CHAOS **18**, 033127 (2008)

Stationary oscillation of an impulsive delayed system and its application to chaotic neural networks

Jitao Sun[1,2,a)] and Hai Lin[2,b)]
[1]*Department of Mathematics, Tongji University, Shanghai 200092, China*
[2]*Department of Electrical and Computer Engineering, National University of Singapore, Singapore*

(Received 9 April 2008; accepted 10 July 2008; published online 5 September 2008)

This paper investigates the stationary oscillation for an impulsive delayed system which represents a class of nonlinear hybrid systems. First, a new concept of S-stability is introduced for nonlinear impulsive delayed systems. Based on this new concept and fixed point theorem, the relationship between S-stability and stationary oscillation (i.e., existence, uniqueness and global stability of periodic solutions) for the nonlinear impulsive delayed system is explored. It is shown that the nonlinear impulsive delayed system has a stationary oscillation if the system is S-stable. Second, an easily verifiable sufficient condition is then obtained for stationary oscillations of nonautonomous neural networks with both time delays and impulses by using the new criterion. Finally, an illustrative example is given to demonstrate the effectiveness of the proposed method. © *2008 American Institute of Physics*. [DOI: 10.1063/1.2966113]

The stability analysis of neural networks, such as Hopfield neural networks, cellular neural networks, and Cohen–Grossberg neural networks, has been extensively investigated. It is known that the existence of time delay may cause oscillations and instability in neural networks. Hence, the investigation of neural networks with time delay is an important issue. In addition, some biological neural networks in biology, bursting rhythm models in pathology, and optimal control models in economics are usually characterized by abrupt changes of state, which are also known as impulsive phenomena. Such a phenomenon, where sudden and sharp changes occur in a continuous process, cannot be well described by either a purely continuous or a purely discrete model. Therefore, it is important and, in fact, necessary to study a new type of neural networks, namely impulsive neural networks with time delay. Although there are a great deal results of periodicity theory for functional differential equations (without impulse) and impulsive differential equations (without delay), respectively, it still remains a very challenging task to investigate the existence of periodic solutions of dynamical systems with both time delays and impulses. The difficulty is mainly due to both the discontinuity of solutions and time delays. This motivates our study of the stationary oscillation for a general nonlinear impulsive delayed system and its application to chaotic neural networks. In this paper, a sufficient condition is presented for the existence of stationary oscillations for an impulsive delayed system based on a new notion of S-stability. The obtained criterion is then used to investigate the stationary oscillation for chaotic neural networks with both time delays and impulses, and an easily verifiable sufficient condition is derived. The introduced notion S-stability could provide a new way to characterize the existence of stationary oscillation for nonlinear impulsive delayed systems, and the new criterion of this paper may be extended to some other chaotic systems.

I. INTRODUCTION

Impulsive effects and time delay exist widely in many dynamical systems involving a many areas, such as population dynamics, economics, automatic control, neural networks, and so on. Impulsive delayed differential equations are effective mathematical models for these kinds of systems and have attracted increasing research attention due to their theoretical importance and application potential. It is well known that the periodic oscillations are important dynamical behaviors for nonlinear dynamical systems. Interested readers may refer to Refs. 1–3 and references cited therein for the periodicity theory for functional differential equations (without impulse), and for impulsive differential equations (without delay) to Refs. 4–8, and references therein. For the case of functional differential equations (without impulse) and impulsive differential equations (without delay), the periodicity theory has been completely developed. However, it still remains a very challenging task to investigate the existence of periodic solutions of dynamical systems with both delays and impulses. This difficulty is mainly due to both the discontinuity of solutions and time delays. Although some interesting results on periodic solutions of impulsive delayed systems have been reported in Refs. 9–16, the existing work focuses mainly on some special delayed differential systems (neural networks) with linear impulses, and the main results of Refs. 15 and 16 are incorrect. Therefore, it is necessary to further study the stationary oscillation for general nonlinear impulsive delayed systems.

On the other hand, neural networks have been extensively and successfully applied in psychophysics, biology, speech, perception, robotics, adaptive pattern recognition, vision, and image processing. But the delays often happen and

[a)]Electronic mail: sunjt@sh163.net.
[b)]Electronic mail: elelh@nus.edu.sg.

they may cause oscillations and instability in neural networks. Hence, there are a lot of research results of neural networks with time delay in recent years; see, for instance, Refs. 17–20 and references cited therein. In addition, most neural networks can be classified into two types, continuous or discrete. However, many real-world systems and natural processes cannot be simply categorized into one of them. They may display both continuous and discrete characteristics. Therefore, it is important and, in fact, necessary to study impulsive neural networks with time delay. To the best of our knowledge, the global stability and existence of periodic solution are seldom considered for neural networks with both time delays and impulses.

