Structural Controllability of High Order Dynamic Multi-agent Systems

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Abstract—**Recently, the controllability problem of multi-agent systems is significantly explored; however, the majority of studies have been focused on the classical controllability approaches. This paper investigates the necessary and sufficient conditions of structural controllability for high order dynamic multi-agent systems. We consider a group of agents in a leader-follower framework under a fixed topology structure. It is assumed that, the agents interconnection is a weighted graph with freely chosen weights and each agent has a high order controllable canonical dynamic. Under this setup we show that the structural controllability of such a network is directly determined by agent interconnection. It is shown that a set of weights can be found which make the entire network controllable if and only if the graph is connected. Finally, we present a numerical example and simulation to illustrate the results.**

Index Terms—**multi-agent systems, structural controllability, high-order dynamic agents, graphs.**

I. Introduction

In the past decade, due to recent significant improvements in communication and network systems, the cooperative and coordination control of multi-agent systems have been widely developed. This research area is supported by a lot of military and industrial applications such as unmanned aerial vehicles(UAVs), distributed sensor networks (DSNs) [1][2], large scale energy systems and so on. Furthermore, we can find many natural and biological inspirations for multi-agent systems including ant colonies, fish schooling, bees flocking and flock of birds [3][4]. The cooperation of multi-agent systems has been addresses from various perspectives such as consensus [5], rendezvous problem [6] and formation [7] [8] [9].

The main idea of multi-agent systems is using a group of simple dynamic agents to accomplish a complicated task. Based on this strategy, behavior of each individual agent not only depends on its dynamics, but also relies on behavior of its neighbors. In order to design a control strategy for a group of mobile agents, exchanging information is necessary; the information can be position, velocity, or other relevant quantities to solve a given task. The aim in this area is to design an appropriate interconnection topology and applying a decentralized local control effort such that the cooperation of whole agents gains a desired global specifications.

Controllability of multi-agent systems as a fundamental concept in this field has gotten significant attention. Many groups have participated in studying this problem. The problem formulation and interconnection control rule, denoted by neighbor-based law, has proposed in [10]. In this work, the topology is assumed to be fixed and agents are supposed to have a single integrator dynamic. This idea has been developed by [11] [12] [13] and several algebraic conditions for controllability of multi-agent systems are achieved. However, it was not clear, how the interconnection topology is related to the network controllability. Ji [14] showed how the topology structure effects on the network controllability and Meng Ji et al. [15] found the graph interpretation conditions for controllability criteria of multi-agent systems. In [16] it is considered the position and velocity states of agents and investigated the controllability of double order integrator dynamic systems. [17] extended this work for high order integrator models. They achieved interesting results which imply that the controllability of a network constructed by high ordered integrator agents has the same properties as networks based on single-integrator agents under the same interconnection and same leaders position. The controllability of weighted graph was first introduced in [18]. In this paper it is assumed that the network is a weighted graph; links weight is not fixed and each agent has a simple one order integrator dynamic. They gave a graph theoretical based conditions for structural controllability of such a network. In [17] the authors worked out this problem for directed graphs and multi-leader framework. Liu et al. in [19] developed these results for switching systems and proved that a network of agents with switching topologies is structurally controllable if union graph of the underlaying communication topologies is connected. The significant finding is that graph connectivity plays an important role in the controllability of multi-agent systems.

In this paper, we will further extend this notion and based on the graph theory we present the sufficient and necessary conditions for structural controllability of high order integrator systems. In the next step, we generalized our results to any controllable canonical dynamics.

The rest of paper is organized as follow: in section 2, we introduce some graph theory preliminaries and definitions; existing results on structural controllability of single integrator system are summarized in section 3. Section 4 contains the problem formulation and main results on structural controllability of multi robot systems based on high order integrator dynamic; more general case is considered in section 5 and it is shown that a network including the controllable canonical model agents has the same structural controllability condition as a high order integrator system based networks. In order to

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illustrate and validate the results, a numerical example is given in section 6.

