ISIS Technical Report: Event-Triggered Cooperative Control For Multi-Agent Systems Based On Passivity Analysis

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Abstract

This paper examines event-triggered communication for cooperative control of multi-agent systems based on passivity analysis. We assume each agent is a passive system and propose a distributed event-triggered communication scheme, where each subsystem broadcasts its output information to its neighbors only when the subsystem's local output measurement error exceeds a specified threshold of its output. The triggering condition is related to the degree and the algebraic connectivity of the underlying communication graph. We have shown that when the underlying communication graph is balanced and strongly connected, the outputs of those interconnected passive agents under our proposed event-triggered cooperative control strategy will reach agreement asymptotically, and the time interval between two consecutive communication updates is strictly positive. Examples illustrate the results.

I. Introduction

Important aspects in the implementation of distributed algorithms for control of multi-agent systems are communication transmissions and actuation update schemes. Most of the work in the literature assumes that the execution of the distributed controller and the scheduling of the communication transmission are implemented in a conservative way, where a tight lower bound is selected as the inter-transmission time to guarantee the performance of the interconnected systems for all possible operating points. This leads to inefficient implementation of distributed control algorithms in terms of processor usage or available communication bandwidth.

To overcome this drawback, several researchers have suggested the idea of event-based control for sensor-actuator networks. In a typical event-based implementation, the control signals are kept constant until the violation of a condition on certain signals of the plant triggers the re-computation of the control signals. The possibility of reducing the number of re-computations, and thus of transmissions, while guaranteeing desired levels of performance makes event-based control very appealing in networked control systems(NCSs). A comparison of time-driven and event-driven control for stochastic systems favoring the latter can be found in [4]; a deterministic event-triggered strategy was introduced in [6]; similar results on deterministic self-triggered feedback control have been reported in [7], [8], [9]; a simple event-triggered real-time scheduling approach for stabilization of passive/output feedback passive systems has been proposed in [17]. In those work, a centralized approach to event-design is taken.

A formal analysis of distributed implementations of event-triggered data transmission in NCSs with packet loss and transmission delays is reported in [13]. The results in [13] show how to schedule local data transmission by using event triggering, but they involve local input-to-state stabilization problems for each subsystem which are not easy in general. Similar techniques to reduce data transmission for distributed control systems by using deadband control and local state estimators have been reported in [15],[16]. However, it is not easy to extend their results to general nonlinear NCSs. An event-triggered control strategy for a class of cooperative control algorithms, namely those that can be reduced to a first order agreement problem, has appeared in [10]; note that the framework in [10] assumes that all the subsystems have the same dynamics(first order integrator) which restricts its applications to NCSs.

In the present paper, we propose a simple event-triggered cooperative control strategy for multi-agent systems based on passivity analysis. By "event", we mean a triggering of communication transmission. We assume that each agent is a passive system and we analyze the interconnected systems (where each subsystem could be linear or nonlinear) from an input-to-output perspective. Many studied systems in control of multi-agent systems can be modeled as passive systems, i.e., robotics and mobile vehicles, so our results could be applied to a large class of systems including the first order agreement problem studied in [10]. The triggering condition is local in the sense that each agent only needs to use its own output information to decide when to trigger a communication transmission. We have also shown that the inter-transmission time for each agent is strictly positive and the length of the time interval is related to the degree of disagreement among interconnected neighbors. Although we did not consider transmission delays and packet dropouts in the current paper, our results can be extended to study those problems in a similar way as shown in [6] and [13]. The rest of this paper is organized as follows: we introduce some background in section II; the problem is stated in section III; our main results are provided in section IV and followed by the examples provided in section V; concluding remarks are given in section VI.

II. Background Material

We first introduce some background on passive systems and graph theory which will be used to derive the results presented in the current paper.

A. *Graph Theory*

We consider finite weighted directed graphs $G := (V, E)$ with no self-loops and *adjacency matrix A*, where V denotes the set of all vertices, E denotes the set of all edges, and $A := [a_{ij}]$ with $a_{ij} > 0$ if there is a directed edge from vertex i into vertex j, and $a_{ij} = 0$ otherwise. The *in-degree* and *out-degree* of vertex k are given by $d_i(k) = \sum_j a_{jk}$ and $d_o(k) = \sum_j a_{kj}$ respectively.

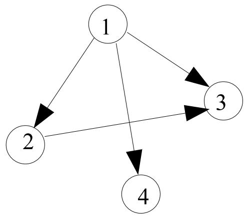

Fig. 1: example on graph Laplacian

The *Laplacian* matrix of a directed graph is defined as $L = D - A$, where D is the diagonal matrix of vertex out-degrees. For example, consider a graph as shown in Fig.1, where we define

$$a_{ij} = \begin{cases} a, & \text{if vertex } i \text{ sends information to vertex } j; \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

and $a \in \mathbb{R}^+$. Then we can get

$$A = \begin{bmatrix} 0 & a & a & a \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 3a & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{2}$$

and the graph Laplacian is given by

$$L = \begin{bmatrix} 3a & -a & -a & -a \\ 0 & a & -a & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{3}$$

Definition 1 (algebraic connectivity)[12]: Let $\mathcal{P}$ be the set $\{x \in \mathbb{R}^n | x \perp 1_n, \|x\| = 1\}$, where $1_n := [1, 1, \dots, 1]^T \in \mathbb{R}^n$. For a directed graph G with Laplacian matrix L, the algebraic connectivity is the real number defined as

$$a(G) = \min_{x \in \mathcal{P}} x^T L x = \min_{x \in \mathcal{P}} \frac{x^T L x}{x^T x}. \tag{4}$$

Definition 2 (strongly connected graph)[11]: A directed graph is strongly connected if for any pair of distinct vertices ν_i and ν_j, there is a directed path from ν_i to ν_j.

