Abstract: One-dimensional continuous piecewise affine (or linear) functions have direct applications to approximation theory and their mathematical structure has been understood quite well. However, their structure in more than one dimension presents some difficulties and their properties (even their definition) have not been presented in a satisfactory manner.

In this talk we shall present a comprehensive study of continuous piecewise linear functions in Euclidean spaces based on the theory of Riesz spaces (vector lattices). In particular, we shall present two (equivalent) definitions of multivariate continuous piecewise affine functions: one topological and the other in terms of lattice properties. The theory we have developed originated from questions regarding the put and call options from the theory of finance.

We shall also present some applications of the theory to the estimation of multivariate continuous piecewise linear regressions. This is accomplished by means of Riesz estimators, i.e., by estimators of the conjunctive Boolean form

\[ \hat{Y} = \bigvee_{j \in J} \bigwedge_{i \in E_j} (r_i^0 + r_i^1 X_1 + r_i^2 X_2 + \cdots + r_i^m X_m), \]

where \((X_1, X_2, \ldots, X_m)\) is some observed vector associated with a random variable \(Y\), \(\{E_j\}_{j \in J}\) is a finite family of finite sets and \(\bigvee\) and \(\bigwedge\) are the vector lattice operations almost sure supremum and almost sure infimum, respectively.