets in the simulation study ( $n = 1000$ ,  $m = 500$ ,  $[c_0, c_1] = [-4, 5]$ )

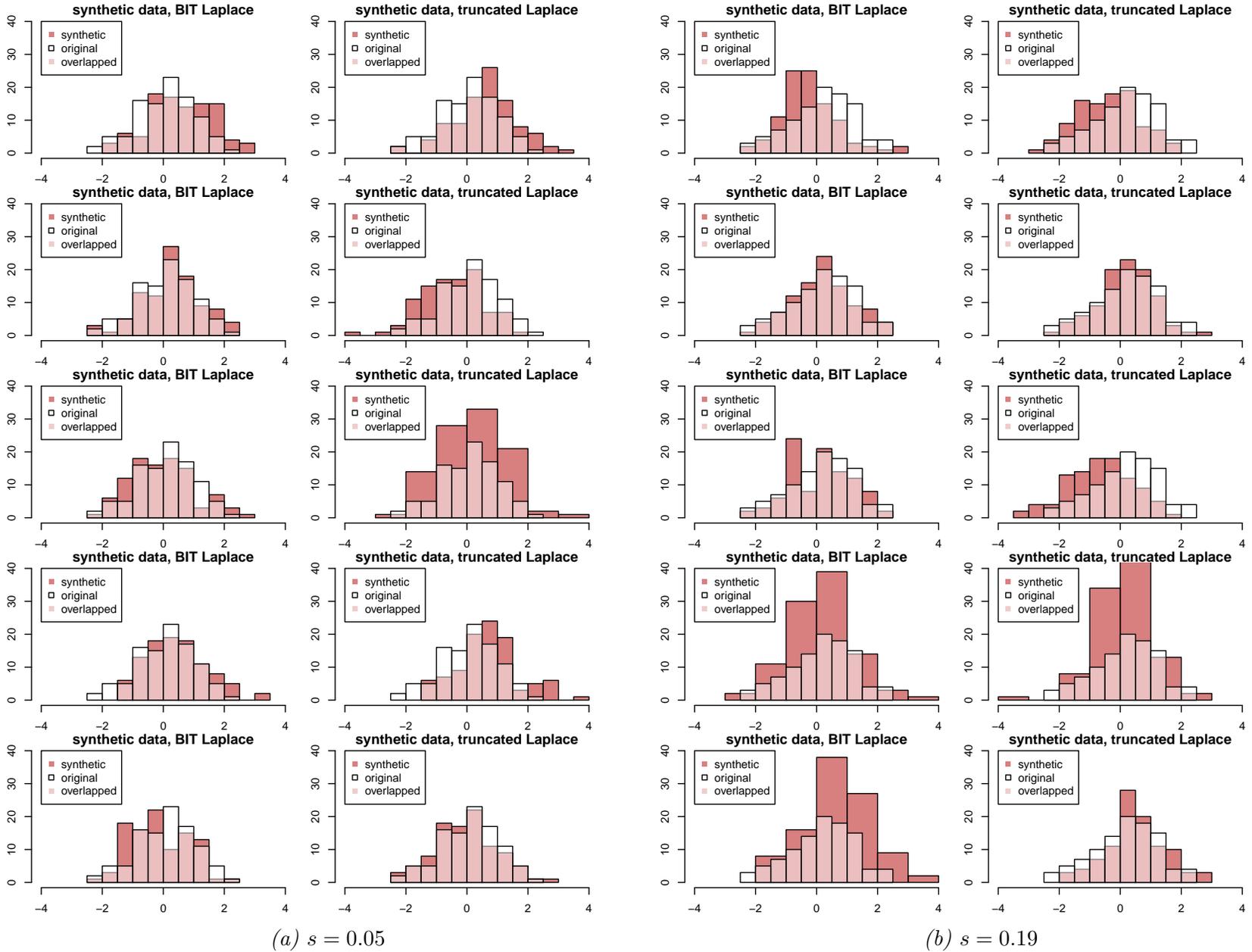


Figure S16: examples of synthetic data via modips in the simulation study ( $n = 100, m = 5, [c_0, c_1] = [-4, 4]$ )

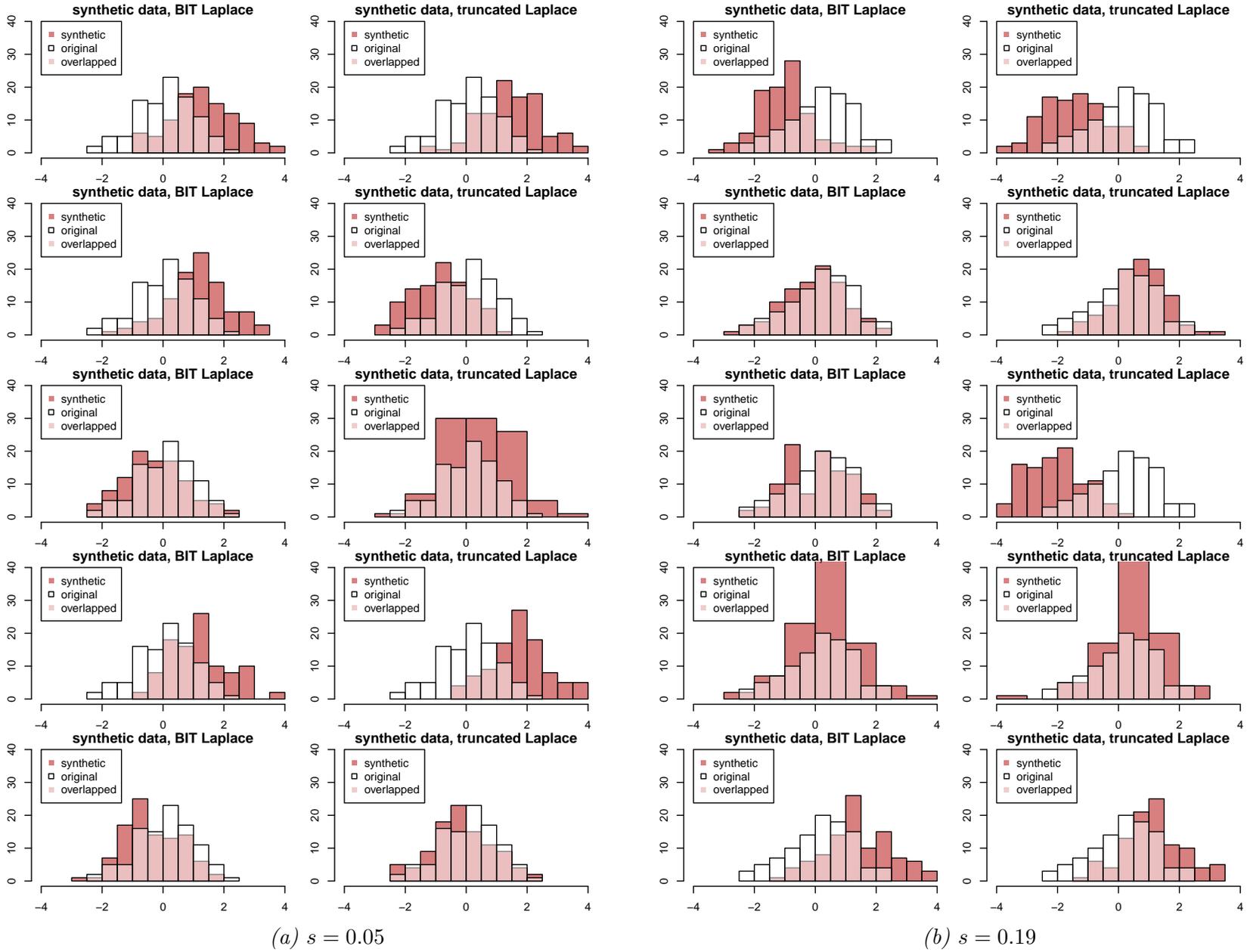


Figure S17: examples of synthetic data via modips in the simulation study ( $n = 100, m = 15, [c_0, c_1] = [-4, 4]$ )