Definition 3 (balanced graph)[11]: A vertex is balanced if its in-degree is equal to its out-degree. A directed graph is balanced if every vertex is balanced.

Lemma 1 [11]: For a balanced graph G with nonnegative weights, $a(G) > 0 \Leftrightarrow G$ is strongly connected.

B. Passivity

Consider the following dynamic system description which can be used to describe both a linear and a nonlinear systems:

$$H : \begin{cases} \dot{x} = f(x, u) \\ y = h(x, u) \end{cases} \tag{5}$$

where $x \in X \subset \mathbb{R}^n$, $u \in U \subset \mathbb{R}^m$ and $y \in Y \subset \mathbb{R}^m$ are the state, input and output variables, respectively, and X, U and Y are the state, input and output spaces, respectively. The representation $x(t) = \phi(t, t_0, x_0, u)$ is used to denote the state at time t reached from the initial state x_0 at t_0.

Definition 4 (supply rate)[1]: The supply rate $\omega(t) = \omega(u(t), y(t))$ is a real valued function defined on $U \times Y$, such that for any $u(t) \in U$ and $x_0 \in X$ and $y(t) = h(\phi(t, t_0, x_0, u))$, $\omega(t)$ satisfies

$$\int_{t_0}^{t_1} |\omega(\tau)| d\tau < \infty \tag{6}$$

Definition 5 (Dissipative System)[1]: System H with supply rate $\omega(t)$ is said to be dissipative if there exists a nonnegative real function $V(x) : X \to \mathbb{R}^+$, called the storage function, such that, for all $t_1 \geq t_0 \geq 0$, $x_0 \in X$ and $u \in U$,

$$V(x_1) - V(x_0) \leq \int_{t_0}^{t_1} \omega(\tau) d\tau \tag{7}$$

where $x_1 = \phi(t_1, t_0, x_0, u)$ and $\mathbb{R}^+$ is a set of nonnegative real numbers.

Definition 6 (Passive System)[1]: System H is said to be **passive** if there exists a storage function $V(x) \geq 0$ such that

$$V(x_1) - V(x_0) \leq \int_{t_0}^{t_1} u(\tau)^T y(\tau) d\tau, \tag{8}$$

if $V(x)$ is $\mathcal{C}^1$, then we have

$$\dot{V}(x) \leq u(t)^T y(t), \ \forall t \geq 0. \tag{9}$$

One can see that passive system is a special case of dissipative system with supply rate $\omega(t) = u(t)^T y(t)$.

III. Problem Formulation

A. System Model

The system considered consists of N agents, and we assume that each of them is a passive system satisfying the following passivity inequality:

$$\dot{V}_i(x_i) \leq u_i^T(t) y_i(t) \quad \forall t \geq 0 \tag{10}$$

where $x_i \in \mathbb{R}^n$ denotes the state of agent i, $V_i : \mathbb{R}^n \to \mathbb{R}^+$ denotes the storage function for agent i, $u_i \in \mathbb{R}^m$ and $y_i \in \mathbb{R}^m$ denote the input and output of agent i respectively.

B. Problem Statement

Consider the cooperative control laws given by

$$u_i(t) = -\sum_{j \in \mathcal{N}_i} (y_i(t) - y_j(t)), \quad \forall t \geq 0 \tag{11}$$

where $\mathcal{N}_i$ denotes the set of neighbors of agent i (agents that send information to agent i). In this paper, we re-formulate the cooperative control action (11) to take into account event-triggered communication transmission and derive a decentralized event-triggered cooperative control strategy.

In our decentralized event-triggered cooperative control strategy, we assume that there are no communication delays between coupled agents and there is no actuation update delay for each subsystem. A monotone increasing sequence of event times $t_0^i, t_1^i, \ldots, t_k^i$ is defined for agent i based on its event triggering condition. For agent i and $\forall t \geq 0$, we introduce an output novelty error $e_i(t)$ which is given by

$$e_i(t) = y_i(t) - y_i(t_k^i), \tag{12}$$

for $t \in [t_k^i, t_{k+1}^i]$, where $y_i(t)$ is the output of agent i and $y_i(t_k^i)$ is the transmitted output information at the event time t_k^i; whenever the output novelty error exceeds a specified threshold of its output, the agent will broadcast its latest sampled output information $y_i(t_k^i)$ to its neighbors(we assume that each agent is facilitated with embedded hardware to monitor when the triggering condition is satisfied); at the same time, it will also update its control actions based on $y_i(t_k^i)$ and the last received information of its neighbors' outputs $y_j(t_{k'}^j)_s, j \in \mathcal{N}_i$, where $t_{k'}^j$ is the latest event time of agent $i's$ neighbors. Thus the control action in this case is given by

$$u_i(t) = -\sum_{j \in \mathcal{N}_i} \big(y_i(t_k^i) - y_j(t_{k'}^j)\big). \tag{13}$$

The control action is kept constant for $t \in [t_k^i, t_{k+1}^i]$ if the neighboring agents do not send in any new information during this time interval, otherwise, it will be piecewise constant. The triggering condition is decentralized in the sense that each agent requires knowledge of its own output measurement information to verify its triggering condition. Note that although the cooperative control action for agent i is updated both at its own event times $t_0^i, t_1^i, \ldots, t_k^i$ as well as the latest event times of its neighbors $t_0^j, t_1^j, \ldots, t_{k'}^j$, $j \in \mathcal{N}_i$, the triggering of a local communication transmission is only requested when the triggering condition for the agent is satisfied.

