Passivity Analysis and Passivation of Event-Triggered Interconnected Systems Using Passivity Indices

Technical Report of the ISIS Group
at the University of Notre Dame
ISIS-2013-009
November, 2013

Feng Zhu, Meng Xia and Panos J. Antsaklis
Department of Electrical Engineering
University of Notre Dame
Notre Dame, IN 46556

Interdisciplinary Studies in Intelligent Systems

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Abstract

We consider the passivity analysis and passivation problems for event-triggered feedback interconnected systems. Based on the location of the event-triggered samplers, we consider two event-triggered control schemes respectively: event-triggered sampler of plant output and event-triggered sampler of controller output. For both schemes, we first derive the conditions to characterize the level of passivity for the interconnected system using passivity indices. The event-triggering condition is proposed to guarantee that these indices can be achieved. Then the passivation problem is considered and the passivation conditions are provided. Considering that the passivation condition depends on the passivity indices of the plant and controller and also the event-triggering condition, the trade off between performance (passivity level) and communication resource utilization is discussed.

I. Introduction

The notion of dissipativity, and its special case of passivity, are characterizations of system input and output behavior based on a generalized notion of energy. The ideas of passivity first emerged from the phenomenon of dissipation of energy across passive components in the circuit theory field [1], [2]. Passive systems can viewed as systems that do not generate energy, but only store or release the energy which was provided. Dissipativity was introduced and formalized in [3], and it is a generalized notion of passivity. Dissipativity and passivity can be applied to the analysis of chemical, mechanical, electromechanical and electrical systems where the definition of energy has both clear physical meaning and concrete mathematical representation. Over the past decades, dissipativity and passivity have received constantly high attention by the systems and control community with plenty of applications in theory and practice [4], [5], [6]. Recent summaries of dissipativity and passivity theory can be found in [7]. The significant benefit of passivity is that when two passive systems are interconnected in parallel or in feedback, the overall system is still passive. Thus passivity is preserved when large-scales systems are combined from components of passive subsystems. Such compositional property is often used in large-scale network design of nonlinear interconnected systems and related topics [8]. The advantage of using this property is that one can always guarantee passivity of the interconnected passive systems and thus stability of the whole system is guaranteed. Recent results [9], [10], [11] also showed its power in compositional design of cyber-physical systems.

Although passivity theory has been applied successfully, this property is vulnerable to discretization, quantization and other factors introduced by digital controllers or communication channels in modern control systems. Results in the literature mainly considered passivity analysis and passivation for an *individual* dynamical system under different network effects. [12] pointed out that passivity is not preserved under discretization and then quantified how much passivity is lost under standard discretization. For quantization effects, passivity analysis and passivation of LTI systems with quantization was treated as an uncertainty described by integral quadratic constraints [13]. Recent work [14] derived the conditions under which the passive structure of an output strictly passive (OSP) nonlinear system can be preserved under quantization. On the other hand, it is also important to study passivity analysis and passivation for *interconnected* systems, considering its advantage in analysis and design of large-scale interconnected systems. As the extension to the well-known compositional property of passivity, [15] considered the passivity analysis and passivation problems for feedback interconnection of two input feed-forward output-feedback (IF-OF) passive systems. [16] considered passivity analysis for discrete-time periodically controlled nonlinear systems, where the system switches between open and closed loop periodically. A maximum allowable transmission ratio (MATR) was found to guarantee passivity for the entire system in each switching period.

Motivated by [17] and [15], in this paper we consider the passivity analysis and passivation problems for event-triggered feedback interconnected systems. Instead of stability [17], we focus on passivity property of the interconnected system. Based on the location of event-triggered sampler implemented, we have two event-triggered control schemes to consider respectively: event-triggered sampler of plant output (Fig. 3) and event-triggered sampler of controller output (Fig. 4). For each control scheme, the condition to characterize the level of passivity for the interconnected system using passivity indices is derived. The event-triggering condition is proposed to guarantee that these indices can be achieved. For the passivation problem, the condition to render the interconnected system passive is given. The condition depends on the passivity indices of the plant and controller and the event-triggering condition. Moreover, we discuss the trade off between performance (passivity level) and resource utilization by choosing appropriate passive controllers and event-triggering conditions. The results presented in this paper are extensions of the corresponding results in [15], by considering, in addition, the effect of event-triggered samplers.

The paper is organized as follows. In Section II, we introduce some background on dissipativity/passivity theory and passivity indices. The passivity analysis and passivation problems are stated in Section III. Section IV considers the two problems for feedback interconnected systems with event-triggered samplers. Based on the location of event-triggered sampler implemented, two event-triggered control schemes are considered, namely event-triggered sampler of plant output and event-triggered sampler of controller output. Two examples are discussed in Section V The conclusion is provided in Section VI.

II. Preliminaries and Background

We first introduce some basic concepts in passive and dissipative system theory. Consider the following nonlinear system G, which is driven by an input $u(t)$ and has an output $y(t)$

$$G : \begin{cases} \dot{x}(t) = f\left(x(t), u(t)\right) \\ y\left(t\right) = h\left(x(t), u(t)\right) \end{cases} \tag{1}$$

where $x(t) \in \mathcal{X} \subset \mathbb{R}^n$, $u(t) \in \mathcal{U} \subset \mathbb{R}^m$ and $y(t) \in \mathcal{Y} \subset \mathbb{R}^p$ are the state, input and output of the system respectively and $\mathcal{X}$, $\mathcal{U}$ and $\mathcal{Y}$ are the state, input and output spaces, respectively.

The definition of a dissipative system is based on a storage function (energy stored in the system) and a supply function (externally supplied energy). The basic idea behind dissipativity is that the increase of the stored energy is bounded by the supplied energy.

Definition 1. [7] System G is said to be dissipative with respect to the supply rate $\omega(x, u, y)$, if there exists a positive semi-definite storage function $V(x)$ such that the (integral) dissipation inequality

$$V(x(t_1)) - V(x(t_0)) \quad \leq \quad \int_{t_0}^{t_1} \omega(x(t), u(t), y(t))dt \tag{2}$$

is satisfied for all t_0, t_1 with $t_0 \leq t_1$ and all solutions $x = x(t), u = u(t), y = y(t)$, $t \in [t_0, t_1]$. If the storage function is differentiable, then the integral dissipation inequality (2) can be rewritten as

$$\dot{V}(x(t)) \leq \omega(x(t), u(t), y(t)), \forall t \tag{3}$$

As a special case of dissipativity, *QSR-dissipativity* was proposed in [18] and developed in [19], [20], [21], [22]. In this case the supply rate is defined as

$$\omega(u, y) = y^T Q y + 2y^T S u + u^T R u \tag{4}$$

where Q, S and R are matrices with proper dimensions. The relation between QSR-dissipativity and $\mathcal{L}_2$ stability has been shown in [18].