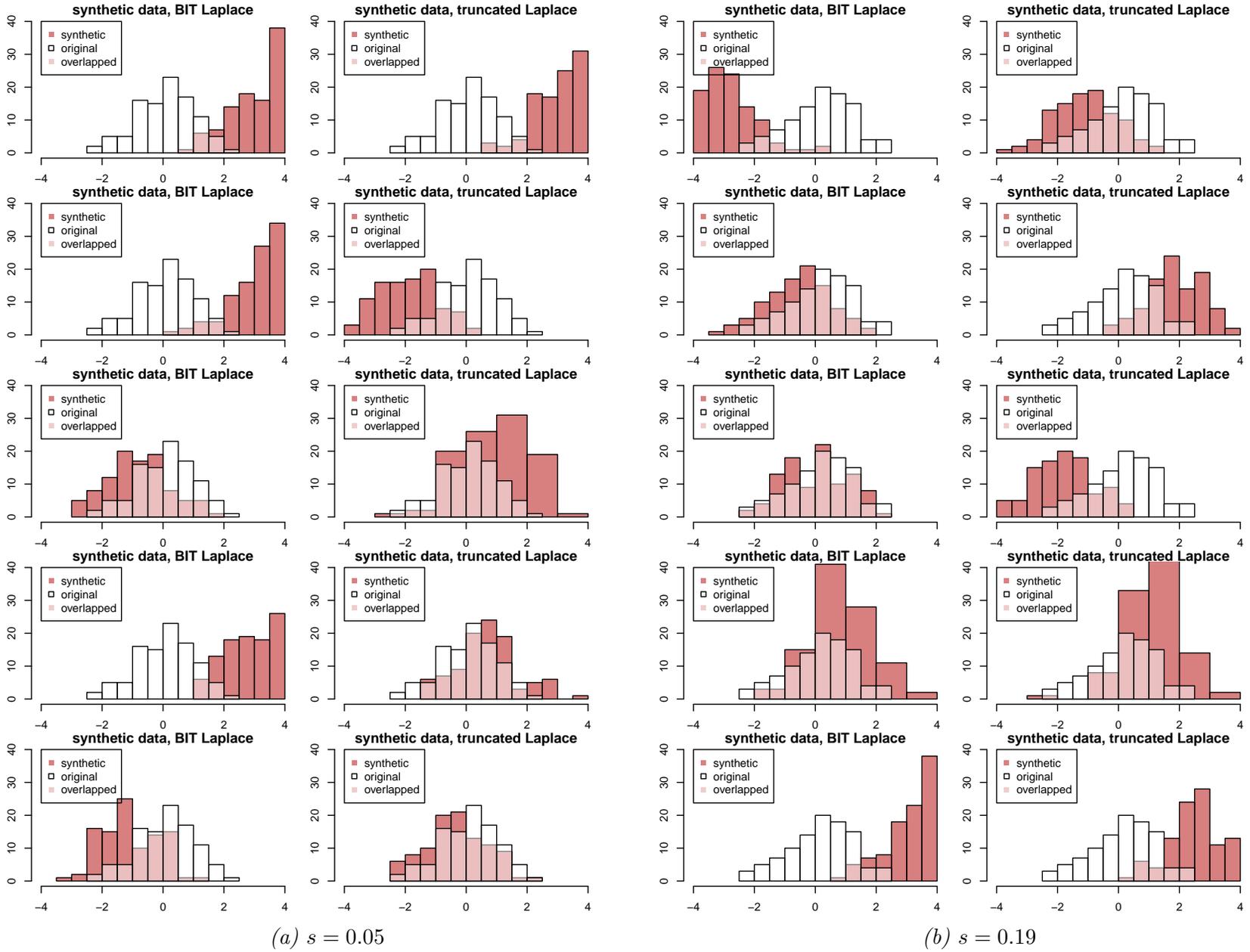
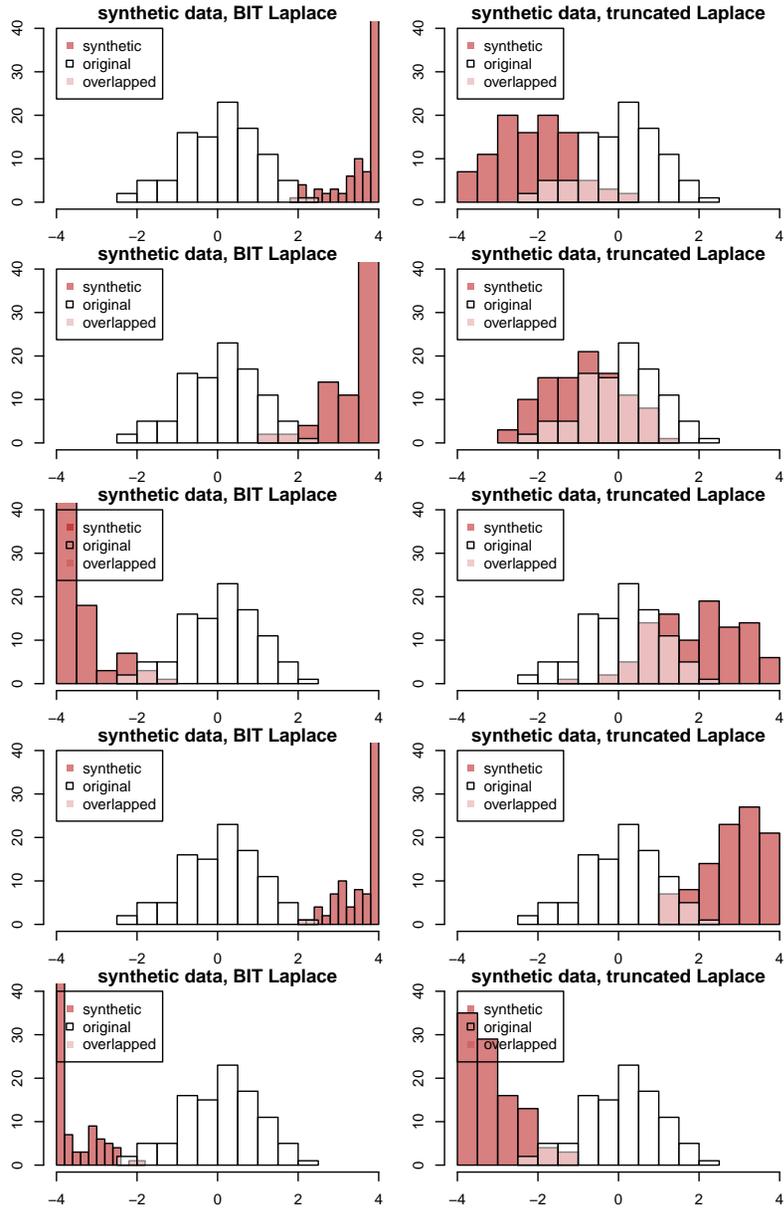
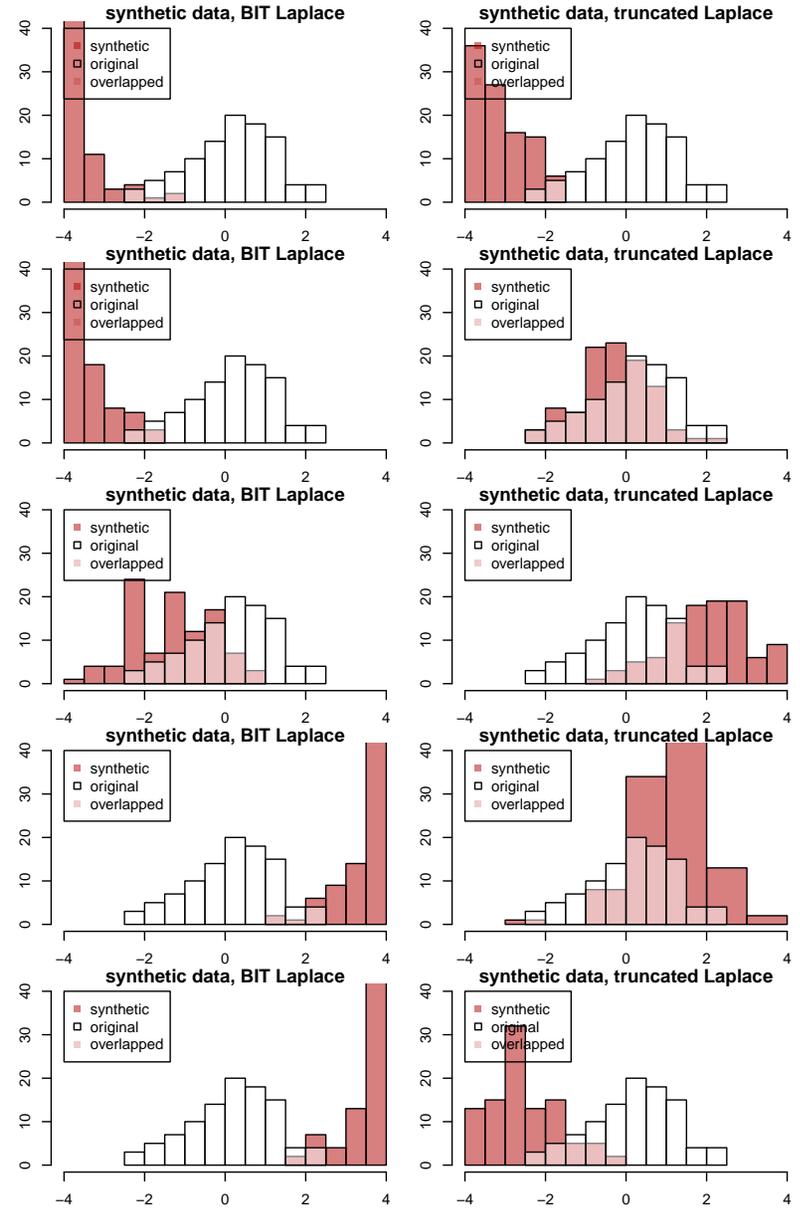