IV. Main Results

In this section, we consider a system consisting of N agents, where each of them is a single input single output(SISO) passive system with input $u_i \in \mathbb{R}$, output $y_i \in \mathbb{R}$, and state $x_i \in \mathbb{R}$ (note that this assumption is only for convenience, and the results in this paper can be easily extended to multi-input-multi-output passive systems; moreover, the dimension of the state dose not need to be the same as the input and the output, which will be discussed later). For notation convenience, we let $Y = [y_1, y_2, \ldots, y_N]^T$ denote the output vector and let $e = [e_1, e_2, \ldots, e_N]^T$ denote the output novelty vector; let $|\Sigma|$ denote the number of the elements in Σ; let $\|\cdot\|_2$ denote the 2-norm of a vector; let L denote the Laplacian of the underlying communication graph and let $a(G)$ denote graph algebraic connectivity.

Theorem 1. Consider a group of N agents, where each agent is a SISO passive system that satisfies the passivity inequality (10), where the dynamics of each agent are given by

$$H_i : \begin{cases} \dot{x}_i = f_i(x_i, u_i) \\ y_i = h_i(x_i). \end{cases} \tag{14}$$

We assume that the underlying communication graph is balanced and strongly connected, and there is no transmission delay and actuation update delay for each agent. Then with the control action given in (13), for any initial condition in $\mathbb{R}$, the output of each agent will reach an agreement asymptotically under the triggering condition given by

$$\|e_i(t)\|_2 > \frac{a(G)}{2|\mathcal{N}_i|}\|y_i(t)\|_2, \ \forall i, \ \forall t \geq 0. \tag{15}$$

Moreover, let the following assumptions be satisfied

1) $f_i : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is Lipschitz continuous on compacts;
2) $h_i : \mathbb{R} \to \mathbb{R}$ is is Lipschitz continuous on compacts and it is also a static nonlinear function of x_i which belongs to a sector $[a_i, b_i]$ such that $a_i x_i^2 \leq x_i h_i(x_i) \leq b_i x_i^2$, where $0 < a_i b_i < \infty$;
3) $\|\frac{\partial h_i(x_i)}{\partial x_i}\|_2 \leq \gamma_i$, where $0 < \gamma_i < \infty$;

then the inter-transmission time $[t_{k+1}^i - t_k^i]$ implicitly determined by the triggering condition (15) is lower bounded by some strictly positive time τ_i.

proof: Since the output novelty error for agent i is defined as

$$e_i(t) = y_i(t) - y_i(t_k^i), \forall k, \forall t \geq 0 \tag{16}$$

then with the control action (13) we can get

$$u_i(t) = -\sum_{j\in\mathcal{N}_i}\big[y_i(t) - y_j(t)\big] + \sum_{j\in\mathcal{N}_i}\big[e_i(t) - e_j(t)\big]. \tag{17}$$

Let $V = \sum_{i=1}^{N} V_i$ denotes the storage function for the entire system with V_i being the storage function for each subsystem, we can get for $t \in [t_k^i, t_{k+1}^i], \forall k$

$$\begin{aligned}\sum_{i=1}^{N}\dot{V}_i \le \sum_{i=1}^{N} u_i y_i &= \sum_{i=1}^{N}\Big(\sum_{j\in\mathcal{N}_i}[e_i - e_j] - \sum_{j\in\mathcal{N}_i}[y_i - y_j]\Big)y_i \\ &= \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}[e_i - e_j]y_i - \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}[y_i - y_j]y_i \\ &= Y^T L^T e - Y^T L^T Y,\end{aligned} \tag{18}$$

so

$$\begin{aligned}\dot{V} &\le -a(G)\|Y\|_2^2 + \sum_{i=1}^{N}\sum_{j\in\mathcal{Z}_i} e_i(y_i - y_j) \\ &= -a(G)\|Y\|_2^2 + \sum_{i=1}^{N}\sum_{j\in\mathcal{Z}_i} e_i y_i - \sum_{i=1}^{N}\sum_{j\in\mathcal{Z}_i} e_i y_j \\ &= -a(G)\|Y\|_2^2 + \sum_{i=1}^{N}|\mathcal{Z}_i| e_i y_i - \sum_{i=1}^{N}\sum_{j\in\mathcal{Z}_i} e_i y_j,\end{aligned} \tag{19}$$

where $\mathcal{Z}_i$ denotes the set of agents receiving information from agent i; since the underlying communication graph is balanced, we have $|\mathcal{Z}_i| = |\mathcal{N}_i|$. Moreover, since $|xy| \le \frac{\varphi}{2}x^2 + \frac{1}{2\varphi}y^2, \forall\varphi > 0$, we can obtain