Theorem 2. *[18] If System G is QSR-dissipative with $Q < 0$, then it is $\mathcal{L}_2$ stable.*

Definition 3. [4] System G with $m = p$ is passive if there exists a positive semi-definite storage function $V(x)$ such that the following inequality holds for all $t_1, t_2 \in [0, \infty)$ such that

$$V(x(t_2)) - V(x(t_1)) \leq \int_{t_1}^{t_2} u^T y dt \tag{5}$$

If the storage function is smooth, then the integral dissipation inequality (5) can be rewritten as $\dot{V}(x(t)) \leq u^T y$.

Note that passivity is also a special case of dissipativity, with supply rate $\omega = u^T y$. One useful property of passive systems in systems theory is the fact that the parallel interconnection and the negative feedback interconnection of two passive systems is again a passive system. Consider the parallel interconnection (Fig. 1) and negative feedback interconnection (Fig. 2) of two passive systems. The following theorems show that passivity is preserved under parallel and negative feedback interconnections.

Theorem 4. [4] *The parallel interconnection of two passive systems (Fig. 1) is passive, with respect to the input u and the output y.*

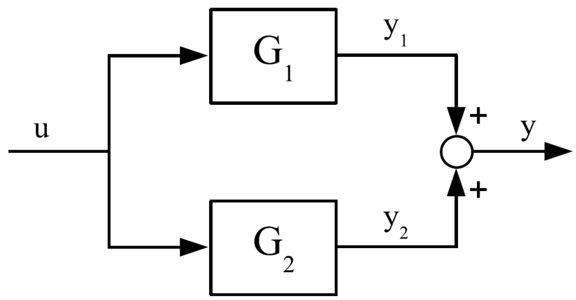

Figure 1. The parallel interconnection of two systems

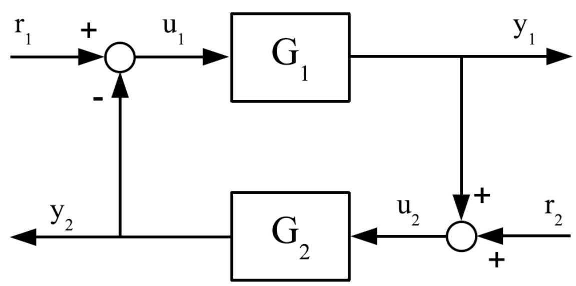

Figure 2. The negative feedback interconnection of two systems

Theorem 5. [4] *The negative feedback interconnection of two passive systems (Fig. 2) is passive, with respect to the input* $\begin{bmatrix} r_1 \\ r_2 \end{bmatrix}$ *and the output* $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$.

The advantage of using this property is that one can always guarantee passivity of the interconnected passive systems and thus stability of the whole system.

In order to measure the excess and shortage of passivity, passivity indices (or passivity levels) [5], [4], [6], [23] are introduced. The indices can be used to render the system passive with feedback and feed-forward, and describe the performance of passive systems.

Definition 6. [5] [6] A system is input feed-forward output feedback passive (IF-OFP) if it is dissipative with respect to the supply rate

$$\omega(u,y) = u^T y - \nu u^T u - \rho y^T y, \, \forall t \geq 0, \tag{6}$$

for some $\rho, \nu \in \mathbb{R}$.

Based on Definition 6, we can denoted an IF-OFP system by IF-OFP(ν, ρ). Definition 6 is often used in passivity analysis, passivation and passivity-based control [23], [24], [17], [14], [25]. It can be seen that when $\rho = \nu = 0$ an IF-OFP system is simply a passive system. one can further have the definitions of input feed-forward (strictly) passive, output feedback (strictly) passive and very strictly passive.

1) When $\rho = 0$ and $\nu \neq 0$, the system is said to be input feed-forward passive (IFP), denoted as IFP(ν). when in addition $\nu > 0$, the system is input feed-forward strictly passive (ISP).
2) When $\rho \neq 0$ and $\nu = 0$, the system is said to be output feedback passive (OFP), denoted as OFP(ρ). When in addition $\rho > 0$, the system is output feedback strictly passive (OSP).
3) When $\rho > 0$ and $\nu > 0$, the system is said to be very strictly passive (VSP).

Note that positive ρ or ν means that the system has an excess of passivity, such as ISP, OSP and VSP. If either ρ or ν is negative, the system has a shortage of passivity and thus is non-passive. When one of indices is zero and the other is non-zero (i.e. IFO and OFP), ρ or ν is called "passivity index", defined as the largest value such that (6) holds for $\forall u$ and $\forall t \geq 0$ [4]. When both of indices are non-zero, the values of ρ and ν may not be unique and are sometimes referred as "passivity levels" [23]. In this paper, we do not distinguish between these two notions as long as there exist ρ and ν such that (6) holds.

The valid domain of ρ and ν has been proposed in [26], [27].

Lemma 7. *[27] The domain of ρ and ν in IF-OFP system is $\Omega = \Omega_1 \cup \Omega_2$ with $\Omega_1 = \left\{\rho, \nu \in \mathbb{R} | \rho\nu < \frac{1}{4}\right\}$ and $\Omega_2 = \left\{\rho, \nu \in \mathbb{R} | \rho\nu = \frac{1}{4}; \rho > 0\right\}$.*

In this paper, we adopt Definition 6 and assume that ρ and ν are in the domain unless otherwise noted.

III. Problem Formulation

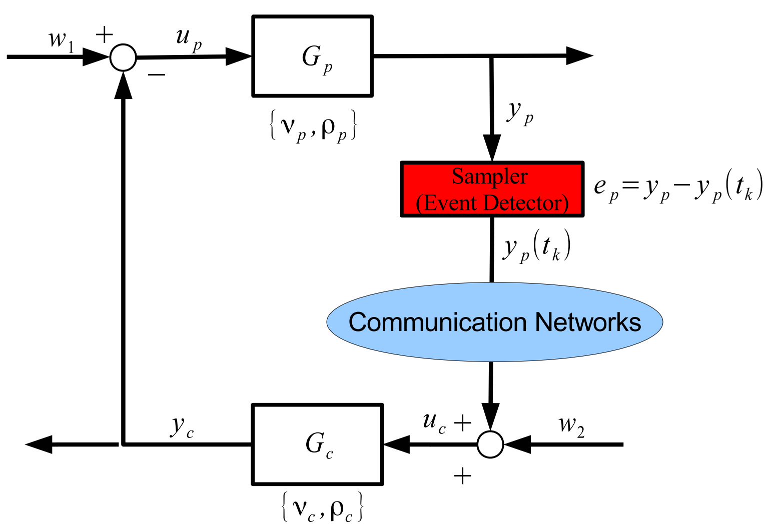

Figure 3. Feedback connection of two IF-OFP systems with event-triggered sampler of plant output

We first consider feedback connection of two systems with an event-triggered sampler of plant output, given in Fig. 3. We assume G_p is IF-OFP(ν_p, ρ_p) and G_c is IF-OFP(ν_c, ρ_c) with known passivity indices. Instead of assuming continuous communication in the feedback loop [15], an event-triggered feedback scheme is introduced.