II. Basic graph theory

In this section, we are going to have an overview on the graph theory concepts. We consider a communication network of agents as a weighted and undirected graph. A graph $\mathcal{G}$ contains a set of nodes $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ and a set of edges $\mathcal{E} \subset V \times V$, $\mathcal{E} = \{e_1, e_2, \ldots, e_N\}$, which is denoted by $\mathcal{G}(\mathcal{V}, \mathcal{E})$. Each agent is represented by a node and each $e_{ij}(v_i, v_j)$ means the agents v_i and v_j share the information about their states. Each node has an individual weight $\mathcal{W}$ and it is assumed that $\mathcal{W}_{ij} \neq \mathcal{W}_{ji}$ and $\mathcal{W}_{ij} \geq 0$. Let's say $\mathcal{G}'$ is a *subgraph* of $\mathcal{G}$, defined as $\mathcal{G}' \subset \mathcal{G}$, if $\mathcal{V}' \subset \mathcal{V}$ and $\mathcal{E}' \subset \mathcal{E}$. Two vertices v_i and v_j are *neighbor*, if $(i, j) \in e$. The neighbors of vertex v_i is denoted by $\mathcal{N}_i = \{v_j : e_{ji} = (v_i, v_j) \in \mathcal{E}\}$. *Valency* of vertex v_i is defined as the number of its neighbors . Vertices x and y are *adjacent* if x is a neighbor of y. A path is an alternating sequence of distinct edges in $\mathcal{E}$ such that all consecutive agents are adjacent. A graph $\mathcal{G}$ is called *connected* if there exists a path between any two different vertices. A subgraph $\mathcal{G}'$ of an undirected graph is called a *connected component* if all its vertices are connected to each other by paths. We say a graph is complete if every pair of (v_i, v_j) are adjacent.

Let's define *weighted adjacency matrix*, $\mathcal{A}$ as

$$\mathcal{A}_{(i,j)} = \begin{cases} \mathcal{W}_{ij} & (i,j) \in e \\ 0 & \text{otherwise} \end{cases}$$

and denote the *Laplacian matrix* $\mathcal{L}(g) = [l_{ij}] \in R^{|\mathcal{V}| \times |\mathcal{V}|}$ as

$$l_{ij} = \begin{cases} \sum\limits_{i \neq j} \mathcal{W}_{ij} & i = j \\ -\mathcal{W}_{ij} & (i,j) \in e \\ 0 & \text{otherwise} \end{cases}.$$

III. Structural controllability of multi-agent systems

A. Single integrator dynamic systems

In this section, our objective is driving N followers by a leader in a fixed topology network. We call the agents with higher capabilities as a leader and rest as followers. It is assumed, the agents are labeled as x_i where $i = 1 \cdots N$ and the N_{th} agent contributes as a leader. Let's suppose each agent has a single integrator dynamic and followers obey neighbor-based control law, defined as

$$\dot{x}_i = u_i = -\sum\nolimits_{j \in \mathcal{N}_i} \mathcal{W}_{ij}(x_i - x_j), \tag{1}$$

In the leader-follower framework, the whole system can be written as follows [10] [18].

$$\begin{bmatrix} \dot{y} \\ \dot{z} \end{bmatrix} = \begin{pmatrix} \mathcal{L}_f & \mathcal{L}_{fl} \\ 0 & 0 \end{pmatrix} \begin{bmatrix} y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ U_E \end{bmatrix} \tag{2}$$

Where $\mathcal{L}_f$ is the followers principal submatrix corresponding to weighted laplacian matrix and $\mathcal{L}_{fl}$ represents the interconnection between leader (y) and followers (z). In order to achieve a formation, leader agent is supposed to change its position such that follower agents converge to the final positions. Controllability of the system (2) guarantees that by applying an appropriate control input U_E followers' states can be steered to any desired values. Since under this setup (1), weight of communication links can be freely chosen, the system (2) should be considered as a structured linear system and accordingly, it is needed to investigate the structural controllability.

As a matter of fact, the problem is summarized to find a set of weights which makes the system controllable. A trivial example is adding weights to an uncontrollable network such as complete graph [10] and turns it to a controllable topology [17].

