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Structural Controllability of Multi-agent Systems

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Abstract— **In this paper, the controllability problem for multi-agent systems is investigated. In particular, the case of a single leader under a fixed topology is considered. In contrast to the existing literature on this topic, we assume that the graph is weighted and we may freely assign the weights. Under this setup, the system is controllable if one may find a set of weights so as to satisfy the classical controllability rank condition. It turns out that this controllability notation purely depends on the topology of the communication scheme, and the multi-agent system is controllable if and only if the graph is connected. Moreover, some simulation results and numerical examples are presented to illustrate the approach.**

I. INTRODUCTION

Motivated by the recent developments in communication and computation technologies, distributed control of networked dynamic agents has rapidly emerged as a hot research area with strong support from both civilian and military applications, such as coordinated surveillance, target acquisition, reconnaissance, underwater or space exploration, assembling and transportation and rapid emergency response.

Cooperative control of multi-agent systems is still in its infancy and poses significant theoretical and technical challenges. Much work has been done on the formation stabilization and consensus seeking, see e.g., [11], [13], [3], [16]. To control such a complicated system, a lot of inspirations has been drawn from natural swarms such as fish schooling, bees flocking and ant colonies. Approaches like graph Laplacian for the associated neighborhood graphs, artificial potential functions, and navigation functions for distributed formation stabilization with collision avoidance constraints have been developed.

The key feature of multi-agent systems is that the group behavior of multiple agents is not simply a summation of the individual agent's behavior. Although each individual agent's dynamics and their interaction rules could be very simple, a large collection of these elementary agents, as a whole, could exhibit remarkable capabilities and display highly complex behaviors. The main challenge in this area is how to design these simple local interaction rules and communication protocols so as to achieve a desirable global behavior as a coherent group. Here, we will focus on the controllability problem and aim to investigate what kinds of communication topology, i.e., the minimum information exchange among agents, is required to make sure that the multi-agent system is controllable. Here, the controllability as a group is the desirable global behavior.

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The controllability problem of multi-agent systems has been investigated in the literature for a while. Tanner proposed this problem in [15] and formulated it as the controllability of a linear system, whose state matrices are induced from the graph Laplacian matrix. Necessary and sufficient algebraic conditions on the state matrices were given based on the well-known linear system theory. Under the same setup, a sufficient condition was derived in [4] and shown that the system is controllable if the null space of the leader set is a subset for the null space of follower set. In [5], it was shown that a necessary and sufficient condition for controllability is not sharing any common eigenvalues between the Laplacian matrix of the follower set and the Laplacian matrix of the whole topology. However, it remains elusive on what exactly the graphical meaning of these rank conditions related to the Laplacian matrix. This motivates several research activities on illuminating the controllability of multi-agent systems from a graph theoretical point of view. For example, a notion of anchored systems was introduced in [14], and it was shown that symmetry with respect to the anchored vertices makes the system uncontrollable. In [7], the authors characterized some necessary conditions for the controllability problem based on a new notation called leader-follower connectedness. While [7] was focused on the case of fixed topology, the corresponding controllability problem under switching topologies was investigated in [8], which employed some recent achievements in the switched system literature.

In contrast to the existing literature, we will consider weighted graph and focus on the case of a single leader under a fixed topology. It is assumed that the graph is weighted and we may freely assign the weighting. Accordingly, a new notion of controllability, structural controllability, for the leader-follower based multi-agent system with weighted topology is introduced. The system is called structurally controllable if one may find a set of weights such that the corresponding multi-agent system is controllable in a classical sense. Based on this notion, a necessary and sufficient condition for controllability of the multi-agent systems is proposed and interpreted in a graphic point of view. It turns out that this controllability notation only depends on the topology of the communication scheme, and the multi-agent systems is controllable if and only if the graph is connected.

The rest of the paper is structured as follows. In the next section, the new notation, structural controllability, for multi-agent systems is proposed, and the problem studied in this paper is formulated. In Section III, a necessary and sufficient condition for the structural controllability problem is given. Section IVpresents some numerical examples to illustrate the derived theoretical results and design methods. Finally, the paper concludes with comments and plans for our further work.