Event-triggered control has been introduced for the possibility of reducing resources usage (i.e., sampling rate, CPU time, network access frequency) [28], [29], [30], [31], [32], [33], [34], [17]. The triggering mechanisms are referring to the situation in which the control signals are kept constant until the violation of a condition on certain signals triggers the re-computation of the control signals. As in Fig. 3, the new output information of G_p is sent to the controller G_c only when the output novelty error $e_p = y_p - y_p(t_k)$ in the event-triggered sampler satisfies a triggering condition. $y_p(t_k)$ denotes the last output information sent to the controller G_c at the event time t_k. Note that [17] considered the same control scheme but focused on deriving the triggering condition to guarantee *stability* of the closed-loop system. In the present paper, we focus on characterizing *dissipativity/passivity* properties of the closed-loop system, which can be viewed as extensions of the results in [17] and [15]. The main problems investigated in the present paper are summarized as follows.

1) Given the passivity indices of G_c and G_p, how can we determine the passivity indices for the closed-loop systems and accordingly, what is the event-triggering condition to guarantee that these indices can be achieved?
2) For a non-passive plant G_p and a passive controller G_c, what condition on the passivity indices of both systems should be satisfied to render the closed-loop system passive and accordingly, what is the event-triggering condition to guarantee that the condition can be satisfied?

In addition to feedback connection with an event-triggered sampler of plant output, another similar scheme can be considered as in Fig. (4), where the event-triggered sampler is implemented in the output path of the controller G_c. The new output information of G_c is sent to the plant G_p only when the output novelty error $e_c = y_c - y_c(t_k)$ in the event-triggered sampler satisfies a triggering condition. $y_c(t_k)$ denotes the last output information sent to the controller G_c at the event time t_k. Analogously, same questions listed above also need to be considered and answered.

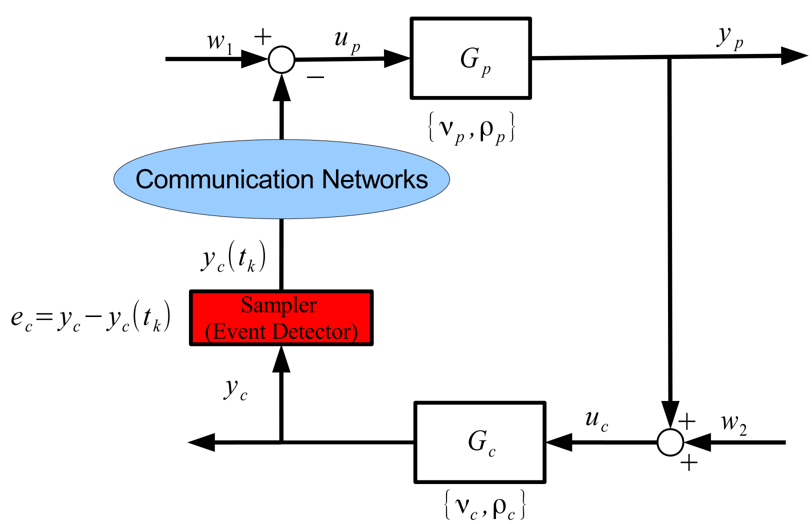

Figure 4. Feedback connection of two IF-OFP systems with event-triggered sampler of controller output

IV. MAIN RESULTS

In this section, we consider the passivity analysis and passivation problems (two problems proposed in Section III) for event-triggered feedback interconnected systems using passivity indices. Based on the location of event-triggered sampler implemented, we have two event-triggered control schemes to consider respectively: event-triggered sampler of plant output and event-triggered sampler of controller output. For both schemes, we first derive the conditions to characterize the level of passivity for the closed-loop system using passivity indices. Then the passivation problem is considered and the passivation conditions are provided.

A. Passivity Analysis and Passivation for Event-Trigerred Sampler of Plant Output

We first consider the passivity analysis problem for the feedback system with an event-triggered sampler of the plant output (Fig. 3). Lemma 8 relates the interconnected system to QSR-dissipative systems.

Lemma 8. *Consider the feedback interconnection of two IF-OF systems with the passivity indices ν_p, ρ_p and ν_c, ρ_c respectively (Fig. 3). If the event time t_k is explicitly determined by the following triggering condition*

$$\|e_p(t)\|_2 = \frac{\beta_p}{\sqrt{\nu_c^2 + m_p\beta_p} + |\nu_c|} \|y_p(t)\|_2 \tag{7}$$

where $m_p = \frac{1}{4\alpha_p} + |\nu_c| - \nu_c$, $\alpha_p > 0$ and $\beta_p > 0$, the interconnection system is QSR-dissipative (with respect to the input $w(t) = \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix}$ and output $y(t) = \begin{bmatrix} y_p(t) \\ y_c(t) \end{bmatrix}$), which satisfies the inequality

$$\dot{V}(t) \leq y(t)^T Q y(t) + 2w(t)^T S y(t) + w(t)^T R w(t) \tag{8}$$

where

$$Q = \begin{bmatrix} -(\rho_p + \nu_c - \beta_p) I & 0I \\ 0I & -(\nu_p + \rho_c - \alpha_p) I \end{bmatrix},$$

$$S = \begin{bmatrix} \frac{1}{2}I & \nu_p I \\ -\nu_c I & \frac{1}{2}I \end{bmatrix},$$

and

$$R = \begin{bmatrix} -\nu_p I & 0I \\ 0I & -(\nu_c - |\nu_c|) I \end{bmatrix}.$$

.

Proof: Since G_p and G_c are IF-OF systems with the passivity indices ν_p, ρ_p, ν_c and ρ_c, there exist $V_p(t)$ and $V_c(t)$ such that

$$\dot{V}_p(t) \leq u_p^T(t) y_p(t) - \nu_p u_p^T(t) u_p(t) - \rho_p y_p^T(t) y_p(t)$$

and

$$\dot{V}_c(t) \leq u_c^T(t) y_c(t) - \nu_c u_c^T(t) u_c(t) - \rho_c y_c^T(t) y_c(t).$$

Consider a storage function for the interconnected system given by $V(t) = V_p(t) + V_c(t)$, we have

$$\dot{V}(t) = \dot{V}_p(t)+\dot{V}_c(t) \leq u_p^T(t)y_p(t)-\nu_p u_p^T(t)u(t)_p-\rho_p y_p^T(t)y_p(t)+u_c^T(t)y_c(t)-\nu_c u_c^T(t)u_c(t)-\rho_c y_c^T(t)y_c(t). \quad (9)$$