In this paper, the authors investigate the stationary oscillation for a nonlinear impulsive delayed system and its application to chaotic neural networks. First, this paper presents a sufficient condition for stationary oscillation of the nonlinear impulsive delayed system based on a new notion of S-stability. The criterion is then used to investigate the stationary oscillation for chaotic neural networks with both time delays and impulses. An easily verifiable criterion for stationary oscillations of the neural networks is obtained through proof that the system is S-stable by using the Lyapunov method. Finally, an illustrative example is given to demonstrate the effectiveness of the proposed method.

The organization of this paper is as follows. In Sec. II, we introduce a new concept of S-stability for general nonlinear impulsive delayed system, and present a criterion for stationary oscillation of the nonlinear impulsive delayed system based on the new concept and fixed point theorem. The criterion is then used to investigate the stationary oscillation for a chaotic neural network with both time delays and impulses. An easily verifiable sufficient condition is obtained for stationary oscillation of the neural network, and the mistakes of the paper[15,16] are pointed out in Sec. III. In Sec. IV, a numerical example is given to illustrate the effectiveness and less conservativeness of the obtained result in Sec. IV. The conclusions are drawn in Sec. V and the possible extensions of the results here are discussed.

II. STATIONARY OSCILLATION

Consider the following general nonlinear impulsive delayed system:

$$\begin{aligned} &\dot{x}(t) = f(t,x_t), \quad t \neq t_k, \quad k = 1,2,\dots, \\ &\Delta x(t) = I_k(x(t_k^-)), \quad t = t_k, \qquad\qquad (1) \\ &x(t) = \phi(t), \quad t \in [-\tau,0], \end{aligned}$$

where $x \in R^n$, $f: R \times R^n \to R^n$ is a piecewise continuous function, and $f(t+\omega,x)=f(t,x)$; $I_k: R^n \to R^n$ are continuous, $x_t(s)=x(t+s)$, $-\tau \le s \le 0$, and there exists a positive integer q such that $t_{k+q}=t_k+\omega$, $I_{k+q}(x)=I_k(x)$ with $t_k \in R$, $t_{k+1}>t_k$, $\lim_{k\to+\infty} t_k=+\infty$, $\Delta x(t_k)=x(t_k^+)-x(t_k^-)=x(t_k^+)-x(t_k)$. For $t_k \neq 0$ $(k=1,2,\dots)$, $[0,\omega]\cap\{t_k\}=\{t_1,t_2,\dots t_q\}$, where $\{t_k\}$ are called the set of jump points.

Let $PC([-\tau,0],R^n)=\{\phi:[-\tau,0]\to R^n \mid \phi$ be piecewise continuous with the first kind of discontinuity at the points $t_k-\tau$, $k\in\{1,2,\dots,q\}$. Moreover, ϕ is left-continuous at each discontinuity point. We can easily prove that $PC([-\tau,0],R^n)$ is a Banach space under $\|\phi\|=\sup_{-\tau\le\theta\le0}\|\phi(\theta)\|$.

Definition: System (1) is said to be S-stable if there exists a non-negative continuous function $\rho(t)$ on R^+, $\lim_{t\to+\infty}\rho(t)\le\rho<1$, such that

$$\|x(t;0,\phi) - x(t;0,\varphi)\| \le \rho(t)\|\phi - \varphi\|,$$

where $x(t;0,\phi)$ and $x(t;0,\varphi)$ are two arbitrary solutions of system (1) through $(0,\phi)$ and $(0,\varphi)$, respectively.

Theorem 1: The nonlinear impulsive delayed system (1) has a stationary oscillation if the system (1) is S-stable.