II. Notations and Problem Formulation

A. Notations

A weighted graph is an appropriate representation for the communication or sensing links among agents because it can represent both existence and strength of these links among agents. The weighted graph $\mathcal{G}$ with N vertices consists of a vertex set $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ and an edge set $\mathcal{I} = \{e_1, e_2, \ldots, e_N\}$, which is the interconnection links among the vertices. Each edge in the weighted graph represents a bidirectional communication or sensing media. The *order* of the weighted graph is denoted to be the cardinality of its vertex set. Similarly, the cardinality of the edge set is defined as its *degree*. Two vertices i and j are known to be *neighbors* if $(i, j) \in e$, and the number of neighbors for each vertex is its *valency*. An alternating sequence of distinct vertices and edges in the weighted graph is called a *path*. The weighted graph is said to be *connected* if there exists at least one path between any distinct vertices, and *complete* if all vertices are neighbors to each other.

The *adjacency matrix*, $\mathcal{A}$, is defined as

$$\mathcal{A}_{(i,j)} = \begin{cases} w_{ij} & (i,j) \in e, \\ 0 & \text{otherwise}, \end{cases}$$

where $w_{ij} \neq 0$ stands for the weight of edge (i, j). Here, the adjacency matrix $\mathcal{A}$ is $|\mathcal{V}| \times |\mathcal{V}|$ and $|.|$ is the cardinality of a set.

Define another $|\mathcal{V}| \times |\mathcal{V}|$ matrix, $\mathcal{D}$, called *degree matrix*, as a diagonal matrix which consists of the degree numbers of all vertices.

The *Laplacian matrix* of a graph $\mathcal{G}$, denoted as $\mathcal{L}(\mathcal{G}) \in \mathbb{R}^{|\mathcal{V}|\times|\mathcal{V}|}$ or $\mathcal{L}$ for simplicity, is defined as

$$\mathcal{L}_{(i,j)} = \begin{cases} \sum_{i\neq j} w_{ij} & i = j, \\ -w_{ij} & (i,j) \in e, \\ 0 & \text{otherwise}. \end{cases}$$

B. Multi-agent Structural Controllability

Our objective in this paper is to control N agents based on the leader-follower framework. Specifically, we will consider the case of a single leader and fixed topology. Without loss of generality, assume the N-th agent serves as the leader and take commands and controls from outside operators directly, while the rest $N - 1$ agents are followers and take controls as the nearest neighbor law.

Mathematically, each agent's dynamics can be seen as a point mass and follows

$$\dot{x}_i = u_i. \tag{1}$$

The control strategy for driving all follower agents is

$$u_i = -\sum\nolimits_{j\in\mathcal{N}_i} w_{ij}(x_i - x_j), \tag{2}$$

where $\mathcal{N}_i$ is the neighbor set of the agent i, and w_{ij} is weight of the edge from agent i to agent j. On the other hand, the leader's control signal is not influenced by the followers and need to be designed, which can be represented as

$$\dot{x}_N = u_N.$$

In other words, the leader affects its nearby agents, but it does not get directly affected from the followers since it only accepts the control input form an outside operator. For simplicity, we will use z to stand for x_N in the sequel.

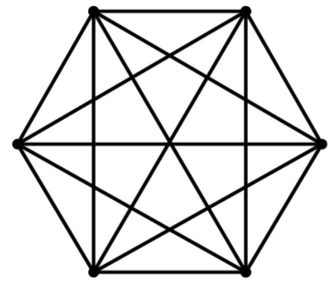

Fig. 1. A complete graph with 6 vertices.

According to the algebraic graph theory [2], it is known that the whole system can be written in a compact form

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A_{aq} & B_{aq} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ u_N \end{bmatrix} . \tag{3}$$

Or, equivalently

$$\begin{cases} \dot{x} &= A_{aq}x + B_{aq}z \\ \dot{z} &= u_N \end{cases} \tag{4}$$

where $A_{aq} \in \mathbb{R}^{(N-1)\times(N-1)}$ and $B_{aq} \in \mathbb{R}^{(N-1)\times 1}$ are both sub-matrices of the corresponding graph Laplacian matrix $\mathcal{L}$. The matrix A_{aq} reflects the interconnection among followers, and the column vector B_{aq} represents the relation between followers and the leader.