Lemma 1: [20] Given a linear system $\dot{x}(t) = Ax(t) + Bu(t)$; where $x(t) \in R^n$ and $u(t) \in R^m$. The matrices A and B are structured, if their elements are either fixed zeros or independent free parameters. The structured matrix pair (A, B) is said to be structurally controllable if the free parameters in A and B can be fixed so that (A, B) is controllable.

The structural controllability of single integrator multi-agent systems was significantly expanded in [18] and developed by [17]. The main result can be summarized as follows.

Lemma 2: [18] The multi-agent system (2) under the communication topology $\mathcal{G}$ is structurally controllable if and only if $\mathcal{G}$ is connected .

B. High order integrator dynamic systems

In this part, we extend structural controllability of multi-agent systems for high order integrator dynamic agents. The following n_{th} order differential equation is considered as a dynamic of each agent.

$$\begin{aligned} &\dot{x}_i^{(1)} = x_i^{(2)}, \ldots, \dot{x}_i^{(n-1)} = x_i^{(n)} \\ &\dot{x}_i^{(n)} = u_i \qquad i = 1 \cdots N+1, \end{aligned} \tag{3}$$

Where n denotes the order of equation (3) ; $x_i^{(m)} \in R$ is m_{th} derivative of agent i state and $u_i \in R$ is the single input control effort. Moreover, it is assumed, the given network has a single leader structure and N follower agents.

The control input is well known neighbor-based law, described by

$$u_i = -\sum_{j \in N_i} \sum_{m=0}^{n-1} k_m (x_i^{(m+1)} - x_j^{(m+1)}) \tag{4}$$

where $k_0 \cdots k_{n-1}$ are nonzero feedback gains of relative states .

It should be noted that, we assume the agents are able to exchange all states information with each other.

By substituting (4) in (3) and partitioning the laplacian matrix to $\mathcal{L}_f$ and $\mathcal{L}_{fl}$ submatrices, the entire network model can be rewritten into the followers and leader dynamics.

The dynamic of follower agents has the following form

$$\dot{x} = \phi_F x + \phi_{fl} z \tag{5}$$

Where

$$\phi_F = \begin{bmatrix} \mathbf{0} & I_N & \mathbf{0} & \mathbf{0} \\ \vdots & & \ddots & \\ \mathbf{0} & & & I_N \\ -k_0 L_f & -k_1 L_f & \cdots & -k_{n-1} L_f \end{bmatrix} \quad (6)$$

and correspondingly,

$$\phi_{fl} = \begin{bmatrix} \mathbf{0} & \cdots & \mathbf{0} \\ -k_0 L_{fl} & \cdots & -k_{n-1} L_{fl} \end{bmatrix}. \quad (7)$$

The dimension of matrices ϕ_F and ϕ_{fl} in system (5) respectively are $\phi_F \in R^{nN \times nN}$ and $\phi_{fl} \in R^{nN \times n}$ and the states of follower group are sorted as $\dot{x} = \begin{bmatrix} \dot{x}_1^{(1)} \cdots \dot{x}_N^{(1)} & \cdots & \dot{x}_1^{(n)} \cdots \dot{x}_N^{(n)} \end{bmatrix}^T$. Noted that, all the leader states are served as control input for system (5).

Accordingly, the leader dynamic is

$$\begin{aligned} \dot{z}^{(1)} &= z^{(2)}, \ldots, \dot{z}^{(n-1)} = z^{(n)} \\ \dot{z}^{(n)} &= U_E \end{aligned} \quad (8)$$

The leader takes control input signal from an external source. This extraneous reference (U_E) is able to drive the leader states to any arbitrary values. Correspondingly, the leader states serve as a control input for followers. In contrast, leader states are supposed to change its states such that followers' states converge to the desire conditions and accordingly, U_E should be able to provide this trend for the leader. The controllability of system (5) guarantees this goal. If the links weight are fixed the system (5) turns to a linear time invariant system and the controllability of such a system can be verified in the classical way.