Consider that $u_p(t) = w_1(t) - y_c(t)$, $u_c(t) = y_p(t_k) + w_2(t)$ and $y_p(t_k) = y_p(t) - e_p(t)$. For any $t \in [t_k, t_{k+1})$, (9) can be rewritten as

$$\begin{aligned}
\dot{V}(t) \quad &\leq \quad \left(w_1^T(t) - y_c(t)\right) y_p(t) - \rho_p y_p^T(t)y_p(t) - \nu_p \left(w_1(t) - y_c(t)\right)^T \left(w_1(t) - y_c(t)\right) + \\
&\quad\quad \left(w_2(t) + y_p(t_k)\right)^T y_c - \rho_c y_c^T(t)y_c(t) - \nu_c \left(w_2(t) + y_p(t_k)\right)^T \left(w_2(t) + y_p(t_k)\right) \\
&= \quad w_1^T(t)y_p(t) + w_2^T(t)y_c(t) + 2\nu_p w_1^T(t)y_c(t) - 2\nu_c w_2^T(t)y_p(t) - \nu_p w_1^T(t)w_1(t) - \nu_c w_2^T(t)w_2(t) \\
&\quad\quad -(\nu_p + \rho_c)y_c^T(t)y_c(t) - (\rho_p + \nu_c)y_p^T(t)y_p(t) \\
&\quad\quad +2\nu_c w_2^T(t)e_p(t) + 2\nu_c y_p^T(t)e_p(t) - \nu_c e_p^T(t)e_p(t) - y_c^T(t)e_p(t).
\end{aligned}$$

Since $2\nu_c w_2^T(t)e_p(t) \leq |\nu_c|\, w_2^T(t)w_2(t) + |\nu_c|\, e_p^T(t)e_p(t)$, we can obtain that

$$\begin{aligned}
\dot{V}(t) \quad \leq \quad & \begin{bmatrix} w_1^T(t) & w_2^T(t) \end{bmatrix} \begin{bmatrix} 1 & 2\nu_p \\ -2\nu_c & 1 \end{bmatrix} \begin{bmatrix} y_p(t) \\ y_c(t) \end{bmatrix} \\
& + \begin{bmatrix} w_1^T(t) & w_2^T(t) \end{bmatrix} \begin{bmatrix} -\nu_p & 0 \\ 0 & -\nu_c + |\nu_c| \end{bmatrix} \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix} \\
& + \begin{bmatrix} y_p^T(t) & y_c^T(t) \end{bmatrix} \begin{bmatrix} -(\rho_p + \nu_c) & 0 \\ 0 & -(\nu_p + \rho_c) \end{bmatrix} \begin{bmatrix} y_p(t) \\ y_c(t) \end{bmatrix} \\
& +2\nu_c y_p^T(t)e_p(t) + (|\nu_c| - \nu_c)e_p^T(t)e_p(t) - y_c^T(t)e_p(t).
\end{aligned}$$

With $y_c^T(t)e_p(t) = \left\| \sqrt{\alpha_p}y_c(t) + \frac{1}{2\sqrt{\alpha_p}}e_p(t) \right\|_2^2 - \alpha_p y_c^T(t)y_c(t) - \frac{1}{4\alpha_p}e_p^T(t)e_p(t)$ where $\alpha > 0$, we can further get

$$\begin{aligned}
\dot{V}(t) \quad \leq \quad & 2w^T(t)Sy(t) + w^T(t)Rw(t) + y^T(t)Qy(t) - \left\| \sqrt{\alpha_p}y_c(t) + \frac{1}{2\sqrt{\alpha_p}}e_p(t) \right\|_2^2 \\
& +m_p \left\|e_p(t)\right\|_2^2 + 2\nu_c y_p^T(t)e_p(t) + \frac{\nu_c^2}{m_p} \left\|y_p(t)\right\|_2^2 - \left(\frac{\nu_c^2}{m_p} + \beta_p\right) \left\|y_p(t)\right\|_2^2
\end{aligned} \quad (10)$$

where

$$Q = \begin{bmatrix} -\left(\rho_p + \nu_c - \beta_p\right) I & 0I \\ 0I & -\left(\nu_p + \rho_c - \alpha_p\right) I \end{bmatrix},$$

$$S = \begin{bmatrix} \frac{1}{2}I & \nu_p I \\ -\nu_c I & \frac{1}{2}I \end{bmatrix},$$

$$R = \begin{bmatrix} -\nu_p I & 0I \\ 0I & -\left(\nu_c - |\nu_c|\right) I \end{bmatrix},$$

$\beta > 0$ and $m_p = \frac{1}{4\alpha_p} + |\nu_c| - \nu_c$.

Note that $2\nu_c y_p^T(t)e_p(t) \leq 2|\nu_c| \|y_p(t)\|_2 \|e_p(t)\|_2$. Then we can show

$$\begin{aligned}
\dot{V}(t) &\leq 2w^T(t)Sy(t) + w^T(t)Rw(t) + y^T(t)Qy(t) \\
&\quad + \left(\sqrt{m_p}\|e_p(t)\|_2 + \frac{|\nu_c|}{\sqrt{m_p}}\|y_p(t)\|_2\right)^2 - \left(\frac{\nu_c^2}{m_p} + \beta_p\right)\|y_p(t)\|_2^2 \\
&= 2w^T(t)Sy(t) + w^T(t)Rw(t) + y^T(t)Qy(t) \\
&\quad + \left(\sqrt{m_p}\|e_p(t)\|_2 + \frac{|\nu_c|}{\sqrt{m_p}}\|y_p(t)\|_2 + \sqrt{\frac{\nu_c^2}{m_p} + \beta_p}\,\|y_p(t)\|_2\right) \times \\
&\quad \left(\sqrt{m_p}\|e_p(t)\|_2 + \frac{|\nu_c|}{\sqrt{m_p}}\|y_p(t)\|_2 - \sqrt{\frac{\nu_c^2}{m_p} + \beta_p}\,\|y_p(t)\|_2\right)
\end{aligned}$$

From (7), one can verify that

$$\dot{V}(t) \leq 2w^T(t)Sy(t) + w^T(t)Rw(t) + y^T(t)Qy(t)$$

which completes the proof. ∎

Remark 9. Although Lemma 8 does not explicitly characterize passivity indices for the closed-loop system, it determines an event-triggering condition (7) which guarantees that the closed-loop system is QSR-dissipative. After preserving QSR-dissipativity of the closed-loop system, same proof techniques used in [15] can be applied to further explore passivity properties of the system.

Remark 10. As pointed out in Theorem 2, the closed-loop system (Fig. (3)) is $\mathcal{L}_2$ stable if $Q < 0$. It can be seen that a sufficient condition for $Q < 0$ is $\nu_p + \rho_c > \alpha_p$ and $\nu_c + \rho_p > \beta_p$, which is similar to the condition derived in [35], [17]. Also note that the triggering condition here is different from the condition in [35], [17].

