ISIS Technical Report: Event-Triggered Real-Time Scheduling For Stabilization of Passive/Output Feedback Passive Systems

Technical Report of the ISIS Group
at the University of Notre Dame
isis-2010-001
October, 2010

Han Yu and Panos J. Antsaklis
Department of Electrical Engineering
University of Notre Dame
Notre Dame, IN 46556

Interdisciplinary Studies in Intelligent Systems

ISIS Technical Report: Event-Triggered Real-Time Scheduling For Stabilization of Passive/Output Feedback Passive Systems

Han Yu and Panos J. Antsaklis
Department of Electrical Engineering
University of Notre Dame
Notre Dame, IN 46556
e-mail: hyu@nd.edu

Abstract

Event-triggered control has been recently proposed as an alternative to the traditional periodic implementation of control tasks. The possibility of reducing the number of executions while guaranteeing stability makes event-triggered control very appealing in the context of sensor/actuator networks. In this paper, we revisit the event-triggered control from an input-to-output perspective and we propose a simple event-triggered control strategy for stabilization of passive/output feedback passive systems. The triggering condition is derived based on the output information of the control system and an estimate of the lower bound of inter-event times is also provided.

I. Introduction

The majority of feedback control laws nowadays are implemented on digital platforms since microprocessors offer many advantages of running real-time operating systems. This creates the possibility of sharing the computational resources among control and other kinds of applications thus reducing the deployment costs of complex control systems [11]. Since we are dealing with resource-limited microprocessors, it becomes important to assess to what extent we can increase the functionality of these embedded devices through novel real-time scheduling algorithms.

In traditional implementations of the control tasks, one designs the controllers under the assumption of no-delayed actuation update and then determines the maximum admissible interval between two consecutive actuation updates. However, the control strategy obtained based on this approach is conservative in the sense that resource usage(i.e., sampling rate, CPU time) is more frequent than necessary to assure a specified performance level, since stability is guaranteed under sufficiently fast periodic execution of control action. To overcome the drawback of periodic paradigm, several researchers suggested the idea of event-based control. The terminology refers to the triggering mechanism as event-based-sampling[13], to event-driven sampling[14], Lebesgue sampling[6], deadband control[15], level-crossing sampling[16], state-triggered sampling[7] and self-triggered sampling[10] with slight different meaning. However in all cases control signals are kept constant until the violation of a condition on certain signals of the plant triggers the re-computation of the control signals. In event-triggered real-time scheduling algorithms, the control tasks are executed whenever a certain error becomes large when compared with the states' norm of the plant[7] (so the triggering condition is based on the full-state information of the plant). The possibility of reducing the number of re-computations, and thus of transmissions, while guaranteeing desired levels of performance makes event-triggered control very appealing in networked control systems(NCSs). One should be aware that the event-triggered technique reduces resource usage while providing a high degree of robustness, since embedded hardware is used to monitor the state of the plant.

Most of results on event-triggered control are obtained under the assumption that the feedback law provides input-to-state stability(ISS) in the sense of [17] with respect to the measurement errors, see [7]-[11]. While the ISS framework provides insight into the triggering condition by exploring the relationship between the demand of stabilizing the system and the current full-state information, it is still a quite restrictive requirement in general, although some results on designing such control laws are available[18]-[21].

In this paper, we explore alternative ways to obtain the triggering condition. We are particularly interested in the class of passive/output feedback passive systems because it is easy to design stabilization controller for these systems, i.e., we can simply stabilize an output feedback passive system via a properly chosen output feedback gain without resorting to the full-state information if additional detectability condition is satisfied. Also note that passive/output feedback passive systems capture a large class of control systems studied in the literature, see [2]-[4]. Here, we propose a simple event-triggered control strategy for stabilization of passive/output feedback passive systems which applies to both linear and nonlinear systems. We take explicitly into account the actuation update delay of the control action and show that the proposed scheduling policy guarantees that the closed-loop system is asymptotically stable. We also provide a lower bound of the inter-sampling time under the proposed scheduling strategy. Based on the work shown in the current paper, we further propose an event-triggered communication strategy for cooperative control of multi-agent systems, see our companion paper [22]. The rest of this paper is organized as follows: we introduce some background on passive/output feedback passive systems in section II; the problem is stated in section III; our main results are provided in section IV and followed by the examples provided in section V; concluding remarks are given in section VI.

II. Background Material

We first introduce some basic concepts on passive systems and output feedback passive systems.

Consider the following control system, which could be linear or nonlinear:

$$H:\begin{cases}\dot{x}=f(x,u)\\ y=h(x,u)\end{cases} \tag{1}$$

where $x\in X\subset\mathbb{R}^n$, $u\in U\subset\mathbb{R}^m$ and $y\in Y\subset\mathbb{R}^m$ are the state, input and output variables, respectively, and X, U and Y are the state, input and output spaces, respectively. The representation $\phi(t,t_0,x_0,u)$ is used to denote the state at time t reached from the initial state x_0 at t_0.

Definition 1(Supply Rate)[1]: The supply rate $\omega(t)=\omega(u(t),y(t))$ is a real valued function defined on $U\times Y$, such that for any $u(t)\in U$ and $x_0\in X$ and $y(t)=h(\phi(t,t_0,x_0,u))$, $\omega(t)$ satisfies

$$\int_{t_0}^{t_1}|\omega(\tau)|d\tau<\infty. \tag{2}$$

Definition 2(Dissipative System)[1]: System H with supply rate $\omega(t)$ is said to be **dissipative** if there exists a nonnegative real function $V(x):X\to\mathbb{R}^+$, called the storage function, such that, for all $t_1\geq t_0\geq 0$, $x_0\in X$ and $u\in U$,

$$V(x_1)-V(x_0)\leq\int_{t_0}^{t_1}\omega(\tau)d\tau, \tag{3}$$

where $x_1=\phi(t_1,t_0,x_0,u)$ and $\mathbb{R}^+$ is a set of nonnegative real numbers.

Definition 3(Passive System)[1]: System H is said to be **passive** if there exists a storage function $V(x)\geq 0$ such that

$$V(x_1)-V(x_0)\leq\int_{t_0}^{t_1}u(\tau)^Ty(\tau)d\tau, \tag{4}$$

if $V(x)$ is $\mathcal{C}^1$, then we have

$$\dot{V}(x)\leq u(t)^Ty(t),\ \forall t\geq 0. \tag{5}$$

One can see that passive system is a special case of dissipative system with supply rate $\omega(t)=u(t)^Ty(t)$.