Proof: For each solution $x(t;0,\phi)$ through $(0,\phi)$ and each $t\ge0$, we can define a function $x_t(0,\phi)$ in this fashion,

$$x_t(0,\phi) = x(t+\theta;0,\phi) \quad \text{for } \theta \in [-\tau,0].$$

On this basis, we can define a mapping $F: PC \to PC$ by $F\phi = x_\omega(0,\phi)$

Let $x(t;0,\phi)$, $x(t;0,\varphi)$ be an arbitrary pair of solutions of Eq. (1). Since impulsive system (1) is S-stable, it is easy to know that F is continuous in R^n. From the definition, there exists $\varepsilon:\varepsilon<1-\rho$, $T\in R^+$ for any $t\ge T$:

$$\rho(t) \le \rho + \varepsilon \triangleq \lambda < 1.$$

Let $m\in N^+$, such that $m\omega>T+\tau$. Then one has

$$\|x(m\omega^+;0,\phi) - x(m\omega^+;0,\varphi)\| \le \lambda\|\phi - \varphi\|$$

which implies $\|F^m\phi - F^m\varphi\| \le \lambda\|\phi-\varphi\|$. Following the fixed-point theorem, F^m possesses a unique fixed point $\phi^* \in PC$. Note that $F^m(F\phi^*)=F(F^m\phi^*)=F\phi^*$, which indicates that $F\phi^* \in PC$ is also a fixed point of F^m. It follows from the uniqueness of the fixed point of F^m that $F\phi^*=\phi^*$, i.e., $x_\omega(0,\phi^*)=\phi^*$. Then

$$x_{t+\omega}(0,\phi^*) = x_t(0,x_\omega(0,\phi^*)) = x_t(0,\phi^*) \quad \text{for all } t \ge 0,$$

which implies

$$\begin{aligned} x(t+\omega;0,\phi^*) &= x_{t+\omega}(0,\phi^*) = x_t(0,\phi^*) \\ &= x(t;0,\phi^*) \quad \text{for all } t \ge 0. \end{aligned}$$

Hence, $x(t;0,\phi^*)$ is ω-periodic.

For any $t>m\omega$, note $t=km\omega+r$, $k\in N^+$, $r\in[0,m\omega)$. For any initial function $\phi\neq\phi^*$, we have

$$\begin{aligned}\|x(km\omega+r;0,\phi)-x(km\omega+r;0,\phi^*)\| &= \|x(m\omega+r;0,x((k-1)m\omega^+;0,\phi))-x(m\omega+r;0,x((k-1)m\omega^+;0,\phi^*))\| \\ &\le \lambda\|x((k-1)m\omega^+;0,\phi)-x((k-1)m\omega^+;0,\phi^*)\| \\ &= \lambda\|x(m\omega^+;0,x((k-2)m\omega^+;0,\phi))-x(m\omega^+;0,x((k-2)m\omega^+;0,\phi^*))\| \\ &\le \lambda^2\|x((k-2)m\omega^+;0,\phi)-x((k-2)m\omega^+;0,\phi^*)\| \le \cdots \\ &\le \lambda^k\|x(0^+;0,\phi)-x(0^+;0,\phi^*)\| = \lambda^k\|\phi-\phi^*\|.\end{aligned}$$

Since $\lambda<1$, we have

$$\|x(km\omega+r;0,\phi)-x(km\omega+r;0,\phi^*)\| \to 0(k\to\infty),$$

which implies that the periodic solution $x(t;0,\phi^*)$ is stable. Hence, the nonlinear impulsive delayed system (1) has a stationary oscillation. Thus the proof is completed.

III. APPLICATION

In this section, we study the stationary oscillations of a Hopfield neural network with both time delays and impulses through proof that the system is S-stable by using the Lyapunov method. This can be seen as an application of the criterion of stationary oscillations for an impulsive delayed system developed in the preceding section. In Refs. 10–16, the authors investigated periodic solutions of some delayed neural networks with linear impulses, where some criteria for stationary oscillation of these neural networks were proposed. It is worth pointing out that the method proposed here is different from those of Refs. 10–16, and can be easily verified.

The Hopfield neural networks subjected to certain impulsive state displacements at fixed moments of time are as follows:

$$\begin{aligned}\frac{dx_i(t)}{dt} &= -a_i(t)x_i(t)+\sum_{j=1}^{m} b_{ij}(t)f_j(x_j(t)) \\ &\quad +\sum_{j=1}^{m} c_{ij}(t)f_j(x_j(t-\tau_{ij}(t)))+I_i(t), \quad t>0, t\ne t_k,\\ \Delta x_i(t_k) &= x_i(t_k^+)-x_i(t_k^-) = -\gamma_{ik}x_i(t_k), \qquad (2)\\ x_i(t) &= \phi_i(t), \quad t\in[-\tau,0], \quad i=1,2,\ldots,m, \quad k=1,2,\ldots,\end{aligned}$$

