A list of statements/theorems that you should be able to prove.

1. Let A, A_k be elementary subsets of $[0,1] \times [0,1]$, such that
$$A \subset \bigcup_{k=1}^{\infty} A_k.$$
Then
$$\tilde{m}(A) \leqslant \sum_{k=1}^{\infty} \tilde{m}(A_k).$$

2. For every $A \subset [0,1] \times [0,1]$ we have $\mu_*(A) \leqslant \mu^*(A)$.

3. Suppose that $A, A_k \subset [0,1] \times [0,1]$, and $A \subset \bigcup_{k=1}^{\infty} A_k$. Then
$$\mu^*(A) \leqslant \sum_{k=1}^{\infty} \mu^*(A_k).$$

4. If $A \subset [0,1] \times [0,1]$ is an elementary set, then A is measurable and $\mu(A) = \tilde{m}(A)$.

5. The union, intersection, difference and symmetric difference of two measurable subsets of $[0,1] \times [0,1]$ is measurable.

6. Suppose that A_1, A_2 are disjoint measurable subsets of $[0,1] \times [0,1]$. Then $\mu(A_1 \cup A_2) = \mu(A_1) + \mu(A_2)$.

7. The union of a countable collection of measurable sets is measurable.

8. If $A = \bigcup_k A_k$ is a disjoint union of a countable collection of measurable sets, then
$$\mu(A) = \sum_{k=1}^{\infty} \mu(A_k).$$

9. Let $f_n : X \to \mathbf{R}$ be measurable, such that the limit $f(x) = \lim_{n\to\infty} f_n(x)$ exists for all x. Then f is measurable.

10. If $f : X \to \mathbf{R}$ is measurable, and $g : \mathbf{R} \to \mathbf{R}$ is continuous, then $g \circ f$ is measurable.

11. A function $f : X \to \mathbf{R}$ is measurable if and only if f is a uniform limit of simple functions.

12. Suppose $f, g : X \to \mathbf{R}$ are measurable and $c \in \mathbf{R}$. Then $f + g$, cf, fg, f/g are measurable if g is nowhere vanishing in the case of f/g.

13. Let $f, g : [0,1] \to \mathbf{R}$ be continuous such that $f(x) = g(x)$ for almost every x (with respect to Lebesgue measure). Then $f(x) = g(x)$ for all x.

14. (Egorov's theorem) Let $f_n : X \to \mathbf{R}$ be a sequence of measurable functions, converging almost everywhere to $f : X \to \mathbf{R}$. For any $\delta > 0$ there exists a set $Y \subset X$ such that $\mu(X \setminus Y) < \delta$ and $f_n \to f$ uniformly on Y.

15. Suppose that $\phi : A \to \mathbf{R}$ is integrable and $f : A \to \mathbf{R}$ satisfies $|f(x)| \leqslant \phi(x)$ for all $x \in A$. Then f is integrable and

$$\left| \int_A f(x)\, \mu \right| \leqslant \int_A \phi(x)\, \mu.$$

16. (Chebyshev's inequality) If $f : A \to \mathbf{R}$ is integrable and $f(x) \geqslant 0$ for all $x \in A$, then

$$\mu\{x\,;\, x \in A,\ f(x) \geqslant c\} \leqslant \frac{1}{c} \int_A f(x)\, d\mu,$$

for all $c > 0$.

17. Let $f : A \to \mathbf{R}$ be integrable. For any $\epsilon > 0$ there is a $\delta > 0$ such that

$$\left| \int_E f(x)\, d\mu \right| \leqslant \epsilon,$$

whenever $E \subset A$ satisfies $\mu(E) < \delta$.

18. (Bounded convergence theorem) Let $f_n \to f$ almost everywhere on A, and let $\phi : A \to \mathbf{R}$ be an integrable function such that $|f_n(x)| \leqslant \phi(x)$ for almost every $x \in A$. Then f is integrable, and

$$\int_A f(x)\, d\mu = \lim_{n\to\infty} \int_A f_n(x)\, d\mu.$$

19. (Monotone convergence theorem) Suppose that $f_1(x) \leqslant f_2(x) \leqslant \ldots$ for all $x \in A$, each f_n is integrable, and

$$\int_A f_n\, d\mu \leqslant M,$$

for some constant M. Then $f(x) = \lim_{n\to\infty} f_n(x)$ is defined almost everywhere on A, f is integrable, and

$$\int_A f\, d\mu = \lim_{n\to\infty} \int_A f_n\, d\mu.$$

20. (Fatou's theorem) Let $f_n \geqslant 0$ be integrableon A, such that for some $M > 0$ we have

$$\int_A f_n\, d\mu \leqslant M,$$

and $\lim_{n\to\infty} f_n(x) = f(x)$ for almost every $x \in A$. Then f is integrable, and

$$\int_A f\, d\mu \leqslant M.$$

21. The space $L^1(X, \mu)$ is complete.

22. The space $L^2(X, \mu)$ is complete.

23. (Lusin's Theorem) Let $f : [a, b] \to \mathbf{R}$ be measurable, with respect to the Lebesgue measure. For every $\epsilon > 0$ there is a set $E \subset [a, b]$ with $\mu([a, b] \setminus E) < \epsilon$ such that the restriction of f to E is continuous.

24. (Riesz representation theorem) For every bounded linear functional $f : H \to \mathbf{C}$ on a Hilbert space H, there is an element $y \in H$ such that

$$f(x) = \langle x, y \rangle, \text{ for all } x \in H.$$

25. If $A : E \to F$ is a bounded linear operator between Banach spaces, then the adjoint A^* is bounded, and $\|A^*\| = \|A\|$.

26. The set of invertible elements in a Banach algebra with unit is open.

27. Any maximal ideal in a Banach algebra with unit is closed.

28. If $a \in A$ is an element in a Banach algebra with unit, then the spectral radius $\nu(a)$ satisfies $\nu(a) \leqslant \|a\|$.

29. If A is a Banach algebra where every non-zero element is invertible, then $A \cong \mathbf{C}$.

30. If $a \in A$ is normal in a C^*-algebra A, then $\nu(a) = \|a\|$.

31. If A is a commutative Banach algebra with unit, then there is a bijection between maximal ideals of A and non-zero homomorphisms $A \to \mathbf{C}$.

32. For a commutative Banach algebra A with unit, the spectrum $\sigma(a)$ of any element is the range of its Gelfand transform $\hat{a}$.