$$\begin{aligned}\dot{V} \le -a(G)\|Y\|_2^2 &+ \sum_{i=1}^{N}\frac{\alpha|\mathcal{N}_i|}{2}e_i^2 + \sum_{i=1}^{N}\frac{|\mathcal{N}_i|}{2\alpha}y_i^2 \\ &+ \sum_{i=1}^{N}\sum_{j\in\mathcal{Z}_i}\frac{\beta}{2}e_i^2 + \sum_{i=1}^{N}\sum_{j\in\mathcal{Z}_i}\frac{1}{2\beta}y_j^2,\end{aligned} \tag{20}$$

thus

$$\begin{aligned}\dot{V} \le -a(G)\|Y\|_2^2 &+ \sum_{i=1}^{N}\frac{\alpha|\mathcal{N}_i|}{2}e_i^2 + \sum_{i=1}^{N}\frac{|\mathcal{N}_i|}{2\alpha}y_i^2 \\ &+ \sum_{i=1}^{N}\frac{\beta}{2}|\mathcal{N}_i|e_i^2 + \sum_{i=1}^{N}\sum_{j\in\mathcal{Z}_i}\frac{1}{2\beta}y_j^2,\ \alpha > 0,\ \beta > 0,\end{aligned} \tag{21}$$

because the underlying communication graph is balanced, we can get

$$\sum_{i=1}^{N}\sum_{j\in\mathcal{Z}_i}\frac{1}{2\beta}y_j^2 = \sum_{i=1}^{N}\frac{|\mathcal{N}_i|}{2\beta}y_i^2, \tag{22}$$

which yeilds

$$\begin{aligned}\dot{V} &\leq -a(G)\|Y\|_2^2 + \frac{\alpha+\beta}{2}\sum_{i=1}^{N}|\mathcal{N}_i|e_i^2 + (\frac{1}{2\alpha}+\frac{1}{2\beta})\sum_{i=1}^{N}|\mathcal{N}_i|y_i^2 \\ &= -a(G)\sum_{i=1}^{N}y_i^2 + \frac{\alpha+\beta}{2}\sum_{i=1}^{N}|\mathcal{N}_i|e_i^2 + (\frac{1}{2\alpha}+\frac{1}{2\beta})\sum_{i=1}^{N}|\mathcal{N}_i|y_i^2 \\ &= -\sum_{i=1}^{N}\Big[a(G) - (\frac{1}{2\alpha}+\frac{1}{2\beta})|\mathcal{N}_i|\Big]y_i^2 + \frac{\alpha+\beta}{2}\sum_{i=1}^{N}|\mathcal{N}_i|e_i^2,\end{aligned} \tag{23}$$

thus a sufficient condition for $\dot{V} \leq 0, \forall t \geq 0$ is given by

$$e_i^2 \leq \frac{a(G) - (\frac{1}{2\alpha}+\frac{1}{2\beta})|\mathcal{N}_i|}{\frac{\alpha+\beta}{2}|\mathcal{N}_i|}y_i^2,\ \forall t \geq 0. \tag{24}$$

Let

$$\sigma_s(\alpha,\beta) = \frac{a(G) - (\frac{1}{2\alpha}+\frac{1}{2\beta})|\mathcal{N}_i|}{\frac{\alpha+\beta}{2}|\mathcal{N}_i|}, \tag{25}$$

one can verify that when $\alpha = \beta > 0$, $\sigma_s(\alpha,\beta)$ will achieve its maximum, so we choose $\alpha = \beta$, and (24) will becomes

$$e_i^2 \leq \frac{a(G) - \frac{1}{\alpha}|\mathcal{N}_i|}{\alpha|\mathcal{N}_i|}y_i^2,\ \forall t \geq 0, \forall i. \tag{26}$$

Let

$$\sigma(\alpha) = \Big(\frac{a(G) - \frac{1}{\alpha}|\mathcal{N}_i|}{\alpha|\mathcal{N}_i|}\Big)^{\frac{1}{2}}, \tag{27}$$

then we can get the maximum of $\sigma(\alpha)$ by taking $\frac{d\sigma(\alpha)}{\alpha} = 0$, which yields

$$\sigma_m = \frac{a(G)}{2|\mathcal{N}_i|}\ , \tag{28}$$

and in this case, (26) becomes

$$\|e_i\|_2 \leq \sigma_m\|y_i\|_2, \forall t \geq 0, \forall i. \tag{29}$$

When we choose

$$\|e_i\|_2 > \sigma_m\|y_i\|_2, \forall t \geq 0, \forall i, \tag{30}$$

as the triggering condition for each agent(note that this is the triggering condition in Theorem 1), then at each event-time which is implicitly defined by (30), agent i will get a new sampled output information and $e_i(t)$ will be reset to zero; at the same time, agent i will broadcast this newly sampled output information to its neighbors; the time of the next communication transmission is determined by when this triggering condition is satisfied again. Moreover, if all the agents trigger their local communication transmissions based on (30), then we have $\dot{V} \leq 0, \forall t \geq 0$. In view of (18) and according to LaSalle's invariance principle [14], we can conclude that $\lim_{t\to\infty}\big(y_i(t) - y_j(t)\big) = 0, \forall i, j$.