Figure S18: examples of synthetic data via modips in the simulation study ( $n = 100, m = 40, [c_0, c_1] = [-4, 4]$ )

(a)  $s = 0.05$ (b)  $s = 0.19$ Figure S19: examples of synthetic data via modips in the simulation study ( $n = 100, m = 500, [c_0, c_1] = [-4, 4]$ )

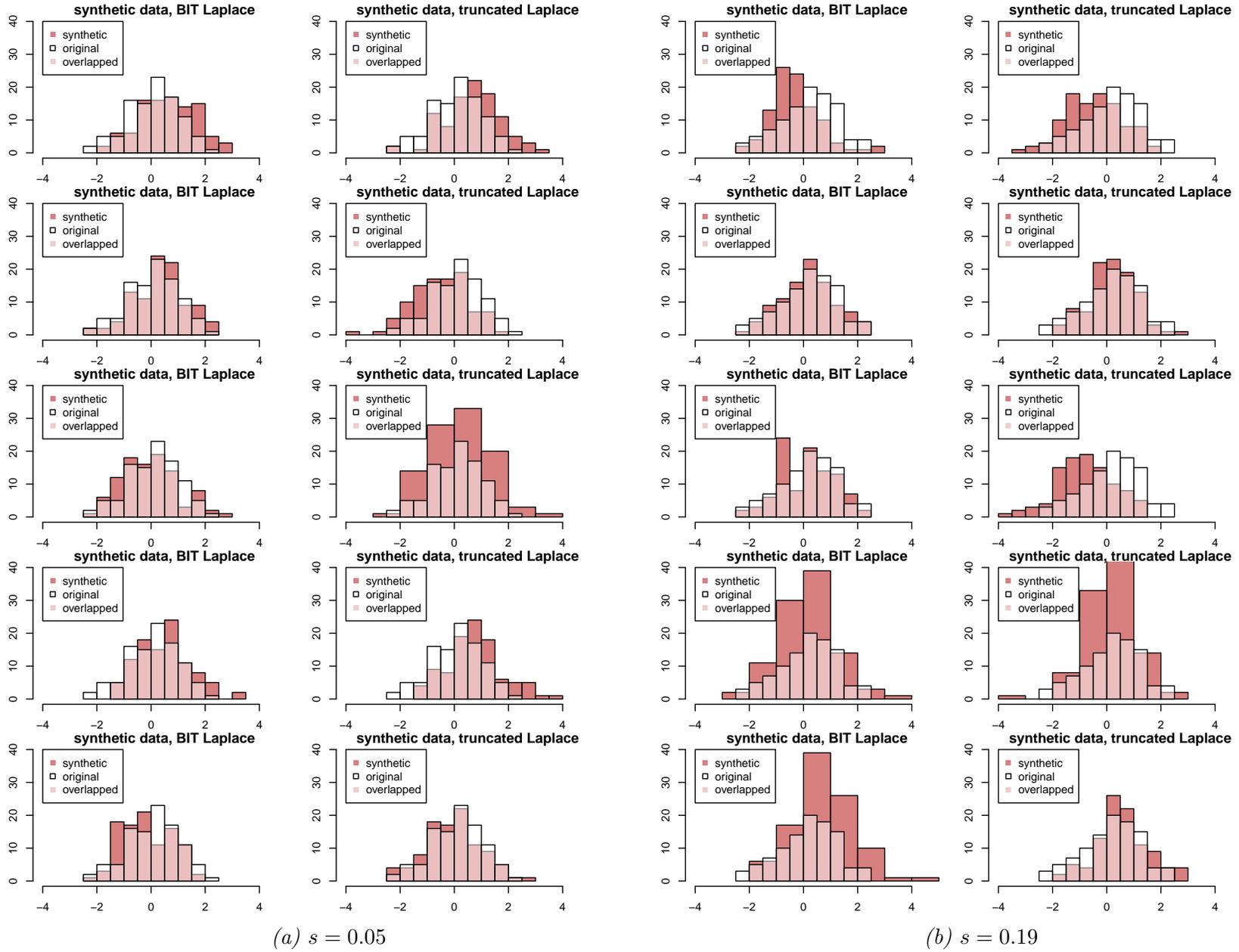


Figure S20: examples of synthetic data via modips in the simulation study ( $n = 100, m = 5, [c_0, c_1] = [-4, 5]$ )