Remark 11. (7) shows that the parameters α_p, β_p and ν_c determine the behavior of the trigger. It can be seen that larger α_p and β_p result in a larger triggering threshold. A large triggering threshold implies less sampling rate and thus less resources usage. Later we will show how these parameters affect passivity of the system.

Next, Theorem 12 shows how to determine the passivity indices for the feedback system with event-triggered sampler of plant output.

Theorem 12. *Consider the feedback interconnected system in Fig. 3. Suppose the passivity indices ν_p, ρ_p, ν_c and ρ_c are known and the triggering condition is determined by* (7)*. If we choose ϵ and δ such that*

$$\begin{cases} \epsilon < \min\{\nu_p, \nu_c - |\nu_c|\} \\ \delta \leq \min\left\{\rho_c - \alpha_p - \frac{\epsilon\nu_p}{\nu_p - \epsilon}, \rho_p - \beta_p - \frac{(|\nu_c| + \epsilon)\nu_c}{\nu_c - |\nu_c| - \epsilon}\right\} \end{cases}, \tag{11}$$

the interconnected system has passivity indices ϵ and δ satisfying

$$\dot{V} \leq w^T(t)y(t) - \epsilon w^T(t)w(t) - \delta y^T(t)y(t) \tag{12}$$

where $w(t) = \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix}$ *and* $y = \begin{bmatrix} y_p(t) \\ y_c(t) \end{bmatrix}$.

Proof: From (8) , we have

$$\dot{V}(t) \leq w^T(t)y(t) - \begin{bmatrix} w_1^T(t) & y_c^T(t) \end{bmatrix} \begin{bmatrix} \nu_p & -\nu_p \\ -\nu_p & \nu_p+\rho_c-\alpha_p \end{bmatrix} \begin{bmatrix} w_1(t) \\ y_c(t) \end{bmatrix} - \begin{bmatrix} w_2^T(t) & y_p^T(t) \end{bmatrix} \begin{bmatrix} \nu_c-|\nu_c| & \nu_c \\ \nu_c & \rho_p+\nu_c-\beta_p \end{bmatrix} \begin{bmatrix} w_2(t) \\ y_p(t) \end{bmatrix}. \tag{13}$$

Since ϵ and δ are chosen such that (11) is satisfied, (14) holds for the chosen ϵ and δ.

$$\begin{cases} \epsilon \leq \nu_p \\ \epsilon \leq \nu_c - |\nu_c| \\ (\nu_p-\epsilon)(\nu_p+\rho_c-\alpha_p-\delta) \geq \nu_p^2 \\ (\nu_c-|\nu_c|-\epsilon)(\rho_p+\nu_c-\beta_p-\delta) \geq \nu_c^2 \\ \nu_p+\rho_c-\alpha_p-\delta \geq 0 \\ \rho_p+\nu_c-\beta_p-\delta \geq 0 \end{cases}. \tag{14}$$

(14) further implies that the matrices

$$M = \begin{bmatrix} \nu_p-\epsilon & -\nu_p \\ -\nu_p & \nu_p+\rho_c-\alpha_p-\delta \end{bmatrix}$$

and

$$N = \begin{bmatrix} \nu_c-|\nu_c|-\epsilon & \nu_c \\ \nu_c & \rho_p+\nu_c-\beta_p-\delta \end{bmatrix}$$

are positive semi-definite. Therefore, we have

$$\begin{bmatrix} w_1^T(t) & y_c^T(t) \end{bmatrix} M \begin{bmatrix} w_1(t) \\ y_c(t) \end{bmatrix} + \begin{bmatrix} w_2^T(t) & y_p^T(t) \end{bmatrix} N \begin{bmatrix} w_2(t) \\ y_p(t) \end{bmatrix} \geq 0 \tag{15}$$

for $\forall w_1(t)$, $w_2(t)$, $y_c(t)$ and $y_p(t)$. After re-arranging the terms in (15), one can obtain

$$-\begin{bmatrix} w_1^T(t) & w_2^T(t) \end{bmatrix} E \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix} - \begin{bmatrix} y_p^T(t) & y_c^T(t) \end{bmatrix} \Delta \begin{bmatrix} y_p(t) \\ y_c(t) \end{bmatrix} \geq -\begin{bmatrix} w_1^T(t) & y_c^T(t) \end{bmatrix} O \begin{bmatrix} w_1(t) \\ y_c(t) \end{bmatrix} - \begin{bmatrix} w_2^T(t) & y_p^T(t) \end{bmatrix} P \begin{bmatrix} w_2(t) \\ y_p(t) \end{bmatrix} \tag{16}$$

where $E = \begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon \end{bmatrix}$, $\Delta = \begin{bmatrix} \delta & 0 \\ 0 & \delta \end{bmatrix}$, $O = \begin{bmatrix} \nu_p & -\nu_p \\ -\nu_p & \nu_p+\rho_c-\alpha_p \end{bmatrix}$ and $P = \begin{bmatrix} \nu_c-|\nu_c| & \nu_c \\ \nu_c & \rho_p+\nu_c-\beta_p \end{bmatrix}$.

From (16) and (13), we can finally show that

$$\begin{aligned} \dot{V}(t) \leq{} & w^T(t)y(t) - \begin{bmatrix} w_1^T(t) & y_c^T(t) \end{bmatrix} \begin{bmatrix} \nu_p & -\nu_p \\ -\nu_p & \nu_p+\rho_c-\alpha_p \end{bmatrix} \begin{bmatrix} w_1(t) \\ y_c(t) \end{bmatrix} \\ & - \begin{bmatrix} w_2^T(t) & y_p^T(t) \end{bmatrix} \begin{bmatrix} \nu_c-|\nu_c| & \nu_c \\ \nu_c & \rho_p+\nu_c-\beta_p \end{bmatrix} \begin{bmatrix} w_2(t) \\ y_p(t) \end{bmatrix} \\ \leq{} & w^T(t)y(t) - \epsilon w^T(t)w(t) - \delta y^T(t)y(t) \end{aligned} \tag{17}$$

■

Remark 13. (11) can be used to obtain an estimate of the passivity indices for the closed-loop system, with respect to the input $w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$ and output $y = \begin{bmatrix} y_p \\ y_c \end{bmatrix}$. The condition is similar to its counterpart in [15]. Additionally, (11) quantifies the impact of triggering condition on the passivity indices of the closed-loop system using the parameters α_p and β_p.

Now we introduce the passivation problem for the feedback system with event-triggered sampler of plant output. For this problem, the goal is to passivate a non-passive plant G_p using a passive controller G_c. Here passivity of the interconnected system is defined on the input w_1 and output y_p. We also assume that w_2 is zero. One may observe from Theorem 11 that passivity with respect to the full input and output (i.e. input w and output y) may not be guaranteed to be reinforced under feedback interconnection and event-triggering scheme. However, since we have selected different inputs and outputs, the corresponding passivity may change accordingly. Theorem 14 shows that it is possible to guarantee passivity for the desired input and output although passivity for full input and output may not hold.