The problem is whether we can find a weighting scheme, i.e., set values for w_{ij}, such that it is possible to drive these agents to any configuration or formation (if the states stand for the positions of agents) by properly designed control signals u_N for the leader. This is related to the controllability of the system (4). Once the weights w_{ij} are all selected and fixed, the system (4) is reduced to a LTI system and its controllability can be directly answered by the well-developed linear system theory, see e.g. [1]. Actually, a special case when all weights $w_{ij} = 1$ (an unweighed graph) has been investigated in the past literature, e.g., [15]. However, Tanner in [15] showed that the complete graph is uncontrollable as illustrates in the following example.

Example 1: Consider a multi-agent system with six agents, whose communication topology is a complete graph with six vertices as shown in Fig. 1. Following the formulation in [15] that the matrices A_{aq} and B_{aq} in (4) can be written as

$$A_{aq} = \begin{bmatrix} 5 & -1 & -1 & -1 & -1 \\ -1 & 5 & -1 & -1 & -1 \\ -1 & -1 & 5 & -1 & -1 \\ -1 & -1 & -1 & 5 & -1 \\ -1 & -1 & -1 & -1 & 5 \end{bmatrix}, \; B_{aq} = \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix}.$$

It is not difficult to see that this pair is uncontrollable. This is quite counter intuitive, since the complete graph is an ideal case which provides the maximum information for the control purpose. It should be the case that more information exchanges among agents imply better control performances. The problem seems to be how we use these information. To treat all available information in an equal way seems not a good choice. One should use the information in a selective way. This motivates us to impose different weights according to the information resources.

With the set-up in (4), a set of weight can be assigned such that the controllability rank is satisfied; for instance, the pair (A_{aq}, B_{aq}) can be written as

$$A_{aq} = \begin{bmatrix} 7 & -2 & -2 & -2 & -1 \\ -2 & 9 & -3 & -2 & -2 \\ -2 & -3 & 13 & -5 & -3 \\ -2 & -2 & -5 & 11 & -2 \\ -1 & -2 & -3 & -2 & 8 \end{bmatrix}, \; B_{aq} = \begin{bmatrix} -1 \\ -2 \\ -5 \\ -3 \\ -1 \end{bmatrix}.$$

One can check that this (A_{aq}, B_{aq}) pair is controllable.

This example motivates us to give a more general definition for controllability of multi-agent systems as follows.

Definition 1: The linear system Σ in (4) is said to be structurally controllable if and only if there exists $w_{ij} \neq 0$ which can make the system (4) controllable.

Here, we are especially interested in a necessary and sufficient condition on the graphical topology of a multi-agent system to make it structurally controllable. That is, under exactly what condition of the graph that we can always find a weighting scheme w_{ij} so as to make the multi-agent system (4) controllable?

III. Structural Controllability

First, a lemma on controllability of (4) when weights are fixed is due.

Lemma 1: For the system (4) with a fixed weighting w_{ij}, the following statements are equivalent:
i) The system (4) is controllable.
ii) The controllability matrix

$$\mathcal{U} = \begin{bmatrix} B_{aq} & A_{aq}B_{aq} & \dots & A_{aq}^{N-2}B_{aq} \end{bmatrix}.$$

is of full row rank.
iii) The controllability grammian

$$W(t_0, t_f) = \int_{t_0}^{t_f} e^{A_{aq}\tau} B_{aq} B_{aq}^T e^{A_{aq}^T \tau} d\tau$$

is nonsingular for all $t > 0$.
iv) The matrix $\begin{bmatrix} A_{aq} - \lambda I & B_{aq} \end{bmatrix}$ has full row rank for all eigenvalues λ of A_{aq}.

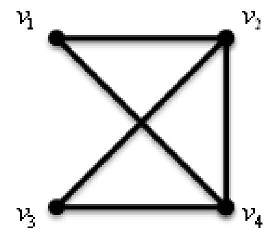

Fig. 2. Topology $\mathcal{G}$

The above lemma is a direct consequence of the well-known linear systems theory, see e.g., [1], due to the fact that the system (4) is reduced to a LTI system once weighting is fixed; however, for the structural controllability of multi-agent system we need the following definitions from [10].