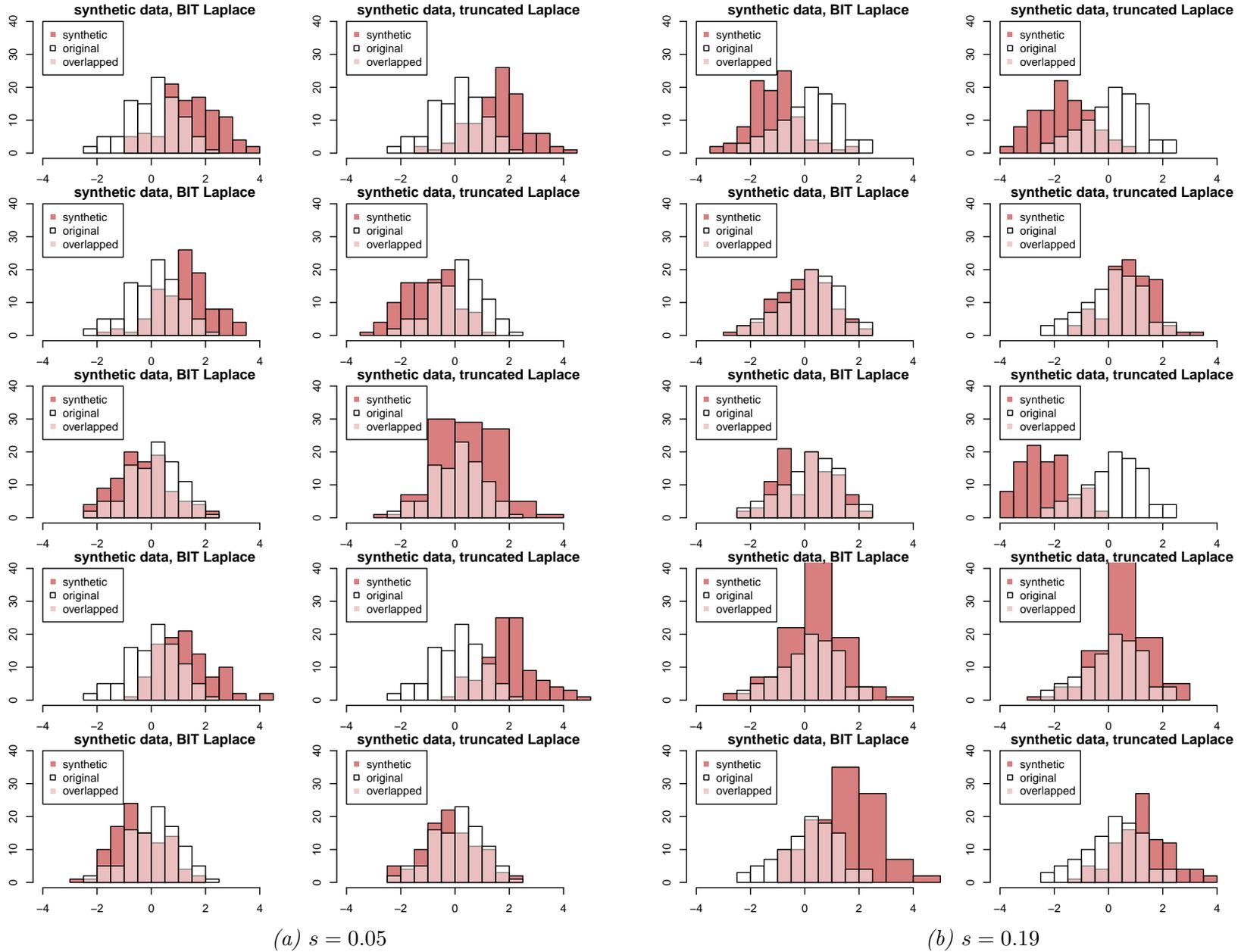


Figure S21: examples of synthetic data via modips in the simulation study ( $n = 100, m = 15, [c_0, c_1] = [-4, 5]$ )

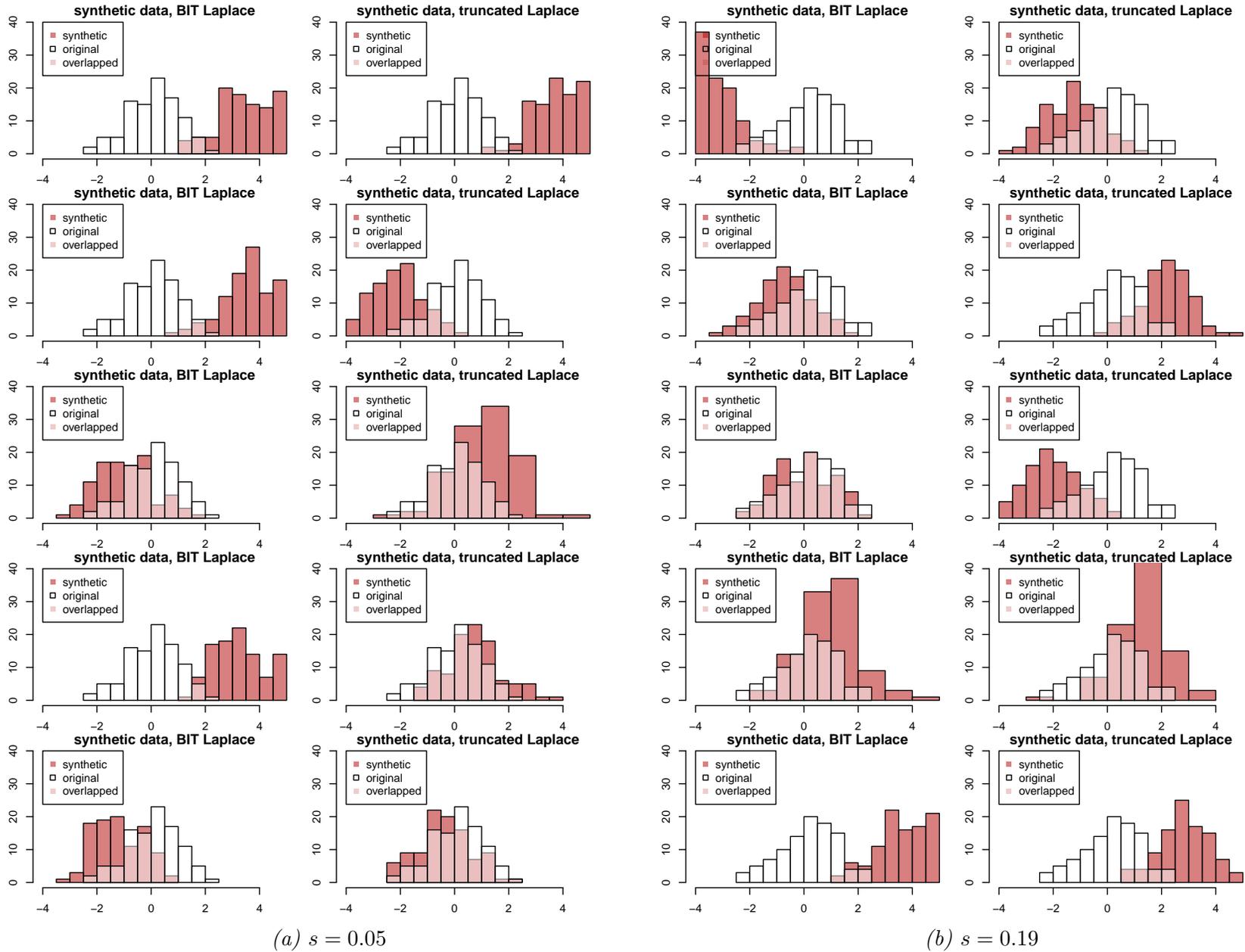


Figure S22: examples of synthetic data via modips in the simulation study ( $n = 100, m = 40, [c_0, c_1] = [-4, 5]$ )