Definition 4(Output Feedback Passive System)[2]: System H is said to be **Output Feedback Passive**(OFP) if it is dissipative with respect to the supply rate

$$\omega(u,y)=u^Ty-\rho y^Ty, \tag{6}$$

for some $\rho\in\mathbb{R}$.

Remark 1: Note that if $\rho>0$, then H is strictly output passive, and H is said to have excessive output feedback passivity of ρ, we denote it as OFP(ρ); if $\rho<0$, H is said to lack output feedback passivity, and we denote it as OFP($-|\rho|$). One can verify that a OFP($-|\rho|$) system can be rendered passive by a negative feedback $|\rho|I$. And clearly, if a system is OFP(ρ), then it is also OFP($\rho-\varepsilon$), $\forall\varepsilon>0$. ■

Definition 5[2]: Consider the system H with zero input, that is $\dot{x}=f(x,0)$, $y=h(x,0)$, and let $Z\subset\mathbb{R}^n$ be its largest positively invariant set contained in $\{x\in\mathbb{R}^n|y=h(x,0)=0\}$. We say H is

Zero-State Detectable(ZSD) if $x = 0$ is asymptotically stable conditionally to Z. If $Z = \{0\}$, we say that H is **Zero-State Observable** (ZSO).

III. Problem Statement

We consider the control system given in (1). We first assume H is a passive system, and there exists a nonnegative storage function $V(x) : \mathbb{R}^n \to \mathbb{R}^+$, such that (5) is satisfied. We know that if H is ZSD, then under the feedback control law

$$u(t) = -Ky(t), \tag{7}$$

where $K > 0$ could be a scalar or an $m \times m$ positive definite matrix, the origin of H is asymptotically stable. For the rest of this paper, we assume for simplicity that $K > 0$ is scalar.

In real time, the implementation of the feedback control law (7) on an embedded processor is typically done by sampling the output $y(t)$ at time instants $t_0,\ t_1,\ t_2,\ t_3,\ t_4,\ \dots,$ computing the control action $u(t) = -Ky(t_i)$ and updating the actuator at time instants $t_0 + \Delta_0,\ t_1 + \Delta_1,\ t_2 + \Delta_2,\ t_3 + \Delta_3,\ t_4 + \Delta_4,\ \dots,$ where $\Delta_i \geq 0$, for $i = 0, 1, 2, \dots$ represents the actuation update delay, which includes the time required to read the output from the sensor, compute the control action and update the actuators. This means a sequence of measurements $y(t_0),\ y(t_1),\ y(t_2),\ y(t_3),\ y(t_4),\ \dots,$ corresponds to a sequence of actuation updates $u(t_0+\Delta_0),\ u(t_1+\Delta_1),\ u(t_2+\Delta_2),\ u(t_3+\Delta_3),\ u(t_4+\Delta_4),\ \dots$. Thus between actuator updates, the control action $u(t)$ is held constant according to

$$u(t) = u(t_i + \Delta_i), t \in [t_i + \Delta_i,\ t_{i+1} + \Delta_{i+1}),\ \forall i. \tag{8}$$

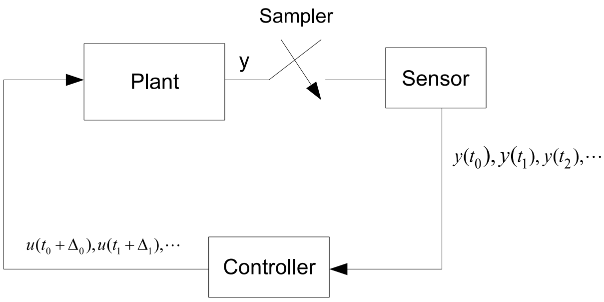

Fig. 1: Implementation of the Feedback Control Action(we assume that the actuator and the controller are collocated with the plant)

If we define the output novelty error at the sensor to be

$$\tilde{e}(t) = y(t) - y(t_i), t \in [t_i, t_{i+1}),\ \forall t \geq 0,\ \forall i, \tag{9}$$

and the output novelty error at the actuator to be

$$e(t) = y(t) - y(t_i), t \in [t_i + \Delta_i, t_{i+1} + \Delta_{i+1}),\ \forall t \geq 0,\ \forall i, \tag{10}$$

then one can see that for $t \in [t_i, t_i + \Delta_i)$, we have $e(t) = y(t) - y(t_{i-1})$ and $\tilde{e}(t) = y(t) - y(t_i)$; for $t \in [t_i + \Delta_i, t_{i+1})$, we have $e(t) = y(t) - y(t_i) = \tilde{e}(t)$; for $t \in [t_{i+1}, t_{i+1} + \Delta_{i+1})$, we have $e(t) = y(t) - y(t_i)$ while $\tilde{e}(t) = y(t) - y(t_{i+1})$.

Let's first consider the case when there is no actuation update delay in the loop, in this case since $\Delta_i = 0, \ \forall i$, we have

$$e(t) = \tilde{e}(t) = y(t) - y(t_i), t \in [t_i, t_{i+1}), \ \forall i. \tag{11}$$

Since the control system H is passive, based on (5),(7),(11), we can obtain

$$\begin{aligned}\dot{V}(x) &\leq u(t)^T y(t) = -K(y(t) - e(t))^T y(t) \\ &= -Ky(t)^T y(t) + Ke(t)^T y(t) \\ &\leq K\|e(t)\|_2 \|y(t)\|_2 - K\|y(t)\|_2^2, \ t \in [t_i, t_{i+1}), \ \forall i.\end{aligned} \tag{12}$$

So if $\|e(t)\|_2 \leq \|y(t)\|_2, \ \forall t \geq 0$, we will have $\dot{V}(x) \leq 0, \ \forall t \geq 0$, and stability of the origin follows from LaSalle's invariance principle[12] and the assumption that system H is ZSD[2].