Lemma 3: If there exists $\mathcal{W}_{ij} = c$ for any nonzero positive constant c, the system (5) is controllable if and only if the controllability matrix (9) is full row rank.

$$CO = \begin{bmatrix} \phi_{fl} & \phi_F \phi_{fl} & \cdots & \phi_F^{nN-1} \phi_{fl} \end{bmatrix} \quad (9)$$

Lemma 3 is observed from well-known LTI system controllability results, see [21]. Here, we are interested to have a freedom to assign the edges weight; therefore, the problem becomes in which condition a network based on the multi-agent systems are structurally controllable. Since the graph theory is very close to our work, we focus on the graph interpretation prospective of structural controllability. First, it is necessary to introduce the system graph representation. Consider a linear system $\dot{x} = Ax + Bu$, where $A \in R^n$ and $B \in R^m$. The graph obtained from the pair (A, B) is defined as follows. Take a set of nodes $X = \{x_1, \cdots, x_n\}$ and $U = \{u_1, \cdots, u_m\}$ into the consideration. A directed link connects x_i and x_j with weight a_{ij} or x_i and u_i with weight b_{ij} if and only if (i, j) element of A or B is a nonzero free parameter respectively. The achieved directed weighted graph is denoted as $G(A, B)$.

Lemma 4: [20] In the graph $G(A, B)$, a path whose initial node is in U(the corresponding nodes to B) is called a *stem*. The initial (terminal) node of a stem is said the root (top) of the stem. For a node in $G(A, B)$ if there exists a stem whose top is the node, the node is said to be *accessible*. If every node in $G(A, B)$ is accessible, we say $G(A, B)$ is accessible .

Lemma 5: [22] The graph of a pair (A, B) contains *dilation* if and only if there is set S of k nodes in the vertex set of the graph not containing the origin nodes(U) such that there are edges no more than $k-1$ nodes V in $T(S)$. (One denotes, $T(S)$ the set of all the nodes x_j with the property that there exists an oriented edge from x_j to a node in S) .

Lemma 6: [20] The following two statements are equivalent.

a) The pair (A, B) is structurally controllable.
b) i) The $G(A, B)$ is accessible.
ii) There exists no dilation in $G(A, B)$.

Based on the Lemma 6, we will show the connectivity of network interconnection plays a major role on structural controllability of the entire system.

Theorem 1: The multi-agent system (5) under the communication topology $\mathcal{G}$ is structurally controllable if and only if $\mathcal{G}$ is connected.

Proof: It is necessary to first obtain the graph representation of $G(\phi_F, \phi_{fl})$. This graph is constructed from three parts. Fig. 1 shows the connection between local states of each individual agents with respect to equation (3).

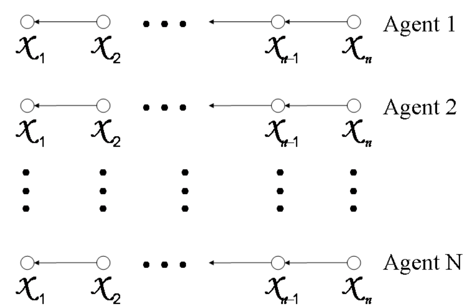

Fig. 1. Graph representation of local states connection

Fig. 2 is the representation of neighbor-based control law (2) which is applied to all $x_i^{(n)}$.

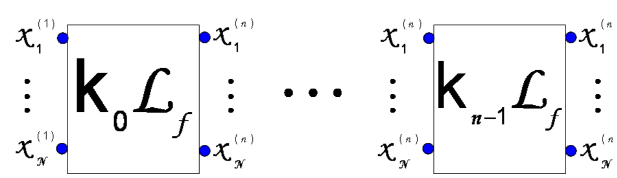

Fig. 2. Graph representation of interconnection between agents states

Similarly, Fig. 3 based on the matrix ϕ_{fl}, gives the agents states and system input interconnection i.e. leader states z_i, and states of ϕ_F.

Now we are ready to prove the necessity and sufficiency parts of Theorem (1).