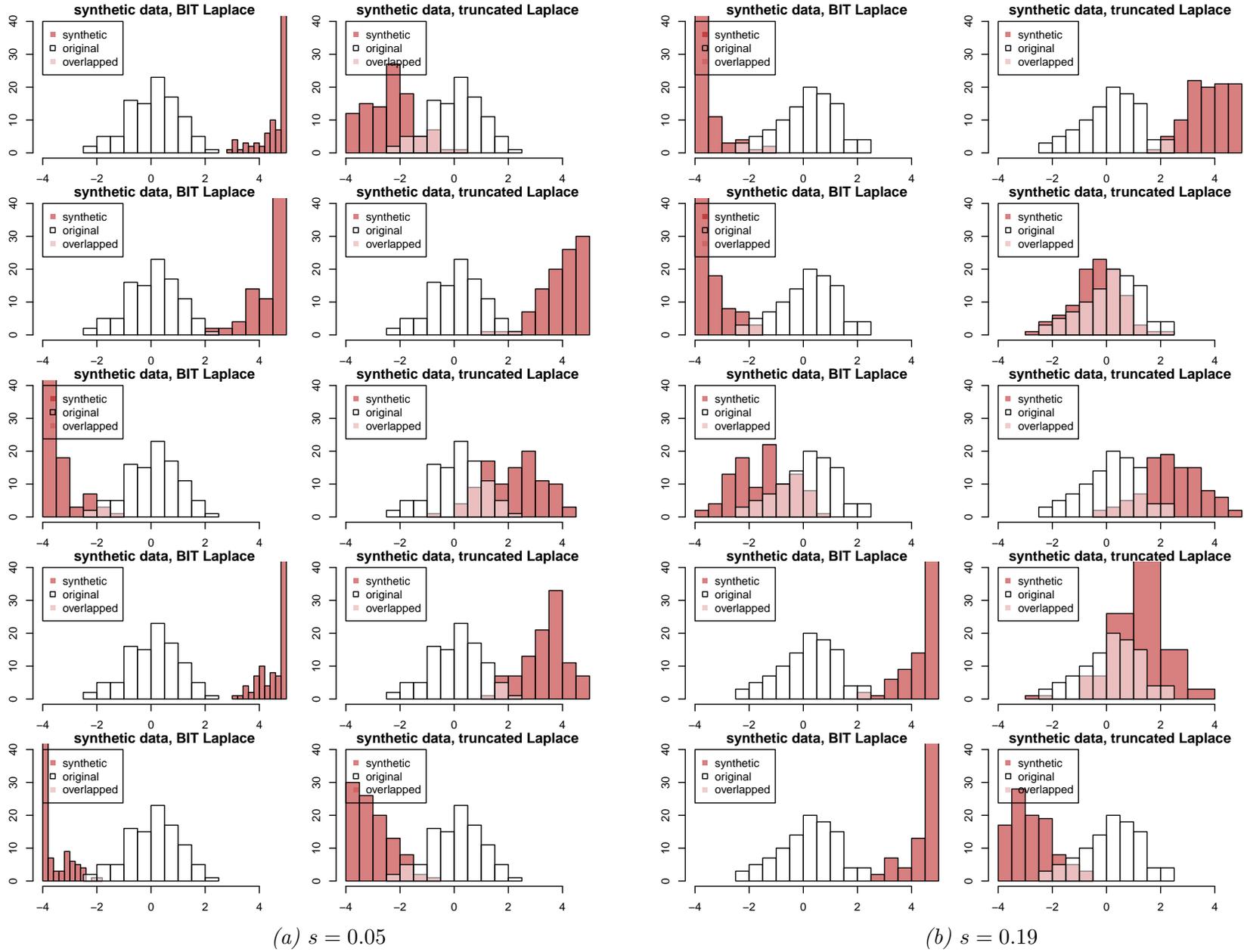


Figure S23: examples of synthetic data via modips in the simulation study ( $n = 100, m = 500, [c_0, c_1] = [-4, 5]$ )