Theorem 14. *Assume $w_2 = 0$ and the triggering condition is determined by* (7)*. The interconnected system (Fig.* (3)*) is passive with respect to the input w_1 and output y_p if the passivity indices satisfy the conditions*

$$\nu_p \geq 0 \tag{18}$$

$$\rho_c \geq \alpha_p \tag{19}$$

$$\rho_p + \nu_c \geq \beta_p. \tag{20}$$

Proof: If $w_2(t) = 0$, (9) becomes

$$\begin{aligned} \dot{V}(t) \leq\ & w_1^T(t)y_p(t) - \begin{bmatrix} w_1^T(t) & y_c^T(t) \end{bmatrix} \begin{bmatrix} \nu_p & -\nu_p \\ -\nu_p & \nu_p + \rho_c - \alpha_p \end{bmatrix} \begin{bmatrix} w_1(t) \\ y_c(t) \end{bmatrix} \\ & - (\rho_p + \nu_c - \beta_p)\, y_p^T(t) y_p(t) \end{aligned}$$

Since we have $\nu_p \geq 0$, $\rho_c \geq \alpha_p$ and $\rho_p + \nu_c \geq \beta_p$, it can be shown that

$$\begin{bmatrix} \nu_p & -\nu_p \\ -\nu_p & \nu_p + \rho_c - \alpha_p \end{bmatrix} \geq 0 \tag{21}$$

$$\rho_p + \nu_c \geq \beta_p. \tag{22}$$

Therefore, we can conclude that

$$\begin{aligned} \dot{V}(t) \leq\ & w_1^T(t)y_p(t) - \begin{bmatrix} w_1^T(t) & y_c^T(t) \end{bmatrix} \begin{bmatrix} \nu_p & -\nu_p \\ -\nu_p & \nu_p + \rho_c - \alpha_p \end{bmatrix} \begin{bmatrix} w_1(t) \\ y_c(t) \end{bmatrix} - (\rho_p + \nu_c - \beta_p)\, y_p^T y_p \\ \leq\ & w_1^T(t)y_p(t). \end{aligned} \tag{23}$$

■

Remark 15. When the plant G_p is non-passive (i.e. $\rho_p < 0$), the closed-loop system can be rendered passive by choosing a passive controller G_c with $\rho_c \geq \alpha_p$ and $\nu_c \geq -\rho_p + \beta_p$. Compared with the passivation conditions in [15], the conditions (18)-(20) imply that one need a passive controller with higher passivity indices to passivate a non-passive plant for a triggering condition with fixed α_p and β_p. On the other hand, the conditions also give the upper bounds for α_p and β_p to guarantee closed-loop passivity for a given plant and controller with known passivity indices. The results provide certain flexibility for designers by trade off between passivity level of the controller and resource utilization.

Moreover, we can also obtain an estimate of passivity indices for the passivated system, as shown in Theorem 16.

Corollary 16. *Assume the triggering condition is determined by* (7)*. Suppose that the conditions* (18)-(20) *are satisfied and $\nu_p + \rho_c > \alpha_p$. If we choose ϵ and δ such that*

$$\begin{cases} 0 \leq \epsilon \leq \frac{\nu_p(\rho_c - \alpha_p)}{\nu_p + \rho_c - \alpha_p} \\ 0 \leq \delta \leq \nu_c + \rho_p - \beta_p \end{cases}, \tag{24}$$

the interconnected system (Fig. (3)*) has passivity indices ϵ and δ satisfying*

$$\dot{V}(t) \leq w_1^T(t) y_p(t) - \epsilon w_1^T(t) w_1(t) - \delta y_p^T(t) y_p(t) \tag{25}$$

Proof: If the condition (18)-(20), (24) and $\nu_p + \rho_c > \alpha$ are satisfied, we have

$$\begin{bmatrix} \nu_p - \epsilon & -\nu_p \\ -\nu_p & \nu_p + \rho_c - \alpha_p \end{bmatrix} \geq 0 \tag{26}$$

$$\nu_c + \rho_p - \beta_p - \delta \geq 0. \tag{27}$$

Then it implies

$$\begin{bmatrix} w_1^T(t) & y_c^T(t) \end{bmatrix} \begin{bmatrix} \nu_p - \epsilon & -\nu_p \\ -\nu_p & \nu_p + \rho_c - \alpha_p \end{bmatrix} \begin{bmatrix} w_1(t) \\ y_c(t) \end{bmatrix} + (\rho_p + \nu_c - \beta_p - \delta)\, y_p^T(t) y_p(t) \geq 0,$$

which can be written as

$$-\begin{bmatrix} w_1^T(t) & y_c^T(t) \end{bmatrix} \begin{bmatrix} \nu_p & -\nu_p \\ -\nu_p & \nu_p + \rho_c - \alpha_p \end{bmatrix} \begin{bmatrix} w_1(t) \\ y_c(t) \end{bmatrix} - (\rho_p + \nu_c - \beta_p)\, y_p^T y_p \leq -\epsilon w_1^T(t) w_1(t) - \delta y_p^T(t) y_p(t). \tag{28}$$

Since it is already known that

$$\dot{V}(t) \leq w_1^T(t) y_p(t) - \begin{bmatrix} w_1^T(t) & y_c^T(t) \end{bmatrix} \begin{bmatrix} \nu_p & -\nu_p \\ -\nu_p & \nu_p + \rho_c - \alpha_p \end{bmatrix} \begin{bmatrix} w_1(t) \\ y_c(t) \end{bmatrix} - (\rho_p + \nu_c - \beta_p)\, y_p^T(t) y_p(t)$$

we can conclude that

$$\dot{V}(t) \leq w_1^T(t) y_p(t) - \epsilon w_1^T(t) w_1(t) - \delta y_p^T(t) y_p(t) \tag{29}$$

holds for $\forall w_1$. ∎

Remark 17. Because of the conditions (18)-(20) and $\nu_p + \rho_c > \alpha_p$, the passivity indices ϵ and δ are upper bounded by positive numbers. (24) provides a way to obtain the desired passivity indices of the closed-loop system by choosing a passive G_c with proper indices and a triggering condition with proper α_p and β_p. As we point out in Remark 15, the trade off between performance (passivity level) and resource utilization can be considered. For instance, if an OFP index given by $\delta = \nu_c + \rho_p - \beta_p$ is desired, one can either choose a passive controller with high ν_c and a triggering condition with low β_p to conserve more communication resources, or a triggering condition with high β_p and a passive controller with low ν_c to impose less restrictions on the controller design.

B. Passivity Analysis and Passivation for Event-Trigerred Sampler of Controller Output

For the feedback system with event-triggered sampler of controller output (Fig. 4), we can follow the same rationale as for the feedback system with event-triggered sampler of plant output. We first consider the passivity analysis problem and then move to the passivation problem.