Necessity : In this part, we will show that if the origin graph $\mathcal{G}$ is disconnected, the $G(A, B)$ is not accessible which implies the system (5) is uncontrollable. In our setup the graph disconnectivity can be due to two reasons; first the

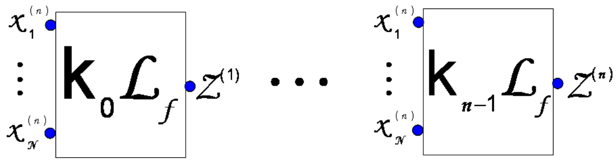

Fig. 3. Graph representation of interconnection between $x_i^{(n)}$ and system inputs (leader states)

leader node is disconnected. In this case $\mathcal{L}_{fl}$ becomes a zero column, therefore there is no path from inputs to $G(A)$ nodes and consequently $G(A, B)$ is not accessible. Second option is when the followers' subgraph is disconnected. Without loss of generality we assume the followers' subgraph is partitioned into two distinct components. By the appropriate permutation, the corresponding $\mathcal{L}_{fl}$ has the following block diagonal matrix form

$$L_f = \begin{bmatrix} \boxed{L_{fC1}} & \mathbf{0} \\ \mathbf{0} & \boxed{L_{fC2}} \end{bmatrix} \tag{10}$$

By substituting (10) in (5), while ϕ_{fl} is persevered, we get the new ϕ_F, described as:

$$\varphi_F = \begin{bmatrix} \mathbf{0} & I_N & \mathbf{0} & \mathbf{0} \\ \vdots & & \ddots & \\ \mathbf{0} & & & I_N \\ -k_0\begin{bmatrix} \boxed{\mathcal{L}_{c1}} & \mathbf{0} \\ \mathbf{0} & \boxed{\mathcal{L}_{c2}} \end{bmatrix} & -k_1\begin{bmatrix} \boxed{\mathcal{L}_{c1}} & \mathbf{0} \\ \mathbf{0} & \boxed{\mathcal{L}_{c2}} \end{bmatrix} & \cdots & -k_{n-1}\begin{bmatrix} \boxed{\mathcal{L}_{c1}} & \mathbf{0} \\ \mathbf{0} & \boxed{\mathcal{L}_{c2}} \end{bmatrix} \end{bmatrix}$$

Tacking Fig. 2 into consideration, the structure of above matrix shows the states of each individual component just affects on its own states. In other words, there is no connection between states of the agents included in component1 to the other component. Consequently, $G(A, B)$ is not accessible if $\mathcal{G}$ is disconnected.

Sufficiency : As previously mentioned, the system LTI system $\dot{x} = Ax + Bu$ is not structural controllable if and only if the $G(A, B)$ is not accessible or there is a dilation in $G(A, B)$. As a matter of fact, in order to handle the sufficiency part it is necessary to check the accessibility and existing of dilation in $G(A, B)$. The first proposition is, if $G(A, B)$ is not accessible, the origin graph $\mathcal{G}$ is disconnected. We are going to prove this statement by contradiction.

Regarding to $\mathcal{L}_f$ structure (see Fig. 1), the states $x_i^{(1)} \cdots x_i^{(n-1)}$ of each individual agent, are accessible from their corresponding x_n. Thus, if there exists at least one path from $x_i^{(n)}$ to the inputs, the $G(A, B)$ is accessible. Furthermore, the submatrix $k_{n-1}L_f$ shows the interconnection between all agents $x_i^{(n)}$ states; so it is easy to realize that if the origin graph $\mathcal{G}$ is connected we can find a path from the system inputs to all $x_i^{(n)}$ states. Hence, if the $G(A, B)$ is not accessible, the origin graph $\mathcal{G}$ must be disconnected.