Now we need to show that based on assumption 1)-3), the inter-transmission time is nontrivial for any agent. Since $\|e_i(t)\|_2 = \|y_i(t) - y_i(t_k^i)\|_2$ for $t \in [t_k^i, t_{k+1}^i], \forall i$, we have

$$\|e_i(t)\|_2 \geq \|y_i(t_k^i)\|_2 - \|y_i(t)\|_2 \Rightarrow \|y_i(t)\|_2 \geq \|y_i(t_k^i)\|_2 - \|e_i(t)\|_2, \tag{31}$$

so a sufficient condition for the triggering condition (30) to hold is given by

$$\|e_i(t)\|_2 < \frac{\sigma_m}{\sigma_m + 1}\|y_i(t_k^i)\|_2, \quad for \quad t \in [t_k^i, t_{k+1}^i], \forall k. \tag{32}$$

For $t \in [t_k^i, t_{k+1}^i]$, we have

$$\begin{aligned}
\frac{d}{dt}\|e_i(t)\|_2 = \frac{d(e_i(t)^T e_i(t))^{\frac{1}{2}}}{dt} &= \frac{e_i(t)^T \dot{e}_i(t)}{\|e_i(t)\|_2} \\
&\leq \|\dot{e}_i(t)\|_2 = \|\dot{y}_i(t)\|_2 \\
&= \|\frac{\partial h_i(x_i)}{\partial x_i}\dot{x}_i\|_2 \leq \|\frac{\partial h_i(x_i)}{\partial x_i}\|_2\|\dot{x}_i\|_2 \\
&= \gamma_i\|f_i(x_i, -\sum_{j\in\mathcal{N}_i}[y_i(t_k^i) - y_j(t_{k'}^j)])\|_2 \\
&\leq \gamma_i L_i\|x_i\|_2 + \gamma_i L_i\|\sum_{j\in\mathcal{N}_i}[y_i(t_k^i) - y_j(t_{k'}^j)]\|_2,
\end{aligned} \tag{33}$$

where L_i is the Lipschitz constant of $f_i(x_i, u_i)$. Moreover, since $a_i x_i^2 \leq x_i h_i(x_i) \leq b_i x_i^2$, where $0 < a_i b_i < \infty$, one can verify that

$$\frac{\|x_i\|_2}{\|y_i\|_2} \leq \max\{\frac{1}{|a_i|}, \frac{1}{|b_i|}\} = \zeta_i, \tag{34}$$

then

$$\begin{aligned}
\frac{d}{dt}\|e_i(t)\|_2 &\leq \gamma_i L_i \zeta_i\|y_i\|_2 + \gamma_i L_i\|\sum_{j\in\mathcal{N}_i}[y_i(t_k^i) - y_j(t_{k'}^j)]\|_2 \\
&= \gamma_i L_i \zeta_i\|e_i + y_i(t_k^i)\|_2 + \gamma_i L_i\|\sum_{j\in\mathcal{N}_i}[y_i(t_k^i) - y_j(t_{k'}^j)]\|_2,
\end{aligned} \tag{35}$$

we can get

$$\begin{aligned}
\frac{d}{dt}\|e_i(t)\|_2 &\leq \gamma_i L_i \zeta_i\|e_i\|_2 + \gamma_i L_i \zeta_i\|y_i(t_k^i)\|_2 \\
&\quad + \gamma_i L_i\|\sum_{j\in\mathcal{N}_i}[y_i(t_k^i) - y_j(t_{k'}^j)]\|_2.
\end{aligned} \tag{36}$$

So the evolution of $\|e_i(t)\|_2$ for $t \in [t_k^i, t_{k+1}^i]$ is bounded by the solution of

$$\dot{p}_i(t) = \gamma_i L_i \zeta_i p_i(t) + \gamma_i L_i \zeta_i\|y_i(t_k^i)\|_2 + \gamma_i L_i\|\sum_{j\in\mathcal{N}_i}[y_i(t_k^i) - y_j(t_{k'}^j)]\|_2, \tag{37}$$

with $p(t_k^i) = 0$ (since at $t = t_k^i$, we have $e_i(t_k^i) = y_i(t_k^i) - y_i(t_k^i) = 0$), the corresponding solution to (37) during $[t_k^i, t_{k+1}^i]$ is given by

$$p_i(t) = \Lambda\left[e^{\gamma_i L_i \zeta_i (t - t_k^i)} - 1\right], \tag{38}$$

with

$$\Lambda = \frac{\gamma_i L_i \zeta_i \|y_i(t_k^i)\|_2 + \gamma_i L_i \| \sum_{j\in\mathcal{N}_i}[y_i(t_k^i) - y_j(t_{k'}^j)]\|_2}{\gamma_i L_i \zeta_i}. \tag{39}$$

So we can get a lower bound of the time for $\|e_i(t)\|_2$ to evolve from 0 to $\frac{\sigma_m}{1+\sigma_m}\|y_i(t_k^i)\|_2$ based on (38) which is given by

$$\tau_i = \frac{1}{\gamma_i L_i \zeta_i} \ln\big(1 + \frac{1}{\Lambda}\frac{\sigma_m}{1+\sigma_m}\|y_i(t_k^i)\|_2\big). \tag{40}$$