where $\phi_i(t)$ is a continuous function, m denotes the number of units in a neural network, $x_i(t)$ corresponds to the state of the ith unit at time t, $f_j(x_j(t))$ denotes the output of the jth unit at time t, $b_{ij}(t)$ denotes the strength of the jth unit on the ith unit at time t, $c_{ij}(t)$ denotes the strength of the jth unit on the ith unit at time $t-\tau_{ij}(t)$, $I_i(t)$ is the external bias on the ith unit at time t, $\tau_{ij}(t)$ denotes the transmission delay along the axon of the jth unit, the delays $0\le\tau_{ij}(t)\le\tau$ are bounded functions. $\Delta x_i(t_k)=x_i(t_k^+)-x_i(t_k^-)=x_i(t_k^+)-x_i(t_k)$ are the impulses at moments t_k and $t_1<t_2<\cdots$ is a strictly increasing sequence such that $\lim_{k\to\infty}t_k=+\infty$.

Throughout this section, we assume that:

(H_1) $a_i(t)>0$, $b_{ij}(t)$, $c_{ij}(t)$, $\tau_{ij}(t)\ge 0$, and $I_i(t)$, $i,j=1,2,\ldots,m$ are all continuous ω-periodic functions.

(H_2) Functions $f_j(u)(j=1,2,\ldots,m)$ satisfy the Lipschitz condition, i.e., there are constants $L_j>0$ such that

$$|f_j(u)-f_j(v)|\le L_j|u-v|, \quad \text{for all } u,v\in R=(-\infty,+\infty).$$

(H_3) There exists a positive integer q such that

$$t_{k+q}=t_k+\omega, \quad \gamma_{i(k+q)}=\gamma_{ik}>0, \quad k=1,2,\ldots, \quad i=1,2,\ldots,m.$$

Now, we introduce a result as follows:

Lemma:[21] Assume that $P(t)$ is a non-negative continuous function on $[t_0-\tau,\beta)$, and satisfies the following inequality:

$$\dot{P}(t)\le -a(t)P(t)+b(t)P_t.$$

Then there exists a positive $\eta>0$, such that $P(t)\le P_{t_0}\exp(-\eta(t-t_0))$ for all $t\in[t_0-\tau,\beta)$ if $a(t)$ or $b(t)$ is bounded, $\eta=\inf_{t\in[t_0,\beta)}\{a(t)-b(t)\}>0$, where $P_t=\sup_{-\tau\le\theta\le 0}P(t+\theta)$.

For the impulsive Hopfield neural networks with time delay [Eq. (2)], we have the following result.

Theorem 2: Under $(H_1)-(H_3)$, the Hopfield neural network with time delay and impulses [Eq. (2)] admits a stationary oscillation if $0<\gamma_{ik}\le 2$, $i=1,2,\ldots,m$, $k=1,2,\ldots$, and

$$\inf_{t>0}\left\{\min_i\Big[a_i(t)-L_i\sum_{j=1}^{m}|b_{ji}(t)|\Big]-\max_i\Big[L_i\sum_{j=1}^{m}|c_{ji}(t)|\Big]\right\}>0.$$

Proof: Suppose that $x(t)=(x_1(t),x_2(t),\ldots,x_m(t))^T$ and $y(t)=(y_1(t),y_2(t),\ldots,y_m(t))^T$ are two arbitrary solutions of system (2) and let $z(t)=x(t)-y(t)$. Then it follows from Eq. (2) that

$$\begin{aligned}\frac{dz_i(t)}{dt} &= -a_i(t)z_i(t)+\sum_{j=1}^{m} b_{ij}(t)g_j(x_j(t),y_j(t)) \\ &\quad +\sum_{j=1}^{m} c_{ij}(t)g_j(x_j(t-\tau_{ij}(t)),y_j(t-\tau_{ij}(t))), \\ &\quad t>0, \quad t\ne t_k,\\ \Delta z_i(t) &= z_i(t_k^+)-z_i(t_k^-) = -\gamma_{ik}z_i(t_k), \qquad (3)\\ z_i(t) &= \phi_i(t)-\varphi_i(t), \quad t\in[-\tau,0], \quad i=1,2,\ldots,m, \\ &\quad k=1,2,\ldots,\end{aligned}$$

where $g_j(x_j(t),y_j(t))=f_j(x_j(t))-f_j(y_j(t))$, $j=1,2,\ldots,m$.