Definition 2: The pair (A_{aq}, B_{aq}) in (4) is said to be reducible if they can be written in the form below;

$$A_{aq} = \begin{bmatrix} A_{aq_{11}} & 0 \\ A_{aq_{21}} & A_{aq_{22}} \end{bmatrix}, \quad B_{aq} = \begin{bmatrix} 0 \\ B_{aq_{22}} \end{bmatrix}, \qquad (5)$$

where $A_{aq_{11}} \in \mathbb{R}^{p \times p}$, $A_{aq_{21}} \in \mathbb{R}^{(N-1-p) \times p}$, $A_{aq_{22}} \in \mathbb{R}^{(N-1-p) \times (N-1-p)}$ and $B_{aq_{22}} \in \mathbb{R}^{(N-1-p)}$.

It was shown in [10] that the controllability matrix for this structure cannot be of full row rank no matter how one chooses the weighting w_{ij}. Hence, the system (4) is not structurally controllable under this situation.
Another obviously uncontrollable scenario is captured as follows.

Lemma 2: [10] The system (4) is not structurally controllable if the matrix $[A_{aq}, B_{aq}]$, which is $N-1 \times N$ matrix, can be written as

$$Q = \begin{pmatrix} Q_{11} \\ Q_{22} \end{pmatrix}, \qquad (6)$$

where Q_{22} is of $(N-1-p) \times N$ and Q_{11} is of $p \times N$ with at most $p-1$ nonzero entries and the rest of columns are all zero.

Interestingly, except these two obviously uncontrollable scenarios, the system (4) will be structurally controllable as the following lemma states.

Lemma 3: [10] The pair (A_{aq}, B_{aq}) is structurally controllable *if and only if* it is neither reducible nor writable into the form of (6) in Lemma 2.

Our next task is to interpret the above results in a graph theory point of view. It has been shown in [2] that the relation of a pair (A_{aq}, B_{aq}) can be depicted in a pictorial representation and the notion of flow structure plays an important role here. Hence, we introduce some necessary notations which we need for further discussions in this paper.

Definition 3: The pair (A_{aq}, B_{aq}) matrix can be represented by a digraph, defined as a flow structure, $\mathcal{F}_{\mathcal{G}}$, with vertex set $\mathcal{V}' = \{v_1', v_2', ...v_N'\}$. There exists an edge from v_i' to v_j' in the flow structure if and only if $A_{aq}(j,i) \neq 0$ and an edge from v_N' to v_i' if and only if $B_{aq}(i) \neq 0$.

Remark 1: Directions of links in flow structure has no dependence on the sign of their corresponding entries in matrix A_{aq}.

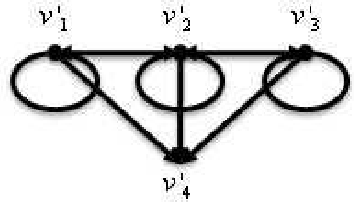

Fig. 3. Flow graph

Fig. 4. Stem

For example, the flow structure for the graph shown in Fig. 2 is depicted in Fig. 3. There are some well known flow structure that have interesting controllability properties, such as the flow structure of an ordered vertex set $\mathcal{V}' = \{v_1', v_2', ..., v_n'\}$ with a sequence of edges, where terminal vertex of each edge is initial vertex of the following edge. This is known as a stem [10], as depicted in Fig. 4. The corresponding state matrices for a stem, denoted as (A_{aq}^*, B_{aq}^*), can be written as

$$A_{aq}^* = \begin{bmatrix} 0 & * & 0 & \dots & 0 \\ & & & & \\ \vdots & & \ddots & \ddots & \\ & & & 0 & * \\ 0 & & \cdots & & 0 \end{bmatrix}, \quad B_{aq}^* = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ * \end{bmatrix},$$

where the symbol $*$ is used to represent the unknown but nonzero elements that depends on the weighting for edges. This falls into the controllable canonical form, so the controllability is obvious for a stem structure.