In view of (40), we can see that τ_i is strictly positive for any $\sigma_m > 0$ and $\|y_i(t_k^i)\|_2 \neq 0$. One should notice that $\lim_{t\to\infty} \| \sum_{j\in\mathcal{N}_i}[y_i(t_k^i) - y_j(t_{k'}^j)]\|_2 = 0$ since $\lim_{t\to\infty}\big(y_i(t) - y_j(t)\big) = 0,\ \forall i,j$, and when $y_i(t)$ approaches origin asymptotically $\| \sum_{j\in\mathcal{N}_i}[y_i(t_k^i) - y_j(t_{k'}^j)]\|_2$ goes to zero at the same time, and τ_i will approach to $\frac{1}{\gamma_i L_i \zeta_i}\ln\big(1+\frac{\sigma_m}{1+\sigma_m}\big)$, so we will still get non-trivial inter-transmission time; moreover, if $y_i(t) \equiv 0$ for some $t \geq t_0 \geq 0$, then no event will be generated for $t \in [t_0, \infty)$, because $e_i(t_k^i) = 0$ for any $t_k^i \in [t_0, \infty)$ and the triggering condition (30) is never satisfied. ■

Remark 1: In view of (40), when $\| \sum_{j\in\mathcal{N}_i}[y_i(t_k^i) - y_j(t_{k'}^j)]\|_2$ is large, τ_i will be small, which implies more frequent communication updates between coupled agents are needed when their outputs are far from agreement. ■

Remark 2: In view of the triggering condition (28),(30), we can see that when $a(G)$ is relatively larger and $|\mathcal{N}_i|$ is relatively smaller, we can obtain a larger threshold for the triggering condition. Notice that $|\mathcal{N}_i|$ is the degree of node i of the underlying communication graph. This implies that if the connectivity of the underlying communication network $a(G)$ is fixed, we may reduce the communication frequency needed for reaching agreement among the interconnected agents by reducing the number of communication links for each node(reduce $|\mathcal{N}_i|$); and with fixed number of communication links for each node(fixed $|\mathcal{N}_i|$), we may reduce the transmission load by improving the connectivity of the communication network(have a larger $a(G)$). So the topology of the underlying communication network actually influences the event-triggered data transmission for the control of multi-agent systems. ■

Remark 3: One may remark that assumption 1) and 2) in Theorem 1 are conservative since we restrict the output of each agent $y_i = h_i(x_i)$ to belong to a bounded sector of full-state and we assume that $f_i(x_i, u_i)$ and $h_i(x_i)$ are Lipschitz continuous on compact set . But this condition can be relaxed as long as

$$\frac{d}{dt}\|e_i(t)\|_2 \leq C_1\|e_i(t)\|_2^p + C_2,\ \ t \in [t_k^i, t_{k+1}^i] \tag{41}$$

for some constant $0 < C_1, C_2 < \infty$, $p \geq 0$, and the way to show that the inter-transmission time is strictly positive will be the same; moreover, the dimension of the state does not need to be the same as the dimension of the input and output(i.e., y_i belongs to a bounded sector of subsystem's observable state while the unobservable state is zero-state detectable, and one can check the example provided in the next section to see how it works).■

Remark 4: For linear passive system, consider the subsystem given by:

$$H_i : \begin{cases} \dot{x}_i = A_i x_i + B_i u_i \\ y_i = C_i x_i, \end{cases} \tag{42}$$

we have

$$\begin{aligned} \frac{d}{dt}\|e_i\|_2 &\le \|\dot{e}_i\|_2 = \|\dot{y}_i\|_2 \le \|C_i A_i x_i\|_2 + \|C_i B_i u_i\|_2 \\ &= \|C_i A_i x_i\|_2 + \|C_i B_i \sum_{j\in\mathcal{N}_i} [y_i(t_k^i) - y_j(t_{k'}^j)]\|_2, \end{aligned} \tag{43}$$

since $\|C_i A_i x_i\|_2 = (x_i^T A_i^T C_i^T C_i A_i x_i)^{\frac{1}{2}}$ and $\|y_i\|_2 = \|C_i x_i\|_2 = (x_i^T C_i^T C_i x_i)^{\frac{1}{2}}$, if $(\frac{x_i^T A_i^T C_i^T C_i A_i x_i}{x_i^T C_i^T C_i x_i})^{\frac{1}{2}}$ is well bounded, such that

$$(\frac{x_i^T A_i^T C_i^T C_i A_i x_i}{x_i^T C_i^T C_i x_i})^{\frac{1}{2}} \le \Gamma, \tag{44}$$

where $0 < \Gamma < \infty$, then we have

$$\begin{aligned} \frac{d}{dt}\|e_i\|_2 &\le \Gamma\|y_i\|_2 + \|C_i B_i \sum_{j\in\mathcal{N}_i} [y_i(t_k^i) - y_j(t_{k'}^j)]\|_2 \\ &= \Gamma\|e_i + y_i(t_k^i)\|_2 + \|C_i B_i \sum_{j\in\mathcal{N}_i} [y_i(t_k^i) - y_j(t_{k'}^j)]\|_2 \\ &\le \Gamma\|e_i\|_2 + \Gamma\|y_i(t_k^i)\|_2 + \|C_i B_i \sum_{j\in\mathcal{N}_i} [y_i(t_k^i) - y_j(t_{k'}^j)]\|_2. \end{aligned} \tag{45}$$

So in this case, we have $\frac{d}{dt}\|e_i\|_2 \le C_{i1}\|e_i\|_2 + C_{i2}$(note that this is the case we have mentioned in Remark 3, with $p = 1$), where

$$C_{i1} = \Gamma, \quad C_{i2} = \Gamma\|y_i(t_k^i)\|_2 + \|C_i B_i \sum_{j\in\mathcal{N}_i} [y_i(t_k^i) - y_j(t_{k'}^j)]\|_2. \tag{46}$$