Lemma 18. *Consider two IF-OF systems with the passivity indices ν_p, ρ_p and ν_c, ρ_c respectively. If the event time t_k is explicitly determined by the following triggering condition*

$$\|e_c(t)\|_2 = \frac{\beta_c}{\sqrt{\nu_p^2 + m_c\beta_c} + |\nu_p|} \|y_c(t)\|_2 \tag{30}$$

where $m_c = \frac{1}{4\alpha_c} + |\nu_p| - \nu_p$, $\alpha_c > 0$ and $\beta_c > 0$, the interconnected system with the event-triggered sampler (Fig. 4) is QSR-dissipative (with respect to the input $w(t) = \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix}$ and output $y(t) = \begin{bmatrix} y_p(t) \\ y_c(t) \end{bmatrix}$), which satisfies the inequality

$$\dot{V}(t) \le y(t)^T Q y(t) + 2w(t)^T S y(t) + w(t)^T R w(t) \tag{31}$$

where

$$Q = \begin{bmatrix} -\left(\rho_p + \nu_c - \alpha_c\right) I & 0I \\ 0I & -\left(\nu_p + \rho_c - \beta_c\right) I \end{bmatrix},$$

$$S = \begin{bmatrix} \frac{1}{2}I & \nu_p I \\ -\nu_c I & \frac{1}{2}I \end{bmatrix},$$

and

$$R = \begin{bmatrix} -\left(\nu_p - |\nu_p|\right) I & 0I \\ 0I & -\nu_c I \end{bmatrix}.$$

.

Proof: Since G_p and G_c are IF-OF systems with the passivity indices ν_p, ρ_p, ν_c and ρ_c, there exist $V_p(t)$ and $V_c(t)$ such that

$$\dot{V}_p(t) \le u_p^T(t) y_p(t) - \nu_p u_p^T(t) u_p(t) - \rho_p y_p^T(t) y_p(t)$$

and

$$\dot{V}_c(t) \leq u_c^T(t)y_c(t) - \nu_c u_c^T(t)u_c(t) - \rho_c y_c^T(t)y_c(t).$$

Consider a storage function for the interconnected system given by $V(t) = V_p(t) + V_c(t)$, we have

$$\begin{aligned}\dot{V}(t) &= \dot{V}_p(t) + \dot{V}_c(t) \leq u_p^T(t)y_p(t) - \nu_p u_p^T(t)u(t)_p - \rho_p y_p^T(t)y_p(t) \\ &\quad + u_c^T(t)y_c(t) - \nu_c u_c^T(t)u_c(t) - \rho_c y_c^T(t)y_c(t). \end{aligned} \tag{32}$$

Consider that $u_p(t) = w_1(t) - y_c(t_k)$, $u_c(t) = y_p(t) + w_2(t)$ and $y_c(t_k) = y_c(t) - e_c(t)$. For any $t \in [t_k, t_{k+1})$, (32) can be rewritten as

$$\begin{aligned}\dot{V}(t) &\leq \left(w_1^T(t) - y_c(t_k)\right) y_p(t) - \rho_p y_p^T(t)y_p(t) - \nu_p \left(w_1(t) - y_c(t_k)\right)^T \left(w_1(t) - y_c(t_k)\right) + \\ &\quad \left(w_2(t) + y_p(t)\right)^T y_c - \rho_c y_c^T(t)y_c(t) - \nu_c \left(w_2(t) + y_p(t)\right)^T \left(w_2(t) + y_p(t)\right) \\ &= w_1^T(t)y_p(t) + w_2^T(t)y_c(t) + 2\nu_p w_1^T(t)y_c(t) - 2\nu_c w_2^T(t)y_p(t) - \nu_p w_1^T(t)w_1(t) - \nu_c w_2^T(t)w_2(t) \\ &\quad -(\nu_p + \rho_c)y_c^T(t)y_c(t) - (\rho_p + \nu_c)y_p^T(t)y_p(t) - 2\nu_p w_1^T(t)e_c(t) + \\ &\quad 2\nu_p y_c^T(t)e_c(t) - \nu_p e_c^T(t)e_c(t) + y_p^T(t)e_c(t).\end{aligned}$$

Since $-2\nu_p w_1^T(t)e_c(t) \leq |\nu_p|\, w_1^T(t)w_1(t) + |\nu_p|\, e_c^T(t)e_c(t)$, we can obtain that

$$\begin{aligned}\dot{V}(t) &\leq \begin{bmatrix} w_1^T(t) & w_2^T(t) \end{bmatrix} \begin{bmatrix} 1 & 2\nu_p \\ -2\nu_c & 1 \end{bmatrix} \begin{bmatrix} y_p(t) \\ y_c(t) \end{bmatrix} \\ &\quad + \begin{bmatrix} w_1^T(t) & w_2^T(t) \end{bmatrix} \begin{bmatrix} -\nu_p + |\nu_p| & 0 \\ 0 & -\nu_c \end{bmatrix} \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix} \\ &\quad + \begin{bmatrix} y_p^T(t) & y_c^T(t) \end{bmatrix} \begin{bmatrix} -(\rho_p + \nu_c) & 0 \\ 0 & -(\nu_p + \rho_c) \end{bmatrix} \begin{bmatrix} y_p(t) \\ y_c(t) \end{bmatrix} \\ &\quad + 2\nu_p y_c^T(t)e_c(t) + (|\nu_p| - \nu_p)e_c^T(t)e_c(t) + y_p^T(t)e_c(t).\end{aligned}$$

With $y_p^T(t)e_c(t) \leq \alpha_c \|y_p(t)\|_2^2 + \frac{1}{4\alpha_c}\|e_c(t)\|_2^2$ where $\alpha_c > 0$, we can further get

$$\begin{aligned}\dot{V}(t) &\leq 2w^T(t)Sy(t) + w^T(t)Rw(t) + y^T(t)Qy(t) + 2\nu_p y_c^T(t)e_c(t) \\ &\quad + (\frac{1}{4\alpha_c} + |\nu_p| - \nu_p)e_c^T(t)e_c(t) - \beta y_c^T(t)y_c\end{aligned} \tag{33}$$

where

$$Q = \begin{bmatrix} -\left(\rho_p + \nu_c - \alpha_c\right) I & 0I \\ 0I & -\left(\nu_p + \rho_c - \beta_c\right) I \end{bmatrix},$$

$$S = \begin{bmatrix} \frac{1}{2}I & \nu_p I \\ -\nu_c I & \frac{1}{2}I \end{bmatrix},$$

$$R = \begin{bmatrix} -\left(\nu_p - |\nu_p|\right) I & 0I \\ 0I & -\nu_c I \end{bmatrix}$$

and $\beta > 0$.