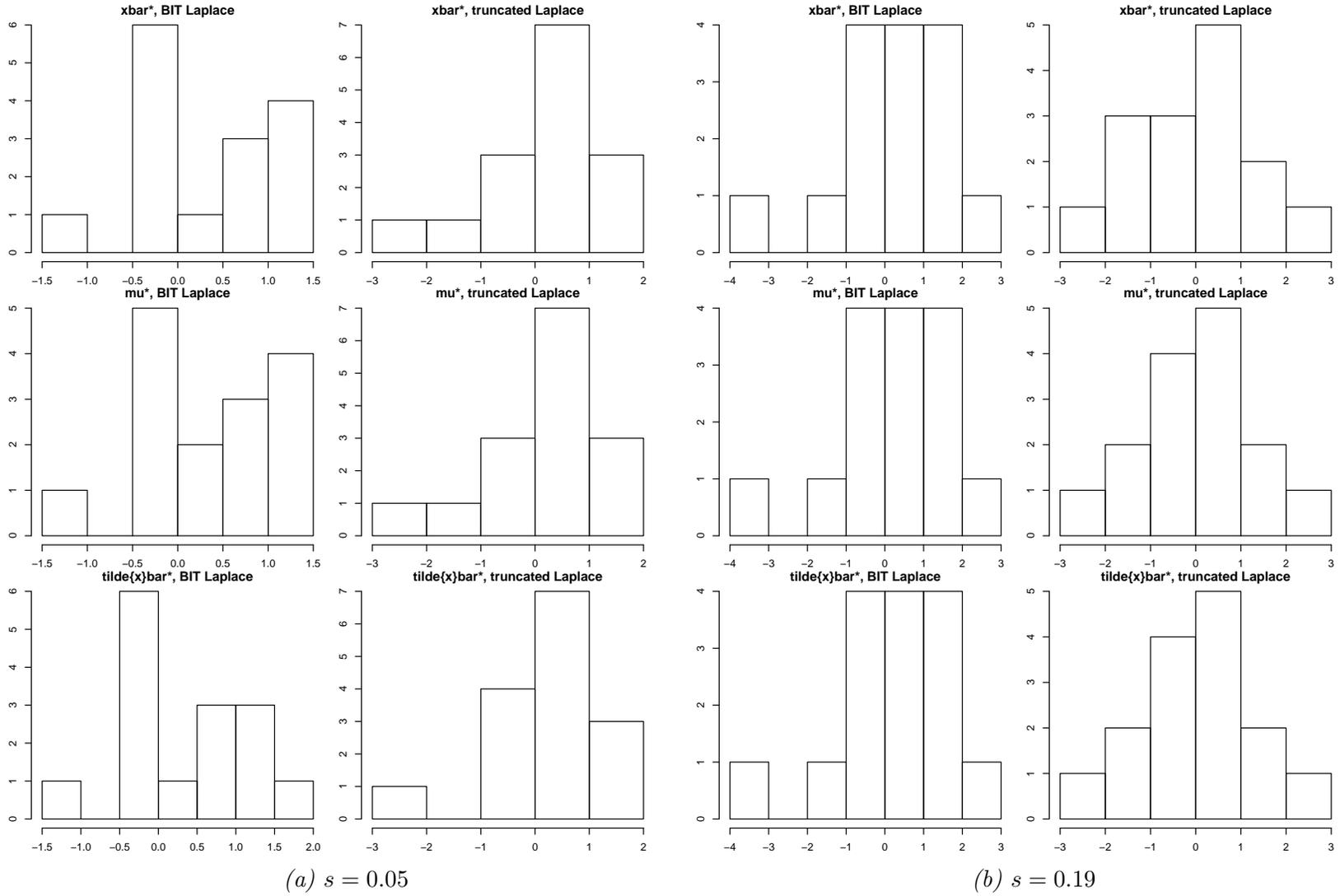


Figure S24: distribution of  $\bar{x}^{*(k)}$ ,  $\mu^{*(k)}$  and  $\tilde{x}^{*(k)}$  across synthetic sets in the simulation study ( $n = 100, m = 15, [c_0, c_1] = [-4, 4]$ )

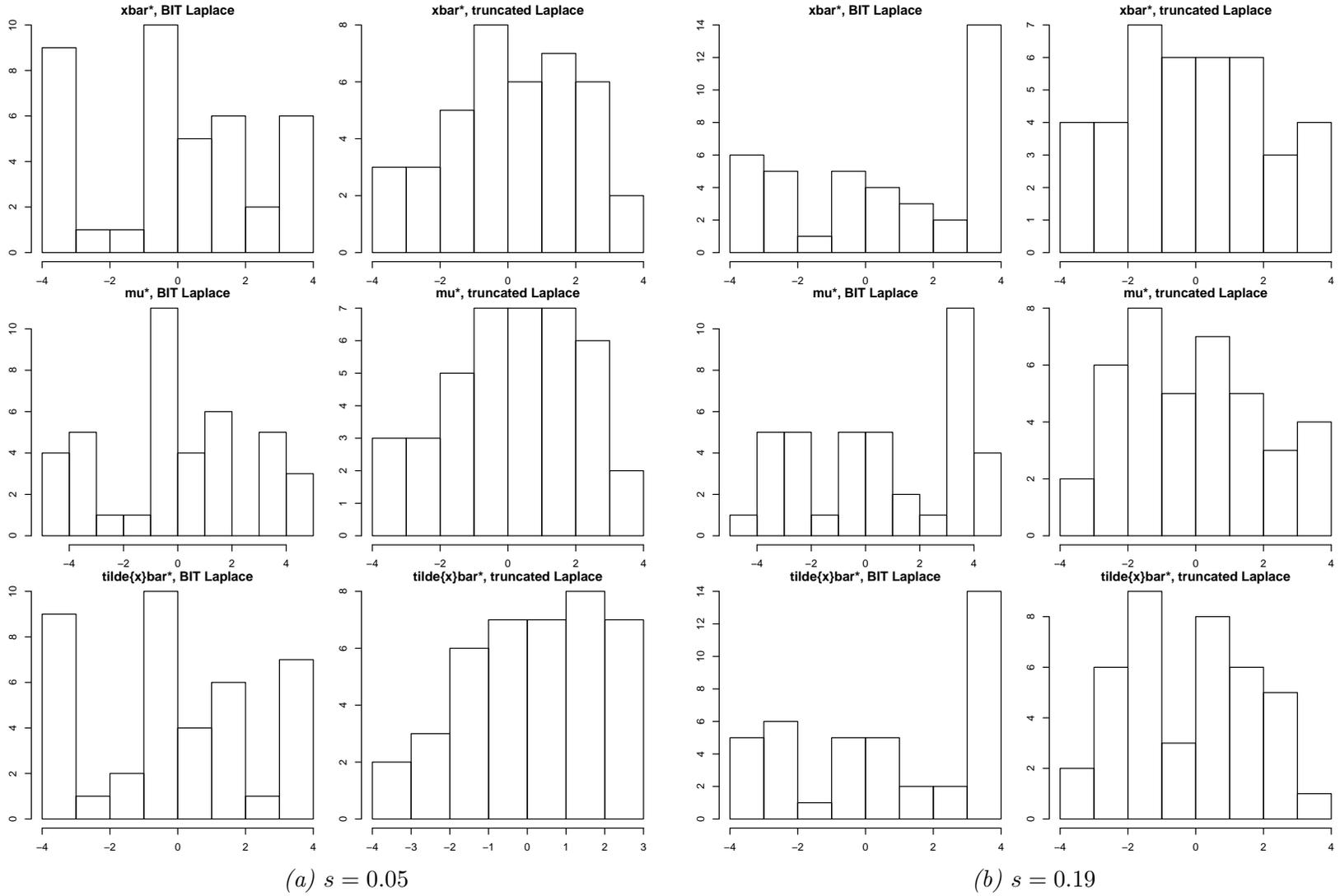


Figure S25: distribution of  $\bar{x}^{*(k)}$ ,  $\mu^{*(k)}$  and  $\tilde{x}^{*(k)}$  across synthetic sets in the simulation study ( $n = 100, m = 40, [c_0, c_1] = [-4, 4]$ )