Another interesting structure grows from a stem. If the vertex v_n' of a stem structure coincides with v_2', the structure is called a bud [10] and its corresponding flow structure is shown in Fig. 5. For a bud, the corresponding pair (A_{aq}^*, B_{aq}^*) can be written as

$$A_{aq}^* = \begin{bmatrix} 0 & * & 0 & \dots & 0 \\ & & & & \\ \vdots & & \ddots & \ddots & \\ & & & 0 & * \\ * & & \cdots & & 0 \end{bmatrix} \quad B_{aq}^* = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ * \end{bmatrix}.$$

A union of a stem $\mathcal{S}$ and and buds $\mathcal{B}_i$, $1 \le i \le d$, is called a cactus if none of the buds $\mathcal{B}_i$ share a common initial vertex

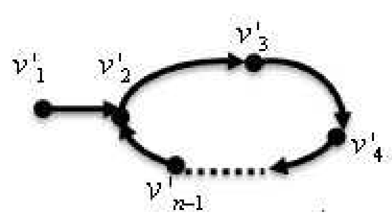

Fig. 5. Bud

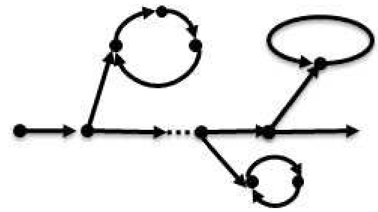

Fig. 6. Cacti

in $\mathcal{S}$. A set of mutually disjoint cactus is called a cacti, as illustrated in Fig. 6.

Based on these notations, we have the following sufficient condition to characterize the structural controllability of the multi-agent system (4).

Proposition 1: The multi-agent system (4) is structurally controllable if its corresponding flow structure can be spanned by a cacti.

Proof: Suppose that the graph can be spanned by a union of mutually disjoint cactus $\mathcal{C}_i$, $1 \le \ i \le p$. Under this scenario all edges equal to zero except those pertaining with one cacti. With the help of the permutation matrix, A_{aq}^* can be written in form I as

$$A_{aq}^* = \left[\begin{array}{c|c|c|c} \begin{bmatrix} 0 & * & & \\ & & \ddots & \\ & & \ddots & * \\ 0 & & & 0 \end{bmatrix} & 0 & & \cdots \\ \hline \begin{bmatrix} 0 & \cdots & 0 \\ & & \\ \ddots & & \\ & * & 0 \end{bmatrix} & \begin{bmatrix} 0 & * & \cdots & 0 \\ & & \ddots & \\ & & \ddots & * \\ * & & & 0 \end{bmatrix} & 0 & \\ \hline \begin{bmatrix} 0 & \cdots & 0 \\ & & \\ \ddots & & \\ & 0 & * \end{bmatrix} & & \begin{bmatrix} 0 & * & \cdots & 0 \\ & & \ddots & \\ & & \ddots & * \\ * & & & 0 \end{bmatrix} & \\ \hline & \begin{bmatrix} 0 & \cdots & 0 \\ & \ddots & \\ & \ddots & \\ * & & 0 \end{bmatrix} & & \begin{bmatrix} 0 & * & \cdots & 0 \\ & & \ddots & \\ & & \ddots & * \\ * & & & 0 \end{bmatrix} \end{array}\right]$$

(Form I)

while B_{aq}^* has the structure in the form;

$$\begin{bmatrix} 0 \\ 0 \\ \vdots \\ * \end{bmatrix}.$$

Hence, the matrix $\begin{bmatrix} A_{aq}^* - \lambda_i I & B_{aq}^* \end{bmatrix}$ has generic full row rank for all λ_i, $1 \le i \le N$, which implies the the structural controllability. ■

The above result is a direct application of some known structural controllability results for linear systems in [10] through the introduction of the flow structure. What does this imply in the original graph? The following theorem answers this and provides a nice graphical interpretation.

Theorem 1: The multi-agent system (4) under the communication topology $\mathcal{G}$ is structurally controllable *if and only if* $\mathcal{G}$ is connected.

Proof: Necessity: Assume that the graph $\mathcal{G}$ is disconnected. For simplicity, we will prove by contradiction for the case that there exists only one disconnected agent. There are two possibilities: First, this isolated agent is the leader. Then, B_{aq} is a null vector in this case , and the system is uncontrollable no matter what the weights are. Secondly, the isolated agent is one follower. For this case, (A^*_{aq}, B^*_{aq}) is reducible, which implies uncontrollability. Both cases end with a contradiction, so the necessity holds.The proof can be straightforwardly extended to more general cases with more than one disconnected agents.