We define a Lyapunov function $V(t)$ by $V(t)=\sum_{i=1}^{m}|z_i(t)|$ for $t\geq 0$. In view of Eq. (3) and condition (H_2), when $t\neq t_k$, we obtain

$$\begin{aligned}\frac{D^+V}{dt} &\leq \sum_{i=1}^{m}\Big[-a_i(t)|z_i(t)|+\sum_{j=1}^{m}(|b_{ij}(t)|L_j|z_j(t)|+|c_{ij}(t)|L_j|z_j(t-\tau_{ij}(t))|)\Big]\\ &=-\sum_{i=1}^{m}a_i(t)|z_i(t)|+\sum_{i=1}^{m}\sum_{j=1}^{m}(|b_{ij}(t)|L_j|z_j(t)|+|c_{ij}(t)|L_j|z_j(t-\tau_{ij}(t))|)\\ &=-\sum_{i=1}^{m}a_i(t)|z_i(t)|+\sum_{i=1}^{m}\sum_{j=1}^{m}(|b_{ji}(t)|L_i|z_i(t)|+|c_{ji}(t)|L_i|z_i(t-\tau_{ji}(t))|)\\ &=-\sum_{i=1}^{m}\Big(a_i(t)-L_i\sum_{j=1}^{m}|b_{ji}(t)|\Big)|z_i(t)|+\sum_{i=1}^{m}\sum_{j=1}^{m}|c_{ji}(t)|L_i|z_i(t-\tau_{ji}(t))|\\ &\leq -\min_i\left\{a_i(t)-L_i\sum_{j=1}^{m}|b_{ji}(t)|\right\}V(t)+\max_i\left\{L_i\sum_{j=1}^{m}|c_{ji}(t)|\right\}V_t. \end{aligned} \tag{4}$$

Also, in view of condition of the theorem, one has

$$V(t_k^+)=\sum_{i=1}^{m}|z_i(t_k^+)|=\sum_{i=1}^{m}|1-\gamma_{ik}||z_i(t_k)|\leq V(t_k). \tag{5}$$

From the Lemma, conditions of Theorem 2, and inequalities (4) and (5), we know that there exists a positive $\eta>0$, such that

$$\sum_{i=1}^{m}|z_i(t)|\leq\sum_{i=1}^{m}|z_i(0)|\exp\{-\eta t\},\quad t>0. \tag{6}$$

Hence, the impulsive system (2) is S-stable. From Theorem 1, the Hopfield neural networks with both time delays and impulses [Eq. (2)] has a stationary oscillation. The proof is completed.

Remark: Reference 15 discussed stationary oscillation of the impulsive delayed Hopfield neural networks. But, in Ref. 15, the inequality (22) is incorrect, thus it leads to the mistake in the proof of Theorem 2, which results in the inaccuracy of the main result of the Ref. 15. In addition, the inequality (17) is also incorrect in Ref. 16, hence the main result of Ref. 16 is false. By using the same method as illustrated in this section, ones can study the stationary oscillations for impulsive delayed neural networks of Refs. 15 and 16, respectively.

IV. AN ILLUSTRATIVE EXAMPLE

Consider the following time-varying impulsive delayed cellular neural networks:

$$\begin{aligned}\frac{dx_1(t)}{dt} &= -(6+\alpha\sin t)x_1(t)+\sin 2tf_1(x_1(t))+\cos 3tf_2(x_2(t))+\sin 3tf_1\left(x_1\left(t-\frac{1+\cos t}{2}\right)\right)+\sin tf_2\left(x_2\left(t-\frac{1+\sin t}{2}\right)\right)\\ &\quad +4\sin t,\quad t\neq 2k\pi,\\ \frac{dx_2(t)}{dt} &= -(7+\alpha\cos t)x_2(t)+\frac{\cos t}{3}f_1(x_1(t))+\frac{\cos 2t}{2}f_2(x_2(t))+\cos tf_1\left(x_1\left(t-\frac{1+\cos t}{2}\right)\right)+\cos 2tf_2\left(x_2\left(t-\frac{1+\sin t}{2}\right)\right)\\ &\quad +2\cos t,\quad t\neq 2k\pi,\\ \Delta x_1(2k\pi) &= x_1(2k\pi^+)-x_1(2k\pi^-)=-0.2x_1(2k\pi),\\ \Delta x_2(2k\pi) &= x_2(2k\pi^+)-x_2(2k\pi^-)=-0.3x_2(2k\pi),\quad k=1,2,\ldots, \end{aligned} \tag{7}$$

where $|\alpha| \leq 2$, the activation function is a piecewise-linear function

$$f_1(x)=f_2(x)=f(x)=\tfrac{1}{2}(|x+1|-|x-1|).$$

According to Theorem 2, the cellular neural networks with both time delays and impulses [Eq. (7)] has a stationary oscillation with 2π period. When $\alpha \neq 0$, the result of the paper[11] cannot be applied for stationary oscillation.