Next claim is, if there is a dilation in $G(A, B)$, the origin graph $\mathcal{G}$ is disconnected. The proof is also conducted by contradiction. It can be seen from Fig. 1 that all of the corresponding states nodes except $x_i^{(n)}$ have at least one incoming edge, therefore there is no dilation in set of $x_i^{(1)} \cdots x_i^{(n-1)}$. It is obvious that if the graph origin $\mathcal{G}$ is connected there is at least one incoming edge to the all of $x_i^{(n)}$ nodes which implies, there is no dilation in $G(A, B)$ if $\mathcal{G}$ is connected. Hence proof is complete. ■

C. General high order dynamic systems

In this section, we extend our result to a more general type. We setup a network based on the agents with controllable canonical dynamic. Let's assume, the agents dynamic is a simple single input model and expressed as

$$\begin{aligned} &\dot{x}_i^{(1)} = x_i^{(2)}, \ldots, \dot{x}_i^{(n-1)} = x_i^{n} \\ &\dot{x}_i^{(n)} = -a_0 x_i^{(1)} - \cdots - a_{n-1} x_i^{(2)} + u_i \qquad i \in N+1, \end{aligned} \tag{11}$$

where n denotes the order of equation (11) ; $x_i^{(m)} \in R$ is m_{th} derivative of agent i state and $u_i \in R$ is the single input control effort. In this setup , it is also assumed that N followers are controled by a leader. By using the control input (4) the entire system is partitioned into followers and leader dynamic.

I) Followers dynamic

$$\dot{x} = \phi_F x + \phi_{fl}, z \;\; i \in N, \tag{12}$$

where z is all leader states and ϕ_F and ϕ_{fl} are respectively defined as

$$\phi_F = \begin{bmatrix} \mathbf{0} & I_N & \mathbf{0} & \mathbf{0} \\ \vdots & & \ddots & \\ \mathbf{0} & & & I_N \\ -k_0 L'_{f0} & -k_1 L'_{f1} & \cdots & -k_{n-1} L'_{fn-1} \end{bmatrix} \tag{13}$$

$$\phi_{fl} = \begin{bmatrix} \mathbf{0} & \cdots & \mathbf{0} \\ -k_0 L_{fl} & \cdots & -k_{n-1} L_{fl} \end{bmatrix}.$$

Noted that, in (13) the matrix $L'_{fm} = -a_m I_N - k_m L_f$ for all $m = 0 \cdots n-1$ and the states vector is $\dot{x} = \begin{bmatrix} \dot{x}_1^{(1)} \cdots \dot{x}_N^{(1)} & \cdots & \dot{x}_1^{(n)} \cdots \dot{x}_N^{(n)} \end{bmatrix}^T$.

Furthermore, the dimension of matrices ϕ_F and ϕ_{fl} in system (12) are $\phi_F \in R^{nN \times nN}$ and $\phi_{fl} \in R^{nN \times n}$ respectively.

II) Leader dynamic

$$\begin{aligned} &\dot{z}^{(1)} = z^{(2)}, \ldots, \dot{z}^{(n-1)} = z^{(n)} \\ &\dot{z}^{(n)} = -a_0 z^{(1)} - \cdots - a_{n-1} z^{(n)} + U_E, \end{aligned}$$

where z is the states of the agent with label $N+1$ and U_E is a external control input.

We are going to investigate the structural controllably for this network.

Theorem 2: The system (12) under the communication topology $\mathcal{G}$ is structurally controllable if and only if $\mathcal{G}$ is connected.

Proof: The differences between general high order dynamic multi-agent systems (12) and high order integrator dynamic systems (5) is just in the followers' subgraph. It is interesting that, the matrices L'_{fm} and (L_f) have differences just in the diagonal elements. Hence, regarding to necessary proof of Theorem 1, a disconnected (L_f) gives block diagonal L'_{fm} and as shown there, it makes $G(A, B)$ not accessible which means the system (12) is uncontrollable. For sufficiency part, as mentioned previously in Theorem 1 proof, a connected $\mathcal{G}$ guaranties that, there exists at least one path between all $x_i^{(n)}$ and all inputs z_i. Consequently the $G(A, B)$ is accessible and there exists no dilation in $G(A, B)$. This fact is especially is concluded from the last term $k_{n-1}L_f$ and since connectivity of terms $k_{n-1}L_f$ in (5) and $k_{n-1}L'_{fn-1}$ in (12) are the same, therefore this conclusion is still true. As a result, if the system (12) is structurally uncontrollable, the origin graph $\mathcal{G}$ is disconnected. Thus the proof is complete. ■