The above discussion gives us an idea as to when the new sampled output information $y(t_i)$ should be sent to the actuator at time t_i for an output feedback control action update when there is no actuation delay in the loop. If we denote a new sampled information update as an *event*, we can see that for the case when there is no network induced delay in the loop, the "event" time is implicitly defined by the following event triggering condition

$$\|\tilde{e}(t)\|_2 = \|y(t)\|_2. \tag{13}$$

The remaining of this paper addresses the following problems:

- Since the event times are implicitly defined by the triggering condition, can we guarantee that they will not becomes arbitrarily close and result in an accumulation-point?
- In the absence of an accumulation-point, can we get an estimate of the time elapsed between any two consecutive updates of the control action?
- If we consider the actuation update delay in the loop, can we get an estimate of the lower bound of the admissible delay in addition to the estimate of the time elapsed between any two consecutive updates of the control action?
- If the plant is not passive but output feedback passive with some negative constant ρ, what could be done?

IV. Main Results

In this section, we present the main results of this paper which show that under certain conditions between the output and the state of the system, our proposed event-triggering condition will assure stability of the closed-loop system while avoiding zeno-sampling. Our results also consider nontrivial actuation update delays. The main results are stated in Theorem 1 which is followed by remarks to show that the assumptions in Theorem 1 can be relaxed in some cases.

For notation convenience, we let $e(t)$ denote the output novelty error at the actuator, and let $\tilde{e}(t)$ denote the output novelty error at the sensor, as we have mentioned before; let t_i denote the *event time* at which a new sampled output information is obtained by the sensor; let Δ_i denote the actuation update delay for the ith event; let $[t_{i+1} - t_i]$ denote the ith inter-event time; let L denote the Lipschitz constant of function f; let $\|\cdot\|_2$ denote the 2-norm of a vector.

Theorem 1. Consider the control system H given by (1) and assume that H is a passive system with state $x \in \mathbb{R}^m$, control input $u \in \mathbb{R}^m$ and output $y \in \mathbb{R}^m$. Let the following assumptions be satisfied:

1) $f : \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}^m$ is Lipschitz continuous on compacts;
2) $h : \mathbb{R}^m \to \mathbb{R}^m$ is Lipschitz continuous on compacts and it is also a static nonlinear function of x, which belongs to a sector $[\alpha, \beta]$ such that $\alpha x^T x \le x^T h(x) \le \beta x^T x$, where $\alpha \in \mathbb{R}$, $\beta \in \mathbb{R}$ and $0 < \alpha\beta < \infty$;
3) $\|\frac{\partial h(x)}{\partial x}\|_2 \le \gamma$, where $0 < \gamma < \infty$;
4) system H is ZSD.

Let $S \subseteq \mathbb{R}^m$ be any compact set containing the origin. Then for any initial condition is S, there exist $\varepsilon_i > 0$ and $\eta > 0$, such that for $\Delta_i \in [0, \varepsilon_i]$ and with control action $u(t) = -Ky(t_i)(K > 0),\ t \in [t_i + \Delta_i, t_{i+1} + \Delta_{i+1})$, the inter-event time $[t_{i+1} - t_i]$ implicitly decided by the triggering condition

$$\|\tilde{e}(t)\|_2 = \hat{\sigma}\|y(t_i)\|_2,\ \hat{\sigma} \in (0, 0.5),\ \forall i, \forall t \ge 0 \tag{14}$$

is lower bounded by $\eta + \Delta_i$ and the origin of system H is asymptotically stable.

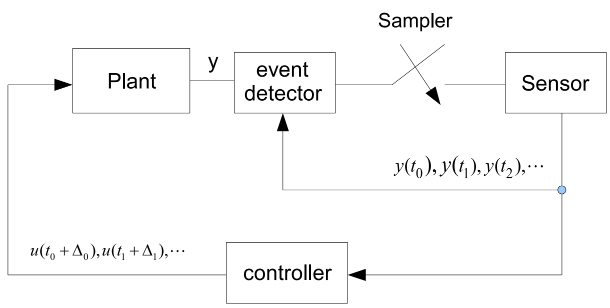

Fig. 2: Implementation of event-triggered real-time scheduling strategy(we assume that the actuator is collocated with the controller)

Proof. The implementation of event-triggered real-time scheduling strategy proposed in Theorem 1 can be illustrated in Fig.2, where we have an “event-detector” located at the plant side to monitor the output of the plant and determine when an “event” should be triggered, this could be done by sampling the output of the plant very fast and we need some sort of buffer to store the last and the latest output information sent to the controller. When there is no actuation update delay in the loop, the triggering condition is simply defined by (13); when there is non-trivial actuation update delay in the loop, the triggering condition is defined by (14).

We will first examine the case when there is no actuation update delay in the loop($\Delta_i = 0, \forall i$). Since the output measurement error $e(t)$ induced by the network at the actuator is defined in (10), based on (12), if we can guarantee that $\|e(t)\|_2 \leq \|y(t)\|_2,\ \forall t \geq 0$, then $\dot{V}(x) \leq 0,\ \forall t \geq 0$, and stability of the origin follows by the assumption that system H is ZSD, as we have discussed before. Consider the triggering condition given by (13), if there is no actuation update delay in the loop($\Delta_i = 0, \forall i$), then as soon as the sensor gets the new sampled information of the output at event time t_i, a new control action $u(t_i) = -Ky(t_i)$ is applied to the plant, and $e(t)$ is reset to zero at $t = t_i$. In this case, since $e(t)$ is reset to zero at each event time, $\|e(t)\|_2 \leq \|y(t)\|_2$ is enforced for $\forall t \geq 0$, and thus we can conclude that the closed-loop system is asymptotically stable. We still need to show that in this case, the inter-event time $[t_{i+1} - t_i]$ is lower bounded by a strictly positive constant.