V. CONCLUSION

We have investigated the stationary oscillation for a nonlinear impulsive delayed system, and the result has been applied to chaotic neural networks in this paper. First, a new concept of S-stability for a nonlinear impulsive delayed system was introduced; the relationship of S-stability and stationary oscillation for the nonlinear impulsive delayed system was explored. Then, an easily verifiable sufficient condition was obtained for stationary oscillation of the Hopfield neural networks with both time delays and impulses based on the new criterion. Finally, we indicated that the main results of the paper[15,16] are incorrect. By using the S-stability and Lyapunov method, one can study the stationary oscillations for impulsive delayed neural networks of Refs. 15 and 16, respectively.

The introduced notion S-stability bridges the existence of stationary oscillations for a nonlinear impulsive delayed system with the well-understood Lyapunov methods. Hence, via exploring S-stability could provide us with a new way to understand the stationary oscillations in general nonlinear impulsive delayed systems. Of course, this needs further exploration and is currently under study, and the main issue is to obtain easily verifiable criteria of S-stability for general impulsive delayed systems. With these new criteria, it will become directly applicable to studying the stationary oscillations in other kinds of neural networks with both time delays and impulses, which still remains challenging using current methods.

ACKNOWLEDGMENTS

The authors would like to thank the editor and the reviewers for their constructive comments and suggestions to improve the quality of the paper. This work is supported by Singapore Ministry of Education's ACRF Tier 1 funding (Grant No. R263000407112/133) and the NNSF of China (Grant No. 60874027).

[1] T. A. Burton, *Stability and Periodic Solutions of Ordinary and Functional Differential Equations* (Academic, Orlando, FL, 1985).
[2] J. K. Hale and S. M. V. Lunel, *Introduction to Functional Differential Equations* (Springer, New York, 1993).
[3] J. J. Wei and M. G. Velarde, Chaos **14**, 940 (2004).
[4] D. D. Bainov and P. S. Simeonov, *Impulsive Differential Equations: Periodic Solutions and Applications* (Longman Scientific Technical Group, New York, 1993).
[5] J. J. Nieto, Nonlinear Anal. Theory, Methods Appl. **51**, 1223 (2002).
[6] X. N. Liu and L. S. Chen, Nonlinear Anal. Theory, Methods Appl. **65**, 1843 (2006).
[7] L. Liu and J. T. Sun, Int. J. Bifurcation Chaos Appl. Sci. Eng. **16**, 3109 (2006).
[8] J. T. Sun, Chaos **17**, 043123 (2007).
[9] X. F. Yang, X. F. Liao, D. J. Evans, and Y. Y. Tang, Phys. Lett. A **343**, 108 (2005).
[10] Z. C. Yang and D. Y. Xu, Nonlinear Anal. Theory, Methods Appl. **64**, 130 (2006).
[11] Y. Q. Yang and J. D. Cao, Nonlinear Anal.: Real World Appl. **8**, 362 (2007).
[12] Y. K. Li and Z. W. Xing, Chaos, Solitons Fractals **33**, 1686 (2007).
[13] H. Wang, X. F. Liao, and C. D. Li, Chaos, Solitons Fractals **33**, 1028 (2007).
[14] Z. W. Ping and J. G. Lu, "Global exponential stability of impulsive Cohen-Grossberg neural networks with continuously distributed delays," Chaos, Solitons Fractals (in press).
[15] Z. J. Gui, X. S. Yang, and W. G. Ge, Math. Comput. Simul. **79**, 14 (2008).
[16] J. Zhang and Z. J. Gui, "Periodic solutions of nonautonomous cellular neural networks with impulses and delays," Nonlinear Anal.: Real World Appl. (in press).
[17] P. V. Den Driessche and X. F. Zou, SIAM J. Appl. Math. **58**, 1878 (1998).
[18] J. Meng and X. Y. Wang, Chaos **17**, 023113 (2007).
[19] Y. Y. Hou, T. L. Liao, C. H. Lien, and J. J. Yan, Chaos **17**, 033120 (2007).
[20] H. B. Zhu and B. T. Cui, Chaos **17**, 043122 (2007).
[21] C. H. Hou and J. X. Qian, IEEE Trans. Autom. Control **44**, 334 (1999).