IV. Numerical Examples

In this section, to illustrate our theoretical results derived in the above section, we provide a numerical example. A formation among a group of mobile robots is our purpose for this example. We consider an undirected and weighted graph as a network. The system is constructed from three followers and one leader. The interconnection topology among the agents is depicted in Fig. 4. It is assumed that each agent has the following controllable canonical dynamic.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

Furthermore, the feedback gain is chosen as $k = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$. The corresponding laplacian matrix of the network is obtained as:

$$\mathcal{L} = \begin{bmatrix} 2 & -2 & 0 & 0 \\ -3 & 8 & -1 & -4 \\ 0 & -1 & 3 & -2 \\ 0 & -4 & -2 & 6 \end{bmatrix}. \tag{14}$$

As discussed in section 4, we can write the followers' model in the form of (12) and construct the controllability matrix as (9). Controllability of the system guarantees that we can find an appropriate control input for the leader to drive followers' states to the desired values.

According to Theorem 2, since the origin graph (14) is connected we expect a controllable system for our example. This phenomena is confirmed since the controllability matrix of network (4) is full row rank. Let's suppose the followers' states initial value is $x_0 = \begin{bmatrix} 1 & 1 & 4 & 2 & 1 & 4 & 3 & 1 & 2 \end{bmatrix}^T$ and the desire formation is formed at $x_f = \begin{bmatrix} 2 & 4 & 6 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$. We design an optimal controller to drive the followers' state to the final value. The simulation results and the state trajectories of each individual follower agent are depicted in Figs. 5, 6 and 7.

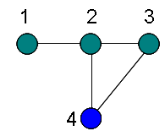

Fig. 4. Network interconnection, agents 1, 2 and 3 are follower and 4 is leader.

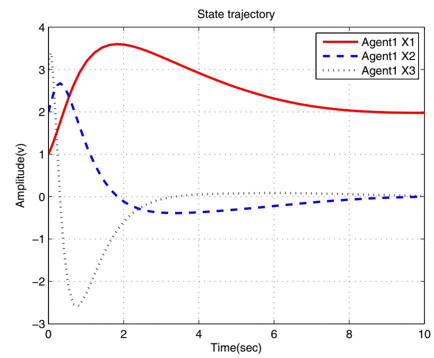

Fig. 5. States trajectory of agent 1.

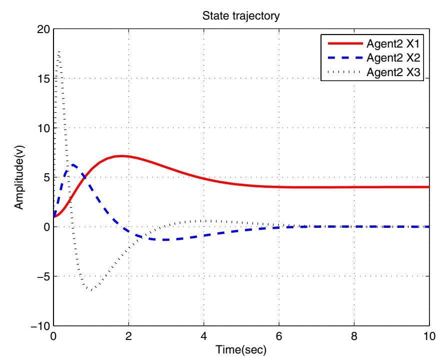

Fig. 6. States trajectory of agent 2.

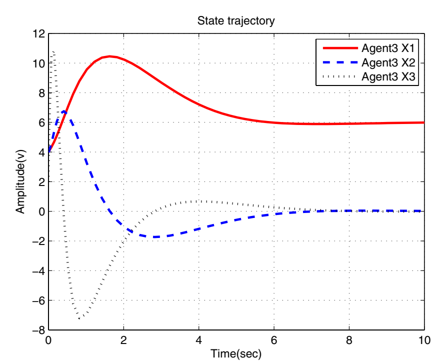

Fig. 7. States trajectory of agent 3.

V. Conclusion

This article focused on the development of controllability problem for multi-agent systems based on a fixed topology and leader-follower framework. The results of this paper highlight the roles of graph connectivity on the controllability of interconnected systems. We have presented the necessary and sufficient condition for structural controllability of the systems modeled by high order integrators. It is shown that a network of agents is structurally controllable as long as the interconnection graph is connected. Furthermore, we have extended our work to the more general case. It is proved that, a group of mobile agents with high order canonical controllable dynamic are structurally controllable if the origin graph is connected. Our future work is to develop these results for a switching network and multi leader structure.

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