Let us look at the dynamics of $\frac{\|e(t)\|_2}{\|y(t)\|_2}$ for $t \in [t_i, t_{i+1})$,

$$\begin{aligned} \frac{d\|e\|_2}{dt\|y\|_2} &= \frac{d(e^Te)^{\frac{1}{2}}}{dt(y^Ty)^{\frac{1}{2}}} = \frac{(e^Te)^{-\frac{1}{2}}e^T\dot{e}(y^Ty)^{\frac{1}{2}} - (y^Ty)^{-\frac{1}{2}}y^T\dot{y}(e^Te)^{\frac{1}{2}}}{y^Ty} \\ &= \frac{e^T\dot{e}}{\|e\|_2\|y\|_2} - \frac{y^T\dot{y}}{\|y\|_2\|y\|_2}\frac{\|e\|_2}{\|y\|_2}, \end{aligned} \tag{15}$$

since $e(t) = y(t) - y(t_i)$ and $y(t_i)$ is kept constant for $t \in [t_i, t_{i+1}), \forall i$, we have $\dot{e}(t) = \dot{y}(t)$, and we can further get

$$\begin{aligned} \frac{d}{dt}\frac{\|e\|_2}{\|y\|_2} &\leq \frac{\|e\|_2\|\dot{y}\|_2}{\|e\|_2\|y\|_2} + \frac{\|y\|_2\|\dot{y}\|_2\|e\|_2}{\|y\|_2\|y\|_2\|y\|_2} \\ &= (1 + \frac{\|e\|_2}{\|y\|_2})\frac{\|\dot{y}\|_2}{\|y\|_2}. \end{aligned} \tag{16}$$

From Lipschitz continuity on compacts of $f(x,u)$ and $h(x)$, we can conclude that $f\big(x, -K(y-e)\big)$ is also Lipschitz continuous on compacts, that is there exists a constant L such that

$$\begin{aligned} \|f(x, -K(y-e))\|_2 &\leq L\|(x,(y-e))\|_2 \leq L\|x\|_2 + L\|y-e\|_2 \\ &\leq L\|x\|_2 + L\|y\|_2 + L\|e\|_2, \end{aligned} \tag{17}$$

thus

$$\begin{aligned} \|\dot{y}\|_2 &= \|\frac{\partial h(x)}{x}\dot{x}\|_2 \leq \|\frac{\partial h(x)}{x}\|_2\|\dot{x}\|_2 \\ &\leq \gamma L\big(\|x\|_2 + \|y\|_2 + \|e\|_2\big). \end{aligned} \tag{18}$$

Moreover, since $y = h(x)$ is static nonlinearity belongs to a sector $[\alpha, \beta]$ such that $\alpha x^Tx \leq x^Th(x) \leq \beta x^Tx$, where $0 < \alpha\beta < \infty$, one can show that

$$\|x\|_2 \leq \max\{\frac{1}{|\alpha|}, \frac{1}{|\beta|}\}\|y\|_2 = \zeta\|y\|_2, \tag{19}$$

so we can obtain

$$\begin{aligned}\frac{d}{dt}\frac{\|e\|_2}{\|y\|_2} &\le \big(1+\frac{\|e\|_2}{\|y\|_2}\big)\frac{\|\dot{y}\|_2}{\|y\|_2}\\ &\le \gamma L\big(1+\frac{\|e\|_2}{\|y\|_2}\big)\big(\frac{\|x\|_2}{\|y\|_2}+\frac{\|e\|_2}{\|y\|_2}+1\big)\\ &\le \gamma L\big(1+\frac{\|e\|_2}{\|y\|_2}\big)\big(1+\zeta+\frac{\|e\|_2}{\|y\|_2}\big).\end{aligned} \tag{20}$$

If we denote $\frac{\|e\|_2}{\|y\|_2}$ by p, then (20) can be rewritten as

$$\dot{p} \le \gamma L(1+p)(1+\zeta+p). \tag{21}$$

Consider the differential equation given by

$$\dot{\xi} = \gamma L(1+\xi)(1+\zeta+\xi), \tag{22}$$

and let $\xi(t, t_0, \xi_0)$ be the solution to (22) defined at time t with the initial condition ξ_0. One can see that for any initial condition $p_0 = \xi_0$, we have $p(t, t_0, p_0) \le \xi(t, t_0, \xi_0)$. Since the inter-event time $[t_{i+1} - t_i]$ is bounded by the time interval it takes for p to evolve from 0 to 1, we can get an estimate of the lower bound of the inter-event time based on (22), which is obtained by the solution $\tau \in \mathbb{R}^+$ of $\xi(\tau, t_0, 0) = 1$, and we get

$$\tau = \frac{1}{\gamma L\zeta}\ln\big(\frac{2+2\zeta}{2+\zeta}\big), \tag{23}$$

one can see that $\tau > 0$ for any $\zeta > 0$.

However, when the actuation update delay in the loop is nontrivial ($\Delta_i > 0, \forall i$), the control action is actually updated at time $t_i + \Delta_i, \forall i$, and notice that at $t = t_i + \Delta_i$, $e(t) = y(t_i + \Delta_i) - y(t_i) \ne 0$, and $\|e(t)\|_2$ maybe not less than or equal to $\|y(t)\|_2$ to enforce $\dot{V}(x) \le 0,\ \forall t \ge 0$, so we need to design the triggering condition carefully for this case. Since for $t \in [t_i + \Delta_i, t_{i+1} + \Delta_{i+1})$, we have

$$\|e(t)\|_2 = \|y(t) - y(t_i)\|_2 \ge \|y(t_i)\|_2 - \|y(t)\|_2, \tag{24}$$

so one could find that a sufficient condition for $\|e(t)\|_2 \le \|y(t)\|_2, \forall t \ge 0$ is given by

$$\|e(t)\|_2 \le 0.5\|y(t_i)\|_2, t \in [t_i + \Delta_i, t_{i+1} + \Delta_{i+1}), \forall i. \tag{25}$$

For $t \in [t_i, t_i + \Delta_i)$, we have $e(t) = y(t) - y(t_{i-1})$ and $\tilde{e}(t) = y(t) - y(t_i)$, at $t = t_i + \Delta_i$, we have $e(t) = \tilde{e}(t) = y(t_i + \Delta_i) - y(t_i)$. At $t = t_i + \Delta_i$, we need

$$\|e(t)\|_2 \le \tilde{\sigma}\|y(t_i)\|_2,\ \tilde{\sigma} \in [0, 0.5) \tag{26}$$

to enforce the stabilization condition (25).