Accordingly, the evolution of $\|e_i(t)\|_2$ during $[t_k^i, t_{k+1}^i]$ is bounded by the solution to

$$\dot{p}_i(t) = C_{i1} p_i(t) + C_{i2} \tag{47}$$

with $p_i(t_k^i) = 0$, and we can obtain the solution to (47) which is given by

$$p_i(t) = \frac{\Gamma\|y_i(t_k^i)\|_2 + \|C_i B_i \sum_{j\in\mathcal{N}_i} [y_i(t_k^i) - y_j(t_{k'}^j)]\|_2}{\Gamma}\big(e^{\Gamma(t-t_k^i)} - 1\big), \tag{48}$$

and in this case, τ_i is given by

$$\tau_i = \frac{1}{\Gamma}\ln\big(1 + \frac{\Gamma\frac{\sigma_m}{1+\sigma_m}\|y_i(t_k^i)\|_2}{\Gamma\|y_i(t_k^i)\|_2 + \|C_i B_i \sum_{j\in\mathcal{N}_i} [y_i(t_k^i) - y_j(t_{k'}^j)]\|_2}\big). \quad ■$$

V. EXAMPLE

In this section, two examples are provided to illustrate our results in this paper. We studied the event-triggered cooperative control for five interconnected agents with two different communication graphs, and compared the communication rate and the performance between them.

Example 1. Consider the system consists of five agents, where the dynamic of each agent is given by

$$H_1 : \begin{cases} \dot{x}_{11}(t) = -x_{11}^3(t) + 0.5x_{11}(t)x_{12}(t) \\ \dot{x}_{12}(t) = -3x_{12}(t) + 2u_1(t) \\ y_1(t) = x_{12}(t); \end{cases} \tag{49}$$

$$H_2 : \begin{cases} \dot{x}_{21}(t) = -2x_{21}^3(t) + 3x_{21}(t)x_{22}(t) \\ \dot{x}_{22}(t) = -3x_{22}(t) + 2u_2(t) \\ y_2(t) = x_{22}(t); \end{cases} \tag{50}$$

$$H_3 : \begin{cases} \dot{x}_{31}(t) = -3x_{31}^3(t) + x_{31}(t)x_{32}(t) \\ \dot{x}_{32}(t) = -3x_{32}(t) + 2u_3(t) \\ y_3(t) = x_{32}(t); \end{cases} \tag{51}$$

$$H_4 : \begin{cases} \dot{x}_{41}(t) = -10x_{41}^3(t) + x_{41}(t)x_{42}(t) \\ \dot{x}_{42}(t) = -3x_{42}(t) + 2u_4(t) \\ y_4(t) = x_{42}(t); \end{cases} \tag{52}$$

$$H_5 : \begin{cases} \dot{x}_{51}(t) = -3x_{51}^3(t) + 10x_{51}(t)x_{52}(t) \\ \dot{x}_{52}(t) = -2x_{52}(t) + 2u_5(t) \\ y_5(t) = x_{52}(t); \end{cases} \tag{53}$$

one could verify that each agent is a passive system. The underlying communication graph is shown in Fig. 2, which is balanced and strongly connected, and the corresponding graph Laplacian is given

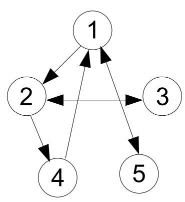

Fig. 2: underlying communication graph

by:

$$L = \begin{bmatrix} 2 & -1 & 0 & 0 & -1 \\ 0 & 2 & -1 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{54}$$

the algebraic connectivity for the graph is $a(G) = 0.6753$. The corresponding triggering condition (15) for each agent is given by

$$\begin{aligned} &\|e_i(t)\|_2 > 0.1688\|y_i(t)\|_2, \forall t \geq 0,\ i = 1, 2 \\ &\|e_i(t)\|_2 > 0.3376\|y_i(t)\|_2, \forall t \geq 0,\ i = 3, 4, 5. \end{aligned} \tag{55}$$

If we denote the evolution of $\frac{\|e_i(t)\|_2}{\|y_i(t)\|_2}$by $\sigma_i(t)$, for $i = 1, 2, 3, 4, 5$, and apply the proposed event-triggered cooperative control strategy as claimed in Theorem 1, we get the simulation results for each agent as shown in Fig.3-Fig.4. The evolution of agent 2 is depicted in Fig.3, where σ_2 shows the evolution of $\frac{\|e_2(t)\|_2}{\|y_2(t)\|_2}$, $[t^2_{k+1} - t^2_k]$ shows the evolution of the inter-transmission time, and we can see that whenever $\sigma_2(t)$ reaches the triggering threshold (depicted by the dashed red line), a communication update is triggered (marked by a dot, with x-axis showing the event time t^2_k and with y-axis showing the time interval from the last communication update); $|e_2|$ shows the evolution of the absolute value of the output novelty error; and y_2 shows the evolution of the output. Fig.4 shows the evolution of outputs of five agents.