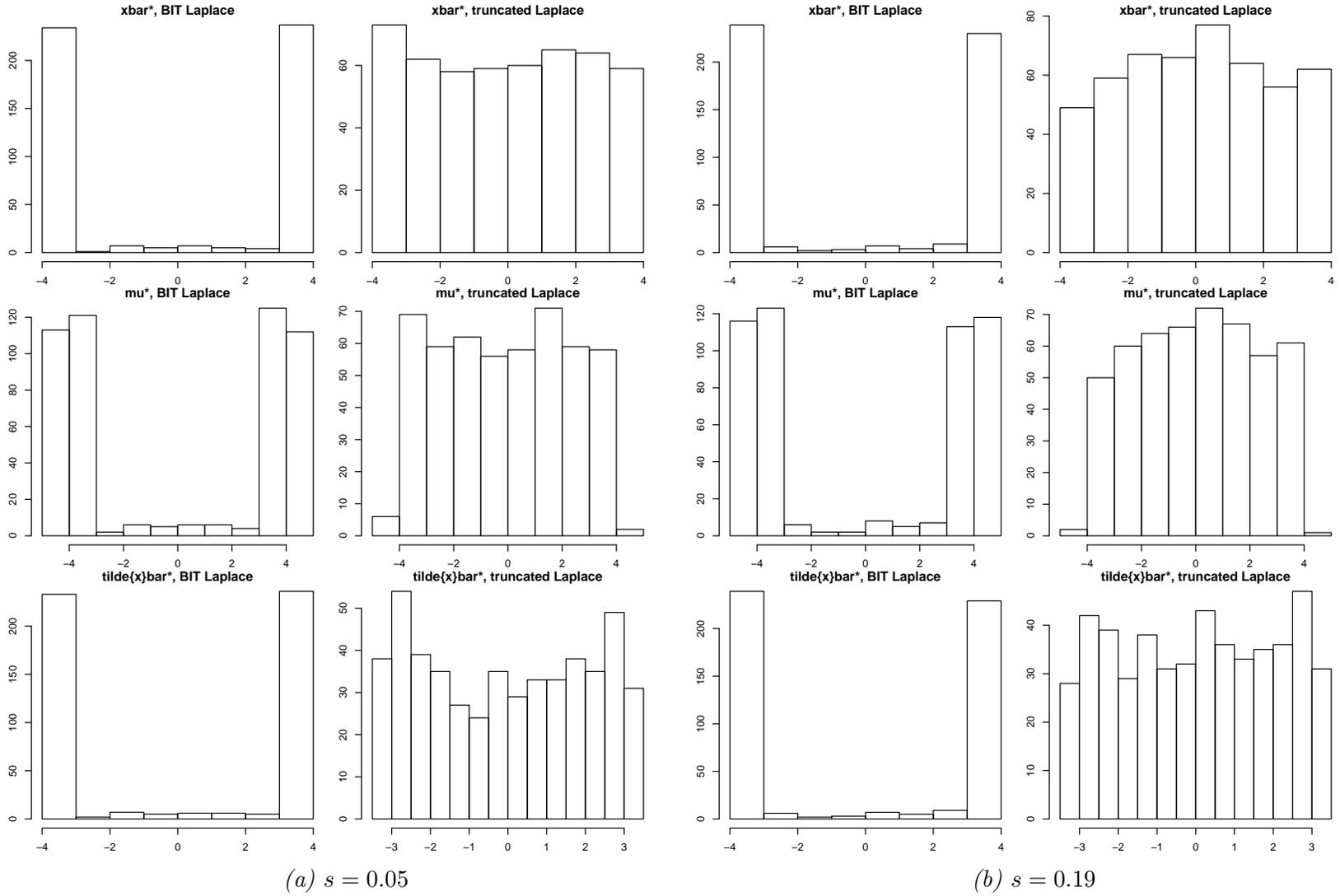


Figure S26: distribution of  $\bar{x}^{*(k)}$ ,  $\mu^{*(k)}$  and  $\tilde{x}^{*(k)}$  across synthetic sets in the simulation study ( $n = 100, m = 500, [c_0, c_1] = [-4, 4]$ )

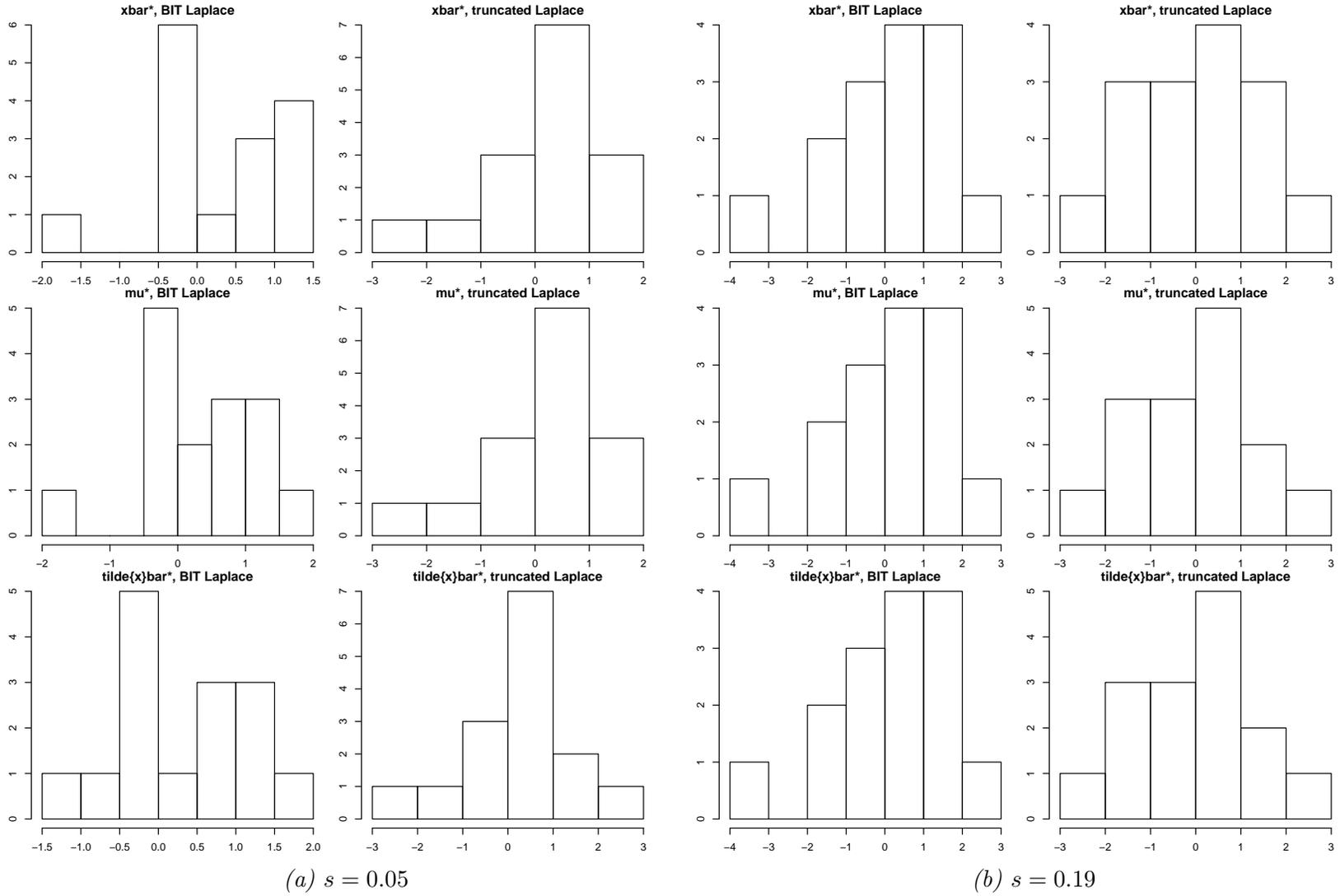


Figure S27: distribution of  $\bar{x}^{*(k)}$ ,  $\mu^{*(k)}$  and  $\tilde{\bar{x}}^{*(k)}$  across synthetic sets in the simulation study ( $n = 100, m = 15, [c_0, c_1] = [-4, 5]$ )