Let's examine the dynamics of $\|\tilde{e}(t)\|_2$ during the time $[t_i, t_i+\Delta_i)$, since

$$\begin{aligned}\frac{d}{dt}\|\tilde{e}(t)\|_2 &\le \|\dot{\tilde{e}}(t)\|_2 = \|\dot{y}(t)\|_2 = \|\frac{\partial y}{\partial x}\dot{x}\|_2 \\ &\le \|\frac{\partial y}{\partial x}\|_2\|\dot{x}\|_2 \le \gamma L\big[\|x(t)\|_2 + \|y(t)\|_2 + \|e(t)\|_2\big] \\ &\le \gamma L[(1+\zeta)\|y(t)\|_2 + \|e(t)\|_2] \\ &= \gamma L\big[(1+\zeta)\|\tilde{e}(t)+y(t_i)\|_2 + \|e(t)\|_2\big] \\ &= \gamma L\big[(1+\zeta)\|\tilde{e}(t)+y(t_i)\|_2 + \|\tilde{e}(t)+y(t_i)-y(t_{i-1})\|_2\big] \\ &\le \gamma L\big[(2+\zeta)\|\tilde{e}(t)\|_2 + (1+\zeta)\|y(t_i)\|_2 \\ &\quad + \|y(t_i)-y(t_{i-1})\|_2\big],\end{aligned} \tag{27}$$

so the evolution of $\|\tilde{e}(t)\|_2$ during the time $[t_i, t_i+\Delta_i)$ is bounded by the solution of

$$\dot{\xi}(t) = \gamma L\big[(2+\zeta)\xi(t) + (1+\zeta)\|y(t_i)\|_2 + \|y(t_i)-y(t_{i-1})\|_2\big], \tag{28}$$

with initial condition $\xi(t_0)=0$. We could get an estimate of the time interval for $\|\tilde{e}(t)\|_2$ to evolve from 0 to $\tilde{\sigma}\|y(t_i)\|_2$ based on (28), which is given by

$$\varepsilon_i^- = \frac{1}{(2+\zeta)\gamma L}\ln\Big[\frac{(2+\zeta)\tilde{\sigma}}{(1+\zeta)+\frac{\|y(t_i)-y(t_{i-1})\|_2}{\|y(t_i)\|_2}} + 1\Big]. \tag{29}$$

One should notice that if we choose $\tilde{\sigma}\in[0, 0.5)$, then the stabilizing condition (25) is enforced, which guarantees that $\|e(t)\|_2 \le \|y(t)\|_2$, for $t\in[t_i, t_i+\Delta_i)$, and we can conclude that at $t=t_i$ we have $\|e(t_i)\|_2 = \|y(t_i)-y(t_{i-1})\|_2 \le \|y(t_i)\|_2$, thus we can further obtain

$$\varepsilon_i^- \ge \frac{1}{(2+\zeta)\gamma L}\ln(\tilde{\sigma}+1), \tag{30}$$

and one can see that $\varepsilon_i^- > 0$ for any $\tilde{\sigma}>0$. So if the control action is applied to the plant at the time $t=t_i+\varepsilon_i^-$, then we can guarantee that $\|e(t)\|_2 \le \|y(t)\|_2$ at $t=t_i+\varepsilon_i^-$ with ε_i^- given in (30).

For $t\in[t_i+\varepsilon_i^-, t_{i+1})$, we have $e(t)=\tilde{e}(t)=y(t)-y(t_i)$, and

$$\begin{aligned}\frac{d}{dt}\|e(t)\|_2 &\le \|\dot{e}(t)\|_2 = \|\dot{y}(t)\|_2 = \|\frac{\partial y}{\partial x}\dot{x}\| \\ &\le \|\frac{\partial y}{\partial x}\|_2\|\dot{x}\|_2 \le \gamma L\big[\|x(t)\|_2 + \|y(t)\|_2 + \|e(t)\|_2\big] \\ &\le \gamma L\big[(1+\zeta)\|y\|_2 + \|e\|_2\big] \\ &\le \gamma L\big[(1+\zeta)\|e(t)+y(t_i)\|_2 + \|e\|_2\big] \\ &\le \gamma L\big[(2+\zeta)\|e(t)\|_2 + (1+\zeta)\|y(t_i)\|_2\big],\end{aligned} \tag{31}$$

so the evolution of $\|e(t)\|_2$ during $[t_i+\varepsilon_i^-, t_{i+1}]$ is bounded by the solution of

$$\dot{\xi}(t) = \gamma L\big[(2+\zeta)\xi(t) + (1+\zeta)\|y(t_i)\|_2\big], \tag{32}$$

with $\xi(t_i+\varepsilon_i^-)=\|y(t_i+\varepsilon_i^-)-y(t_i)\|_2=\tilde{\sigma}\|y(t_i)\|_2$. Choose $\hat{\sigma}\in(\tilde{\sigma},0.5)$, then an estimate of the time it takes for $\|e(t)\|_2$ to evolve from $\tilde{\sigma}\|y(t_i)\|_2$ to $\hat{\sigma}\|y(t_i)\|_2$ is given by

$$\eta=\frac{1}{\gamma L(2+\zeta)}\ln\big(\frac{\frac{1+\zeta}{2+\zeta}+\hat{\sigma}}{\frac{1+\zeta}{2+\zeta}+\tilde{\sigma}}\big), \tag{33}$$

and notice that for any $\hat{\sigma}>\tilde{\sigma}>0$, we have $\eta>0$.

Assuming at $t=(t_{i+1}+\Delta_{i+1})^-$, we have $\|e(t)\|_2=0.5\|y(t_i)\|_2$, then we could get an estimate of the time it takes for $\|e(t)\|_2$ to evolve from $\hat{\sigma}\|y(t_i)\|_2$ to $0.5\|y(t_i)\|_2$ based on (32) with initial condition $\xi(t_{i+1})=\hat{\sigma}\|y(t_i)\|_2$, and the estimate of the time interval is given by

$$\varepsilon_i^+=\frac{1}{\gamma L(2+\zeta)}\ln\big(\frac{\frac{1+\zeta}{2+\zeta}+0.5}{\frac{1+\zeta}{2+\zeta}+\hat{\sigma}}\big). \tag{34}$$

If $\Delta_i\in\big[0,\min\{\varepsilon_i^-,\varepsilon_i^+\}\big]$, then we can conclude that $\|e(t)\|_2\le\|y(t)\|_2$ is enforced for $\forall t\ge0$, and the inter-event time $[t_{i+1}-t_i]$ is lower bounded by $\tau=\eta+\Delta_i$. The proof is completed. ■

Remark 2: From the above analysis, one can see that when there is no actuation update delay in the loop, the inter-event time is lower bounded by (23), which is strictly positive; when there is nontrivial actuation update delay in the loop, we use a more tight triggering condition (14); to assure the stability of the closed-loop, the actuation update delay for each event should be properly bounded, and one can see that a lower bound of the delay is related to two event-design parameters: $\tilde{\sigma}$ and $\hat{\sigma}$. A lower bound of the inter-event time $\tau=\eta+\Delta_i$ is also directly related to these two design parameters. ■