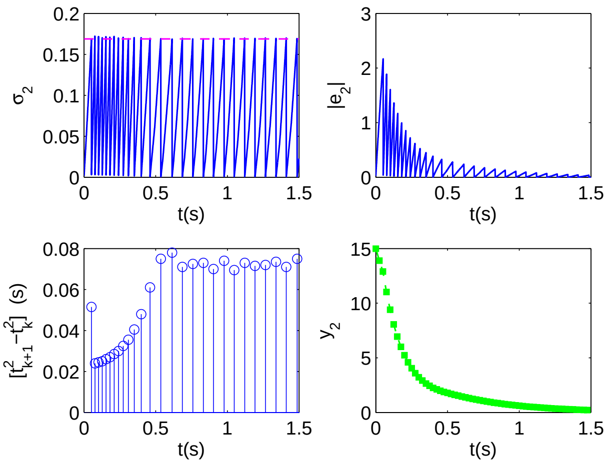

Fig. 3: Agent2

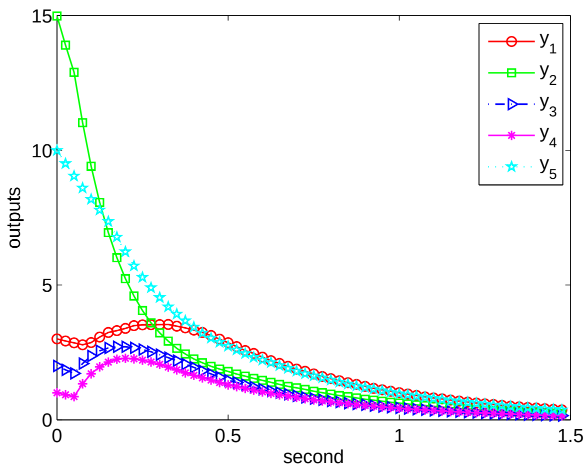

Fig. 4: Outputs of five agents

Example 2. In Remark 2, we have mentioned that the topology of the underlying communication graph will influence the threshold of the triggering condition: in view of (28),(30), if we can enhance the connectivity while reducing the degree of each node of the underlying communication graph, then we can get a larger threshold for the triggering condition. This is illustrated through simulation shown in this example, where we still consider the same interconnected five agents as discussed in Example 1(the initial conditions for each agent is also the same), but now they are interconnected in a different way and the underlying communication graph is shown in Fig.5, with the corresponding

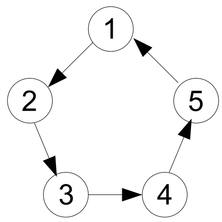

Fig. 5: underlying communication graph

graph Laplacian given by:

$$L = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{56}$$

the algebraic connectivity for the graph is $a(G) = 0.6910$. The corresponding triggering condition (15) for each agent in this case is given by

$$\|e_i(t)\|_2 > 0.3455\|y_i(t)\|_2, \forall t \geq 0, \forall i, \tag{57}$$

and we get the simulation results. We can see that the frequency of communication is reduced significantly for the interconnected agents in this case (check the simulation results comparing the communication rate and performance of agent 1 in Example 1 and Example 2 from Fig.6), Fig.7 shows the evolution of outputs of the five agents.

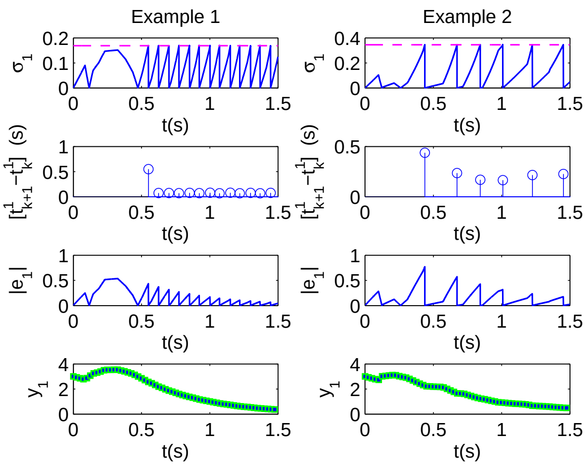

Fig. 6: comparison of agent 1 in Example 1 and Example 2

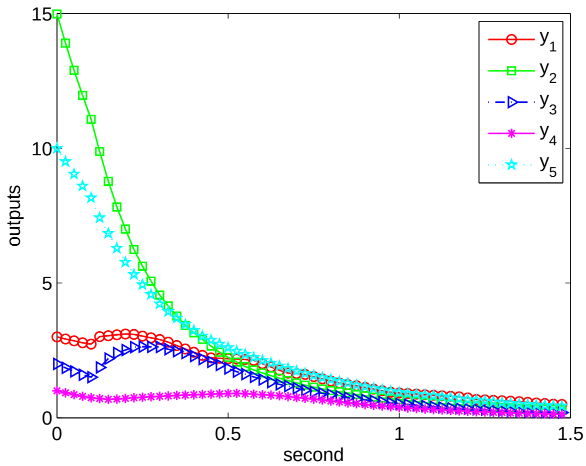

Fig. 7: Outputs of five agents

VI. Conclusion

In the present paper, we propose an event-triggered cooperative control strategy for multi-agent systems based on passivity analysis. We assume that each agent is a passive system, and the triggering condition is local in the sense that each agent only needs to use its own output information to decide when to trigger a communication transmission. At each agent's event time, the agent will broadcast its latest output information to its neighbors, and the neighboring agents will update their control actions accordingly. We also show the inter-transmission time for each agent is strictly positive. Simulations are used to illustrate these results. Future work will take packet loss and transmission delays into consideration.

VII. Acknowledgments

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