Sufficiency: For the sufficiency part, we show that a connected graph cannot be written either in a reducible form or in the form of (6). Note that $w_{ij} \neq 0$ if and only if $w_{ji} \neq 0$. Then, (A^*_{aq}, B^*_{aq}) is in a reducible form if and only if A^*_{aq} is of a block diagonal matrix, which implies that the graph is disconnected. This contradicts with our assumption on the graph connectivity. On the other hand, the graph contains isolated vertex if and only if $\mathcal{D}$ matrix contains zero diagonal element. So, (A_{aq}, B_{aq}) pair can be written in the form of (6) in Lemma 2 if and only if it has a group of isolated agents. Therefore, according to Lemma 3, the graph is structurally controllable. ■

IV. Numerical examples

In this section, we give some numerical examples to illustrate the theoretical results demonstrated in the earlier sections. In section II we just mentioned the controllability for one dimensional cases. However, all the results can be readily extended to higher dimensions by Kronecker product, as argued in [15].

Example 2: A star graph is shown in Fig. 7. It is assumed that the central agent which is denoted with bold point in Fig. 7 serves as the leader and reset are just followers. This structure can be steered to any desired configuration because leader has direct access to all followers. Under the notion of structural controllability one can find a set of weight to make controllability rank condition satisfied; for example, the pair can be written as

$$A_{aq} = \begin{bmatrix} 1 & & & \\ & 5 & & \\ & & 3 & \\ & & & 2 \end{bmatrix} \quad B_{aq} = \begin{bmatrix} -1 \\ -5 \\ -3 \\ -2 \end{bmatrix}$$

Another interesting phenomenon is demonstrated in the following example.

Example 3: The graph shown in Fig. 8. The middle agent, depicted with bold dot is the leader. It is claimed in [14] that symmetry with respect to the sufficient condition for a system to be uncontrollable. However, under the set-up in (4), pair (A_{aq}, B_{aq}) can be written as the following form:

$$A_{aq} = \begin{bmatrix} 3 & -2 & 0 & 0 & 0 & 0 \\ -2 & 7 & -4 & 0 & 0 & 0 \\ 0 & -4 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6 & -2 & -3 \\ 0 & 0 & 0 & -2 & 2 & 0 \\ 0 & 0 & 0 & -3 & 0 & 3 \end{bmatrix} B_{aq} = \begin{bmatrix} 0 \\ -1 \\ 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

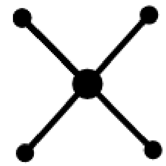

Fig. 7. Star graph

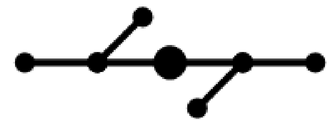

Fig. 8. Symmetrical structure

Next, we will consider the formation control among a group of agents on the plane, while each agent's state is of three dimensions, the x, y positions and its heading angle. Assume that interconnected topology is as depicted in Fig. 2, where the vertex v_1 is selected to be the leader and the remaining three are followers. Thus, the corresponding (A_{aq}, B_{aq}) with proper weighting selections is

$$A_{aq} = \begin{bmatrix} 5 & -1 & -2 \\ -1 & 4 & -2 \\ -2 & -2 & 4 \end{bmatrix}, B_{aq} = \begin{bmatrix} -1 \\ -3 \\ 0 \end{bmatrix}.$$

Some desired formation such as horizontal line, vertical

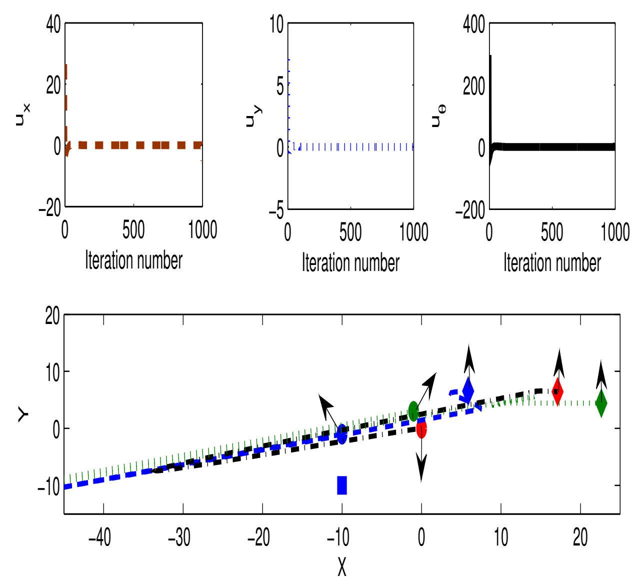

Fig. 9. Horizontal line formation. Heading control effort (solid line), X position control effort (dashed line), Y position control effort (dotted line). Initial position (circle), final position (diamond), the leader (square)

line and triangular shape are applied to this topology. The initial position and final position are denoted with circle and diamond, respectively and the leader is denoted by a square.