Remark 3: For linear systems, consider a linear passive system given by:

$$H:\begin{cases}\dot{x}=Ax+Bu\\ y=Cx\end{cases} \tag{35}$$

then we have

$$\begin{aligned}\|\dot{y}\|_2&=\|C\dot{x}\|_2=\|C(Ax+Bu)\|_2\\ &=\|CAx-CBK(y-e)\|_2=\|CAx-CBKy+CBKe\|_2,\end{aligned} \tag{36}$$

$$\begin{aligned}\Rightarrow\frac{\|\dot{y}\|_2}{\|y\|_2}&=\frac{\|CAx-CBKy+CBKe\|_2}{\|y\|_2}\\ &\le\frac{\|CAx\|_2}{\|y\|_2}+\frac{\|CBKy\|_2}{\|y\|_2}+\frac{\|CBKe\|_2}{\|y\|_2}\\ &\le\frac{\|CAx\|_2}{\|y\|_2}+\frac{\|CBK\|_2\|y\|_2}{\|y\|_2}+\frac{\|CBK\|_2\|e\|_2}{\|y\|_2},\end{aligned} \tag{37}$$

since $\frac{\|CAx\|_2}{\|y\|_2}=\big(\frac{x^TA^TC^TCAx}{x^TC^TCx}\big)^{\frac{1}{2}}$, if $\big(\frac{x^TA^TC^TCAx}{x^TC^TCx}\big)^{\frac{1}{2}}$ is well bounded, such that

$$\big(\frac{x^TA^TC^TCAx}{x^TC^TCx}\big)^{\frac{1}{2}}\le\alpha \tag{38}$$

where $0 < \alpha < \infty$, then we have

$$\begin{aligned}\frac{\|\dot{y}\|_2}{\|y\|_2} &\leq \alpha + \frac{\|CBK\|_2\|y\|_2}{\|y\|_2} + \frac{\|CBK\|_2\|e\|_2}{\|y\|_2} \\ &\leq \|C\|_2\|BK\|_2\big(\frac{\alpha}{\|C\|_2\|BK\|_2} + 1 + \frac{\|e\|_2}{\|y\|_2}\big),\end{aligned} \tag{39}$$

in view of (16), we can get

$$\begin{aligned}\frac{d}{dt}\frac{\|e\|_2}{\|y\|_2} &\leq \big(1 + \frac{\|e\|_2}{\|y\|_2}\big)\frac{\|\dot{y}\|_2}{\|y\|_2} \\ &\leq \|C\|_2\|BK\|_2\big(1 + \frac{\|e\|_2}{\|y\|_2}\big)\big(1 + \frac{\alpha}{\|C\|_2\|BK\|_2} + \frac{\|e\|_2}{\|y\|_2}\big).\end{aligned} \tag{40}$$

So when there is no actuation update delay in the loop, the evolution of $\frac{\|e\|_2}{\|y\|_2}$ is bounded by (40), and one can show that the inter-event time is strictly positive in this case. We can further get a lower bound of the inter-event time when there is nontrivial actuation update delay in the loop by following the similar analysis as shown in the proof of Theorem 1. ■

Remark 4: It can be shown that similar event-triggering approach can be applied to stabilization of output feedback passive systems. Consider an OFP($-|\rho|$) system H with storage function $V(x)$, the following dissipative equality is satisfied:

$$\dot{V}(x) \leq u^T y - \rho y^T y, \quad \rho < 0, \tag{41}$$

notice that ρ is the smallest constant such that (41) is satisfied. In this case, H is non-passive and unstable. By applying negative output feedback $u = -Ky$, where $K > -\rho I > 0$, we can directly stabilize the system H if it is ZSD. And one can show that the stabilization condition in this case is given by:

$$\|e(t)\|_2 \leq \|K^{-1}(K - |\rho|I)\|_2\|y(t)\|_2, \ \forall t \geq 0, \tag{42}$$

and based on this one can derive the triggering condition and the rest of the analysis should be the same as shown in the proof of Theorem 1. ■

Remark 5: One may remark that assumptions 1) and 2) in Theorem 1 are conservative, and by assuming that the output of system H belongs to a bounded sector of the state, we restrict the output to have the same dimension as the state. However, in many cases, those assumptions can be relaxed as long as

$$\frac{\|\dot{y}\|_2}{\|y\|_2} \leq C_1(C_2 + \frac{\|e\|_2}{\|y\|_2}) \tag{43}$$

is satisfied for some constant C_1, C_2, where $0 < C_1 < \infty$ and $0 \leq C_2 < \infty$, and one can check how this works from the examples provided in the next section. ■

V. EXAMPLE

In this section, instead of coming up with examples to verify our main results shown in Theorem 1, we would like to show two examples based on our discussions provided in Remark 3-Remark 5.

Example 1. Consider the linear passive system given by

$$\begin{aligned} \dot{x}_1(t) &= -5x_1(t) - x_2(t) \\ \dot{x}_2(t) &= -x_2(t) + u(t) \\ y(t) &= x_2(t), \end{aligned} \tag{44}$$

we can see that the system is stable and ZSD. With

$$A = \begin{bmatrix} -5 & -1 \\ 0 & -1 \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ C = \begin{bmatrix} 0 & 1 \end{bmatrix}, \tag{45}$$

we can see that $\frac{\|CAx\|_2}{\|y\|_2} = \big(\frac{x^T A^T C^T CAx}{x^T C^T Cx}\big)^{\frac{1}{2}} = 1$. So if we choose K=0.2, based on the discussion shown in Remark 3 and assume that there is no actuation update delay in the loop, we can obtain

$$\begin{aligned} \frac{d}{dt}\frac{\|e(t)\|_2}{\|y(t)\|_2} &\leq \big(1 + \frac{\|e\|_2}{\|y\|_2}\big)\frac{\|\dot{y}\|_2}{\|y\|_2} \\ &\leq \|C\|_2\|BK\|_2\big(1 + \frac{\|e\|_2}{\|y\|_2}\big)\big(1 + \frac{1}{\|C\|_2\|BK\|_2} + \frac{\|e\|_2}{\|y\|_2}\big) \\ &= 0.2\big(1 + \frac{\|e\|_2}{\|y\|_2}\big)\big(6 + \frac{\|e\|_2}{\|y\|_2}\big), \end{aligned} \tag{46}$$

so we could get an estimate of the lower bound of the inter-event time τ, and in this case we have $\tau \geq 0.5390s$. Notice that in this example, the output dose not belong to a bounded sector of the full state, but it belongs to a bounded sector of the observable state x_2, and the unobservable state x_1 is ZSD. The simulation results is shown in Fig.3, where $\sigma(t)$ shows the evolution of $\frac{\|e(t)\|_2}{\|y(t)\|_2}$, $[t^i_{k+1} - t^i_k]$ shows the evolution of the inter-event time, and we can see that whenever the triggering condition is satisfied (when $\sigma(t) = 1$, depicted by the dashed red line), a new event is generated (marked by a dot, with x-axis showing the event time t^i_k and with y-axis showing the time interval from the last event time). The inter-event time is larger than 0.5390s and the system is asymptotically stable.

