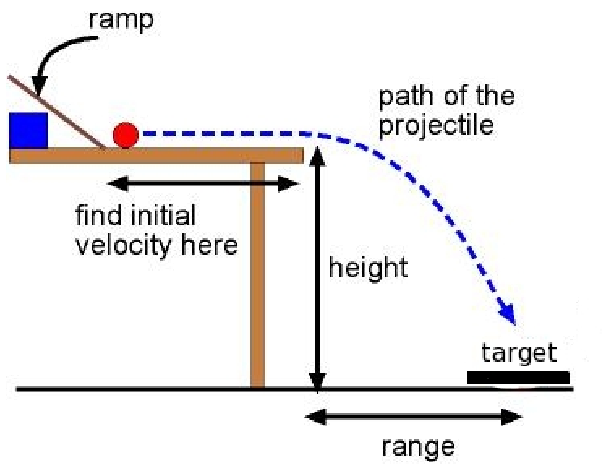
PROJECTILE MOTION

Introduction: By rolling a steel marble down a ramp and measuring its horizontal range, you can calculate the marble's launch velocity. To confirm this velocity with an independent measurement, you can use a photogate. Once your experiment is well aligned and your calculations and measurements agree, you get to design an experiment.

Equipment: steel marbles, polycarbonate ramp, block, photogate (with lift), 2 rulers (one with open center), meterstick, meter tape, catch box, white paper, carbon paper, blue painter's tape, plumb bob, Vernier Caliper

Experimental Set-up:



1. Make a Launcher: the ruler with the open center is perfect for creating repeatable, straight launches down the ramp. Allow the ruler to stick off the edge of the ramp to smooth the transition of the ball to the table. Using tape, fix the ruler to the ramp so that it is centered and you can easily allow the marble to roll down it.

2. The marble has a little bounce coming off the ramp that you want to eliminate before it passes through the photogate at the table's edge. Roll the marble off the ramp a few times and decide how far to set the bottom of the ramp from the edge of the table. Be safe and make sure it is at least 50 cm back.

3. Once you've decided where to place your ramp, use blue tape to mark it's location. Mark the location of the block and the edge of the ramp. *The ramp will get bumped at some point during the experiment and you may have to retake your measurements if you can't exactly return the ramp to its original location and angle.*

4. Use the catch box as a backstop so your marble will stay contained when it rolls off the table.

5. Align the ramp with the photogate (the small black u-shaped detector attached to the lift). A meterstick is really helpful for this. Until aligned, make sure the marble doesn't hit the photogate. Use something (your lab partner's hand, a small book, a pencil case, a block of paper from the classroom), to set in front of the black plastic edges.

6. Make sure the marble rolls down the ramp and through the photogate without hitting anything. It should cleanly land on the floor.

7. Choose a release point for your marble(a fixed location on the ruler), it should be the same for every trial. At this point, you should have a repeatable experiment: the ramp should be at a fixed angle, the ball is always the same size and released from the same height, and the ball should cleanly pass through the photogate.

Study I: Range and Initial Velocity of the ball.

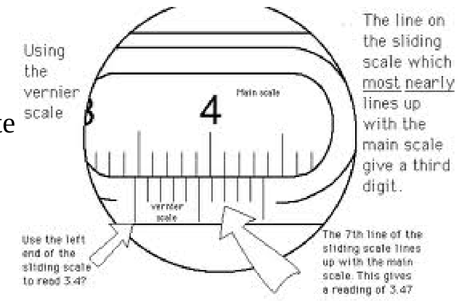
You are going to launch the marble from the same height 10 times and measure its horizontal range. Using the kinematic equations, you can calculate the marble's initial velocity as it leaves the table.

1. Tape a plain piece of paper to the floor so that the ball will land on it – line its edges up with a floor tile to get it straight. Place a piece of carbon paper (no tape) black-side-down on the plain paper. The carbon paper will make a mark on the paper when the ball strikes it.

2. Use the plumb bob to determine the point in the floor directly below the point that the ball leaves the table. Place a piece of tape on the floor and mark the tape with a pencil mark.

3. Launch the ball ten times (from the same height) onto the carbon paper target. Afterwards, lift the target. You should have a cluster of 10 points.

4. Measure the horizontal distance from the center of the cluster of points to the point the ball leaves the table. Estimate the uncertainty from the width of the cluster of points. Leave the paper on the floor for later measurements.



5. You will need to combine the next two measurements to calculate the total change of height of the ball Δy :
   i) height from the floor to the top of the table.

   ii) radius of the ball. Get the diameter using a Vernier Caliper.

   Combine (i) the table height and (ii) the marble radius to get Δy .

6. Given the equation $\Delta y = -1/2 g t^2 + v_{oy} t$, solve for the time of flight of the ball.

7. Given the equation $\Delta x = 1/2 a_x t^2 + v_{ox} t$, solve for the initial velocity of the ball.

Study II: Measuring the Initial Velocity

A photogate has a beam that trips when something passes through it. Once you calibrate, the photogate will directly measure the velocity of the ball. You can compare this to your previous value.

1. Make sure your photogate is plugged in both on the photogate and into the digital plug in the black **ScienceWorkshop 750** box on your lab bench.

2. Open **DataStudio**. If you don't immediately see the black **ScienceWorkshop 750** box on the screen, you will need to hit the "Change Interface" tab and select it.

3. Double-click on the image of the digital plug (corresponding to the physical plug) and add a "photogate" from the list of options.

4. Under the "Measurements" tab select "Velocity in Ch. 1", and under the "Constants" tab enter the diameter of the ball IN METERS into a box called a "Flag Length". This is how the software calculates a velocity. The photogate measures the time it take the ball to pass through and the software calculates the velocity through the photogate by: $v_{photogate} = \frac{flag\ length}{time\ measured}$.