The control input for the x position, y position and heading are depicted with the dashed line, the dotted line and the solid line, correspondingly. The magnitude of control effort

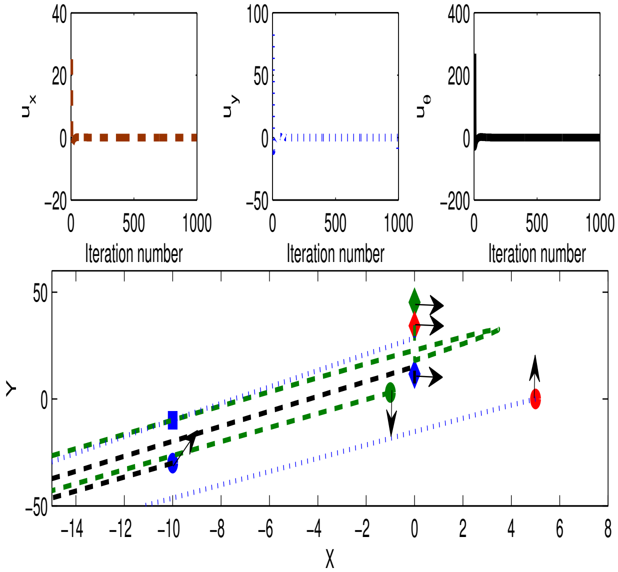

Fig. 10. Vertical line formation. Heading control effort (solid line), X position control effort (dashed line), Y position control effort (dotted line). Initial position (circle), final position (diamond), the leader (square)

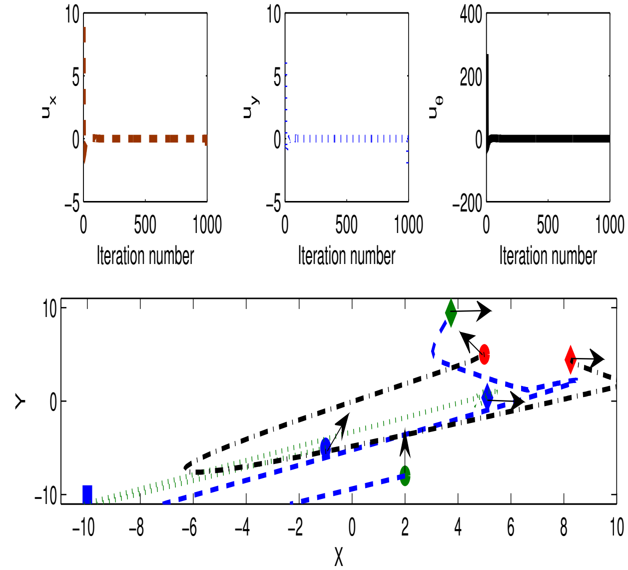

Fig. 11. Triangular shape formation. Heading control effort (solid line), X position control effort (dashed line), Y position control effort (dotted line). Initial position (circle), final position (diamond), the leader (square)

needed for the heading is relatively large comparing to the positioning control efforts.

V. Conclusion

In this paper, the controllability problem for multi-agent systems interconnected via a fixed weighted topology was investigated. A novel notion of multi-agent structural controllability was proposed, and a necessary and sufficient condition was derived accordingly. It was shown that the connectivity is not only necessary, but also sufficient for structural controllability of interconnected systems. The simulation results seem promising and underscore their theoretical counterparts.

Further research is directed on developing an algorithm to preserve the connectivity among the agents; moreover, an optimal paradigm needs to be developed for weights' assignment among agents such that the total energy given to the system be minimized. In addition, assuming more than one leader in a group and high order dynamics realization for each agent are future challenges.

VI. Acknowledgement

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