For the case when there is actuation update delay in the loop, we use the more conservative triggering condition $\|e(t)\|_2 = \hat{\sigma}\|y(t_i)\|_2, \ t \in [t_i + \Delta_i, t_{i+1} + \Delta_{i+1}]$, where we have $\hat{\sigma} \in (0, 0.5)$. The bound on the actuation update delay and the bound on the inter-event time are depended on two event design parameters $\hat{\sigma}$ and $\tilde{\sigma}$, as we have discussed in Remark 2. For this case, if we choose $\tilde{\sigma} = 0.05$ and $\hat{\sigma} = 0.48$, one can show that a tight bound on the actuation update delay is 0.0118s, and the inter-event time is lower bounded by 0.3075s. We get the simulation results shown in Fig.4, where $\sigma_d(t)$ shows the evolution of $\frac{\|e(t)\|_2}{\|y(t_i)\|_2}$. One can see that the actual inter-event time is larger than 0.3075s.

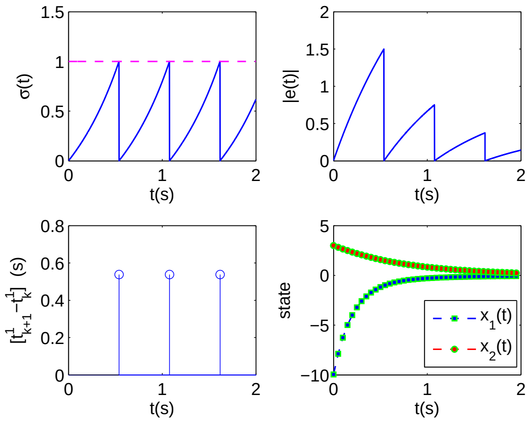

Fig. 3: simulation result of Example 1 with no actuation update delay

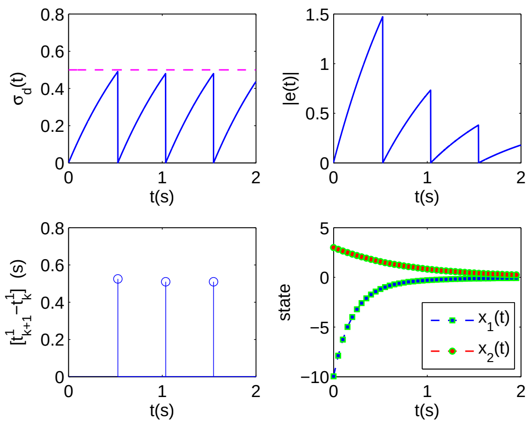

Fig. 4: simulation result of Example 1 with nontrivial actuation update delay

Example 2. Consider the output feedback passive system given by

$$\begin{aligned} \dot{x}_1(t) &= -3x_1^3(t) + x_1(t)x_2(t) \\ \dot{x}_2(t) &= 3x_2(t) + 2u(t) \\ y(t) &= x_2(t), \end{aligned} \tag{47}$$

we can see that the system is ZSD but unstable. If we choose the storage function $V(x) = \frac{1}{4}x_2^2(t)$, we can get

$$\dot{V}(x) = u(t)y(t) + 1.5y^2(t), \tag{48}$$

and in this case $\rho = -1.5$. Since the output $y = h(x)$ is only a linear function of x_2, so y does not belong to a bounded sector of the full state. However, we have

$$\begin{aligned}\frac{\|\dot y\|_2}{\|y\|_2} &= \frac{\|\dot x_2\|_2}{\|y\|_2} = \frac{\|3x_2 - 3K(y-e)\|_2}{\|y\|_2}\\ &\le 3\frac{\|x_2\|_2}{\|y\|_2} + 3K + 3K\frac{\|e\|_2}{\|y\|_2} = 3 + 3K + 3K\frac{\|e\|_2}{\|y\|_2},\end{aligned} \tag{49}$$

so for no actuation update delay case, we can still bound $\frac{\|e\|_2}{\|y\|_2}$ by

$$\begin{aligned}\frac{d}{dt}\frac{\|e\|_2}{\|y\|_2} &\le \big(1+\frac{\|e\|_2}{\|y\|_2}\big)\frac{\|\dot y\|_2}{\|y\|_2}\\ &\le \big(1+\frac{\|e\|_2}{\|y\|_2}\big)\big(3+3K+3K\frac{\|e\|_2}{\|y\|_2}\big)\\ &\le 3K\big(1+\frac{\|e\|_2}{\|y\|_2}\big)\big(1+\frac{1}{K}+\frac{\|e\|_2}{\|y\|_2}\big).\end{aligned} \tag{50}$$

According to Remark 4, we need to choose $K > -\rho > 0$ as the stabilization feedback gain. If we choose $K = 3$, then the triggering condition becomes $\|e(t)\|_2 = K^{-1}(K - |\rho|)\|y(t)\|_2 = 0.5\|y(t)\|_2$, and one can show that the inter-event time is lower bounded by 0.08s. The simulation result is shown in Fig.5. We also did simulation for the nontrivial actuation update delay case, where we choose $\hat\sigma = 0.05$ and $\tilde\sigma = 0.3233$, and the simulation results is shown in Fig.6.