   If you mis-align the photogate, the "flag length" will be wrong and the calculated velocity will be too fast. The best way to align the photogate is to make sure the ball lines up with the hole on the inside of the photogate.

5. Drag the Table icon (in the lower left) to "Velocity in Gate Ch. 1" (in the upper left).

6. Hit **Start** and launch the ball. You should get a velocity that is very close to your calculated initial velocity. Here are several things that could cause a discrepancy:
   i. You may need to align your photogate. You will never get velocities that are too slow, only too fast.
   ii. As long as you haven't disturbed your experiment, your new runs should fall exactly on your old runs (check this with the carbon paper). If something has moved and you think your initial velocity has changed, don't panic. Just measure the new range after you've completed the next step and redo the calculations from Study I.
   iii. You made a mistake entering your "flag length" in DataStudio.

7. Once your calculations and your measured velocities match, launch the ball 10 times: hit the Start (and stop) button in DataStudio before each run. Take a quick average of your data and compare it to your calculated value from Study I.

Study III: Choose your own adventure *With a little luck and patience, you should have a fairly accurate experimental apparatus. It is time to have some fun! Design an experiment using the following guidelines:*

A) **Math challenge:** Get your TA to give you a new height Δy . They will pick an object from around the room (catch box, block of paper, something), and it will be your catch pad. Using your initial velocity, predict precisely where the ball will land (with error bars to define a range of possible values). **Do all of your calculations to predict where the ball will land, and get your TA. You need a witness BEFORE launching the marble with the new Δy .**

B) **Modify one of the independent variables:** size of ball, release height of ball, angle of ramp or anything else you decide. Record how the change affects the dependent variables (range and initial velocity). Remember that you carefully calibrated the photogate with the diameter of one ball.

C) **Change the angle:** With the supplies at your lab bench, design an apparatus to change the launch angle of the ball off the table. Study the effects of changing the angle on the range and initial velocity.

Be sure to address the following:

1. What is your experimental question?

2. Isolate independent variables and measure their effects on dependent variables (what did you change and what did it affect? Don't change too many things so you can't isolate causes/effects).

3. Estimate uncertainty – particularly watch for the greatest source of uncertainty.
4. Record your data in a table if appropriate. If you do either experiments B or C, make a GA plot with the independent variable on the x-axis (angle, height, ball diameter) and the dependent variable on the y-axis (range, initial velocity).

Estimation of Uncertainty (Studies I & II)

1. Presumably you've done a very accurate job measuring Δy, so the error in your calculated time of flight is very small (less than 1%). The range of the ball had an uncertainty due to small variations in the experiment. Using the rules of error propagation, calculate the error in v_{ox}.

2. Use Graphical Analysis to calculate the average and standard deviation of your velocity data. Because you did multiple trials of something that had a small variance, a statistical analysis is appropriate.

   i. Make a column (named “index”) that simple reads 1,2,3,...10.
   ii. Enter your velocity data in a second column.
   iii. Plot “velocity” vs. “index” (so velocity is on the y-axis). Hit the STAT button and read off the average and the standard deviation. Enter these in the provided data table
   iv. By hand, calculate “the standard deviation of the mean” using N = 10. Present the best value of your velocity for the initial velocity using the statistical uncertainty δv_{ox} is the standard deviation of the mean.
   v. This GA plot was merely used for a statistics calculation. It isn't interesting enough to print and include in your lab report.

Before you leave the lab:
You should have a completed data tables for Studies I&II – including the uncertainty estimations/calculations. For Study III – it should be clear what your experimental question was and how you answered it with your data. You must have this data looked over and initialed by your T.A. before leaving the lab.

For the lab write-up: consider Studies I&II as a calibration step that everyone did. You don't have to formally write them up (or their error analysis). Don't bother writing the lab objective: “Our purpose was to predict the initial velocity of a projectile using its range and compare it to the value measured with the photogate.”

Instead focus on Study III. What was your objective? Greatest source of uncertainty? Present your data and calculations clearly. Spend plenty of time on the theory section: clearly draw the physics of a projectile in motion and show the relevant equations. It should be clear that you know how to calculate v_{ox} given the range Δx and change of height Δy.

Discussion questions for the report:

1. Did your values for v_{ox} agree (within error) between Studies I&II?

2. What was the spread of your values for v_{ox} from Study II? How did it compare with the spread in the values of the range Δx ?

3. Please discuss your results from Study III. What did you learn? Were you surprised? How did you plot your data, and what sort of relationship (curve fit) did you get? How would you improve your experiment for next time? What additional equipment would you want?

Study I: Range and Initial Velocity

Release point of marble (distance on ruler)	Distance from marble to photogate
diameter of steel marble, radius of steel marble	height of table
Δy = height + radius	$\Delta x \pm \delta(\Delta x)$ (measured/estimated)
Time of flight (calculated)	V_{ox} (calculated)

Study II: Measuring the Initial Velocity

Trial Number N	1	2	3	4	5	6	7	8	9	10
$\mathbf{V_{ox}}$ **(m/s)**										

Estimation of Uncertainty:
Given $\delta(\Delta x)$ your estimated uncertainty in the range (and ignoring $\delta(\Delta y)$) , calculate the uncertainty in the initial velocity $\mathbf{V_{ox}}$ from Study I.

Statistical Analysis Data (from Graphical Analysis)

Average $\mathbf{V_{ox}}$	Standard deviation of the mean $\sigma(\overline{V_{ox}}) = \sigma(V_{ox})/\sqrt{N}$
Standard deviation $\sigma(V_{ox})$	Best Value $\mathbf{V_{ox}}$ +/- $\sigma(\overline{V_{ox}})$

Study III:
Which option (A, B, or C) did you choose? What is your experimental question?