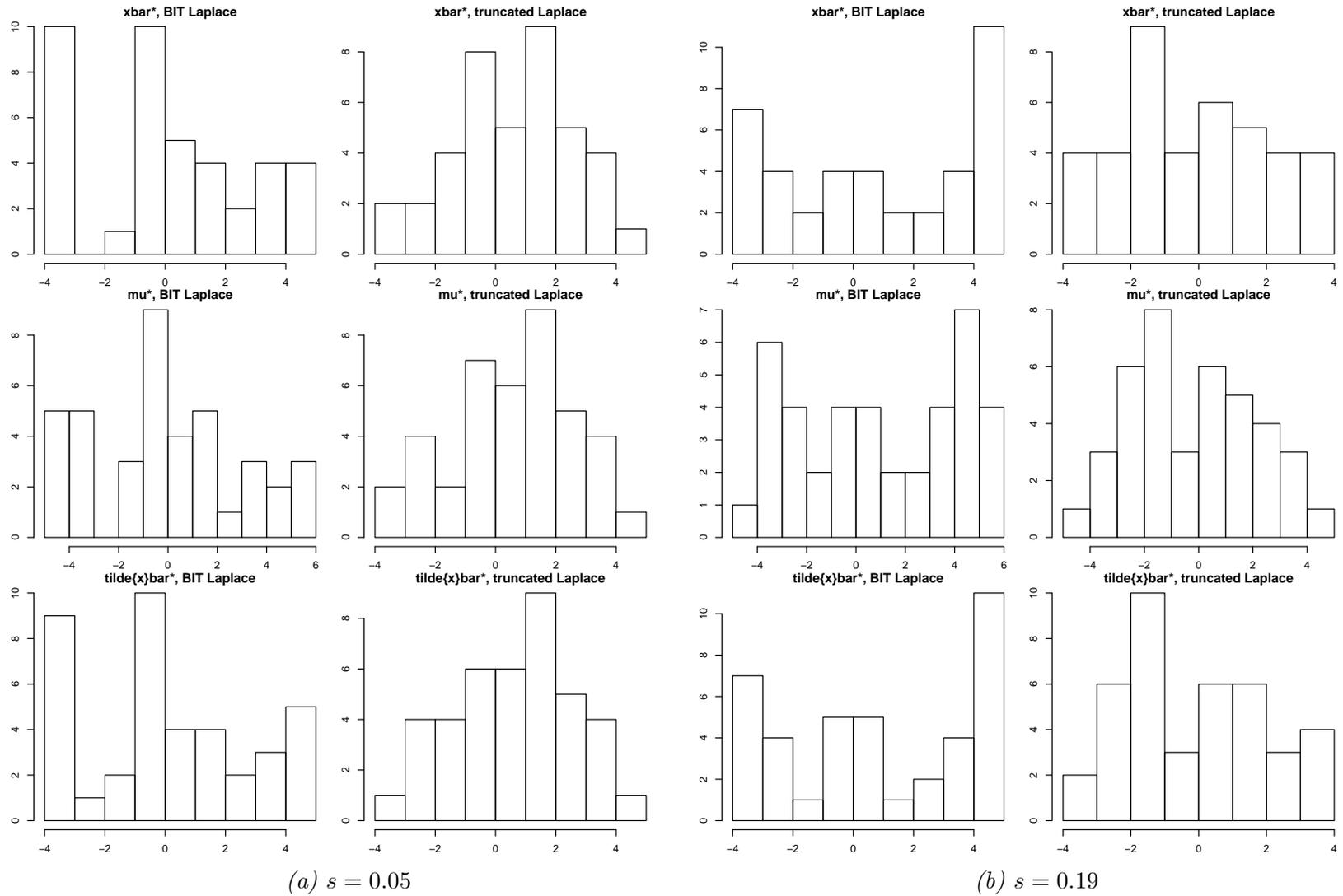


Figure S28: distribution of  $\bar{x}^{*(k)}$ ,  $\mu^{*(k)}$  and  $\tilde{\bar{x}}^{*(k)}$  across synthetic sets in the simulation study ( $n = 100, m = 40, [c_0, c_1] = [-4, 5]$ )

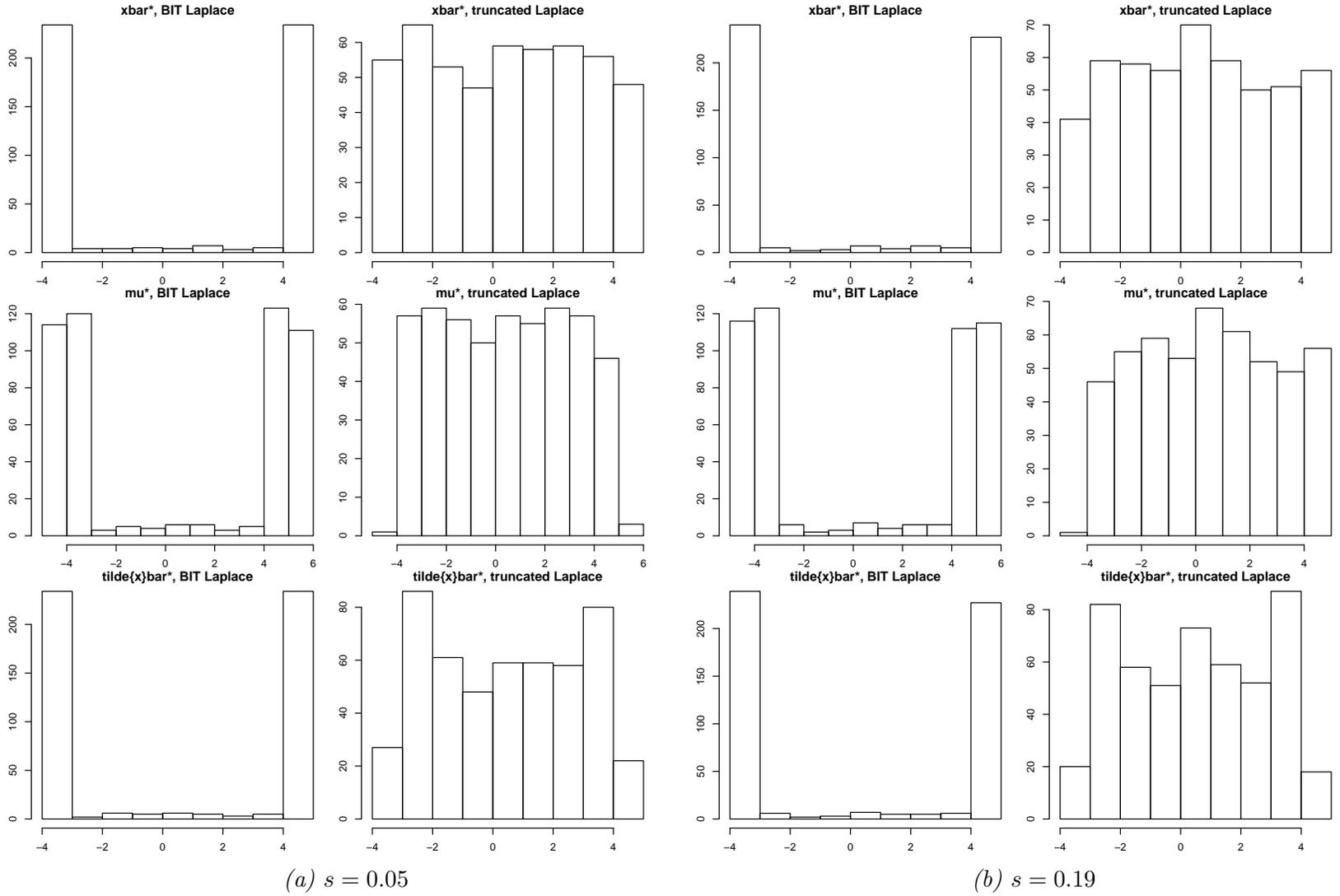


Figure S29: distribution of  $\bar{x}^{*(k)}$ ,  $\mu^{*(k)}$  and  $\tilde{x}^{*(k)}$  across synthetic sets in the simulation study ( $n = 100, m = 500, [c_0, c_1] = [-4, 5]$ )