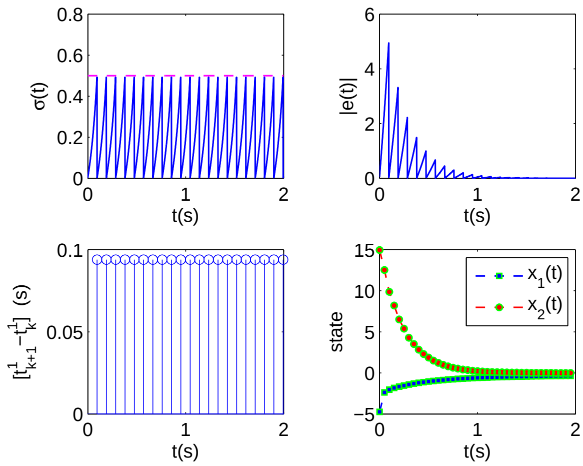

Fig. 5: simulation result of Example 2 with no actuation update delay

VI. Conclusion

In this paper, we proposed a very simple event-triggered control strategy for stabilization of passive/output feedback passive systems. We take explicitly into account the actuation update delay of the control action and show that the proposed scheduling strategy guarantees that the closed-loop system is asymptotically stable. Lower bound of the inter-sampling time under the proposed scheduling strategy is analyzed in detail.

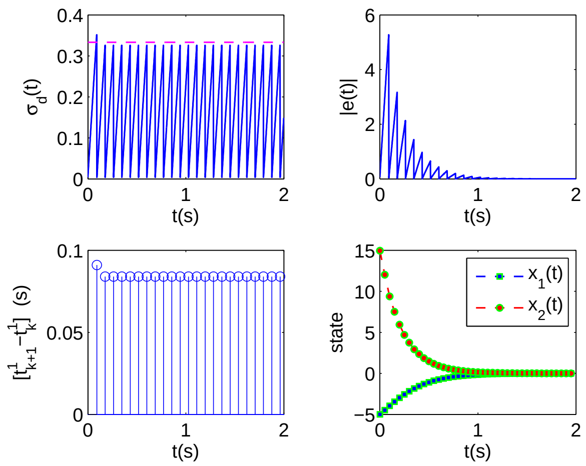

Fig. 6: simulation result of Example 2 with nontrivial actuation update delay

VII. Acknowledgments

The support of the National Science Foundation under Grant No. CCF-0819865 is gratefully acknowledged.

References

[1] J. C. Willems, "Dissipative dynamical systems part I: General theory", *Archive for Rational Mechanics and Analysis*, Springer Berlin, Volume 45, Number 5, Page 321-351, January, 1972.

[2] R. Sepulchre, M. Jankovic and P. Kokotovic, *Constructive Nonlinear Control*, Springer-Verlag, 1997.

[3] R. Ortega, "Passivity-based control of Euler-Lagrange systems: mechanical, electrical, and electromechanical applications", Springer, 1998.

[4] A. J. van der Schaft, "$\mathcal{L}_2$-gain and passivity techniques in nonlinear control", Springer, 2000.

[5] K. E. Årzén, "A simple event based PID controller", *Proceedings of 14th IFAC World Congress*, pp.423-428, vol.18, 1999.

[6] K. J. Aström and B. M. Bernhardsson, "Comparison of Riemann and Lebesgue sampling for first order stochastic systems (I)", *Proceedings of the 41st IEEE Conference on Decision and Control*, vol.2, pp.2011-pp.2016, 2002.

[7] P. Tabuada, "Event-triggered real-time scheduling of stabilizing control tasks", *IEEE Transaction on Automatic Control*, pp.1680-pp.1685, vol.52, no.9, September 2007.

[8] M. Mazo and P. Tabuada, "On event-triggered and self-triggered control over sensor/actuator networks" , in Proceedings of the 47th Conference on Decision and Control, 2008.

[9] X. Wang and M. Lemmon, "Event-triggering in distributed networked systems with data dropouts and delays" , in Hybrid Systems: Computation and Control, 2009.

[10] X. Wang and M. D. Lemmon, "Self-triggered feedback control systems with finite-gain $\mathcal{L}_2$ stability", IEEE Transactions on Automatic Control, pp.452Cpp.467, vol.54, no.3 , March 2009.

[11] A. Anta and P. Tabuada, "To sample or not to sample: Self-triggered control for nonlinear systems", To appear in IEEE Transactions on Automatic Control.

[12] J. P. LaSalle, “Some extensions of Liapunovs second method”, *IRE Transactions on Circuit Theory*, CT-7, pp.520-527, 1960.

[13] K. J. Aström, “Event Based Control”Analysis and Design of Nonlinear Control Systems, Part 3, pp.127-147, 2008.

[14] W. P. M. H. Heemelsa, J. H. Sandeeb, P. P. J. Van Den Boscha, “Analysis of event-driven controllers for linear systems”, International Journal of Control, Volume 81, Issue 4, April 2008, pages 571-590.

[15] P. G. Otanez, J. R. Moyne, D. M. Tilbury, “Using deadbands to reduce communication in networked control systems”, in Proceedings of the American Control Conference, pp.3015-3020, 2002.

[16] E. Kofman, J. H. Braslavsky, “Level Crossing Sampling in Feedback Stabilization under Data-Rate Constraints”, in Proceedings of the IEEE Conference on Decision and Control, pp.4423-4428, 2006.

[17] E. D. Sontag, “Smooth stabilization implies coprime factorization,”IEEE Transactions on Automatic Control, vol.34, no.4, pp.435-443, Apr. 1989.

[18] N. C. S. Fah, “Input-to-state stability with respect to measurement disturbances for one-dimensional systems”, ESAIM J. Control, Optim. Calculus Variations, vol.4, pp.99-122, 1999.

[19] R. A. Freeman, “Global internal stabilizability does not imply global external stabilizability for small sensor disturbances,”IEEE Transactions on Automatic Control, vol.40, no.12, pp. 2119-2122, Dec. 1995.

[20] R. A. Freeman, P. V. Kokotovic “Robust Nonlinear Control Design: State-Space and Lyapunov Techniques”, Boston, MA: Birkhäuser,1996. 1996.

[21] Z. P. Jiang, I. M. Y. Mareels, and D. Hill, “Robust control of uncertain nonlinear systems via measurement feedback,”IEEE Transactions on Automatic Control, vol.44, no.4, pp.807-812, Apr. 1999.

[22] H. Yu, F. Zhu, P. J. Antsaklis, “Event-Triggered Cooperative Control For Multi-Agent Systems Based On Passivity Analysis”, submitted for 2011 American Control Conference.