Note that $2\nu_p y_c^T(t)e_c(t) \le 2|\nu_p| \|y_c(t)\|_2 \|e_c(t)\|_2$. Then we can show

$$\begin{aligned}\dot{V}(t) &\le 2w^T(t)Sy(t) + w^T(t)Rw(t) + y^T(t)Qy(t) \\ &\quad + \left(\sqrt{m_c}\|e_c(t)\|_2 + \frac{|\nu_p|}{\sqrt{m_c}}\|y_c(t)\|_2\right)^2 - \left(\frac{\nu_p^2}{m_c} + \beta_c\right)\|y_c(t)\|_2^2 \\ &= 2w^T(t)Sy(t) + w^T(t)Rw(t) + y^T(t)Qy(t) + \\ &\quad \left(\sqrt{m_c}\|e_c(t)\|_2 + \frac{|\nu_p|}{\sqrt{m_c}}\|y_c(t)\|_2 + \sqrt{\frac{\nu_p^2}{m_c} + \beta_c}\,\|y_c(t)\|_2\right) \\ &\quad \times \left(\sqrt{m_c}\|e_c(t)\|_2 + \frac{|\nu_p|}{\sqrt{m_c}}\|y_c(t)\|_2 - \sqrt{\frac{\nu_p^2}{m_c} + \beta_c}\,\|y_c(t)\|_2\right)\end{aligned}$$

From (30), one can verify that

$$\dot{V}(t) \le 2w^T(t)Sy(t) + w^T(t)Rw(t) + y^T(t)Qy(t)$$

which completes the proof. ■

Theorem 19. *Suppose that the passivity indices ν_p, ρ_p, ν_c and ρ_c are known and the triggering condition is determined by* (30). *If we choose ϵ and δ such that*

$$\begin{cases} \epsilon < \min\{\nu_p - |\nu_p|, \nu_c\} \\ \delta \le \min\left\{\rho_p - \alpha_c - \frac{\epsilon\nu_c}{\nu_c - \epsilon}, \rho_c - \beta_c - \frac{(|\nu_p|+\epsilon)\nu_p}{\nu_p - |\nu_p| - \epsilon}\right\} \end{cases}, \tag{34}$$

the interconnected system with the event-triggered sampler (Fig. 4) has the passivity indices ϵ and δ satisfying

$$\dot{V} \le w^T(t)y(t) - \epsilon w^T(t)w(t) - \delta y^T(t)y(t) \tag{35}$$

where $w(t) = \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix}$ *and* $y = \begin{bmatrix} y_p(t) \\ y_c(t) \end{bmatrix}$.

Proof: From (31) , we have

$$\begin{aligned}\dot{V}(t) &\le w^T(t)y(t) - \begin{bmatrix} w_1^T(t) & y_c^T(t) \end{bmatrix} \begin{bmatrix} \nu_p - |\nu_p| & -\nu_p \\ -\nu_p & \nu_p + \rho_c - \beta_c \end{bmatrix} \begin{bmatrix} w_1(t) \\ y_c(t) \end{bmatrix} \\ &\quad - \begin{bmatrix} w_2^T(t) & y_p^T(t) \end{bmatrix} \begin{bmatrix} \nu_c & \nu_c \\ \nu_c & \rho_p + \nu_c - \alpha_c \end{bmatrix} \begin{bmatrix} w_2(t) \\ y_p(t) \end{bmatrix}.\end{aligned} \tag{36}$$

Since ϵ and δ are chosen such that (34) is satisfied, (37) holds for the chosen ϵ and δ.

$$\begin{cases} \epsilon \le \nu_p - |\nu_p| \\ \epsilon \le \nu_c \\ (\nu_p - |\nu_p| - \epsilon)(\nu_p + \rho_c - \beta_c - \delta) \ge \nu_p^2 \\ (\nu_c - \epsilon)(\rho_p + \nu_c - \alpha_c - \delta) \ge \nu_c^2 \\ \nu_p + \rho_c - \beta_c - \delta \ge 0 \\ \rho_p + \nu_c - \alpha_c - \delta \ge 0 \end{cases}. \tag{37}$$

(37) further implies that the matrices

$$M = \begin{bmatrix} \nu_p - |\nu_p| - \epsilon & -\nu_p \\ -\nu_p & \nu_p + \rho_c - \beta_c - \delta \end{bmatrix}$$

and

$$N = \begin{bmatrix} \nu_c & \nu_c \\ \nu_c & \rho_p + \nu_c - \alpha_c - \delta \end{bmatrix}$$

are positive semi-definite. Therefore, we have

$$\begin{bmatrix} w_1^T(t) & y_c^T(t) \end{bmatrix} M \begin{bmatrix} w_1(t) \\ y_c(t) \end{bmatrix} + \begin{bmatrix} w_2^T(t) & y_p^T(t) \end{bmatrix} N \begin{bmatrix} w_2(t) \\ y_p(t) \end{bmatrix} \geq 0 \tag{38}$$

for $\forall\ w_1(t)$, $w_2(t)$, $y_c(t)$ and $y_p(t)$. After re-arranging the terms in (38), one can obtain

$$\begin{aligned} -w^T(t)Ew(t) - y^T(t)\Delta y(t) \quad &\geq \\ -\begin{bmatrix} w_1^T(t) & y_c^T(t) \end{bmatrix} O \begin{bmatrix} w_1(t) \\ y_c(t) \end{bmatrix} - \begin{bmatrix} w_2^T(t) & y_p^T(t) \end{bmatrix} P \begin{bmatrix} w_2(t) \\ y_p(t) \end{bmatrix}&. \end{aligned} \tag{39}$$

where $E = \begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon \end{bmatrix}$, $\Delta = \begin{bmatrix} \delta & 0 \\ 0 & \delta \end{bmatrix}$, $O = \begin{bmatrix} \nu_p - |\nu_p| & -\nu_p \\ -\nu_p & \nu_p + \rho_c - \beta_c \end{bmatrix}$, and $P = \begin{bmatrix} \nu_c & \nu_c \\ \nu_c & \rho_p + \nu_c - \alpha_c \end{bmatrix}$.

From (39) and (36), we can finally show that

$$\begin{aligned} \dot{V}\quad(t) \leq \quad & w^T(t)y(t) - \begin{bmatrix} w_1^T(t) & y_c^T(t) \end{bmatrix} \begin{bmatrix} \nu_p - |\nu_p| & -\nu_p \\ -\nu_p & \nu_p + \rho_c - \beta_c \end{bmatrix} \begin{bmatrix} w_1(t) \\ y_c(t) \end{bmatrix} \\ & - \begin{bmatrix} w_2^T(t) & y_p^T(t) \end{bmatrix} \begin{bmatrix} \nu_c & \nu_c \\ \nu_c & \rho_p + \nu_c - \alpha_c \end{bmatrix} \begin{bmatrix} w_2(t) \\ y_p(t) \end{bmatrix} \\ \leq \quad & w^T(t)y(t) - \epsilon w^T(t)w(t) - \delta y^T(t)y(t) \end{aligned} \tag{40}$$

∎

Remark 20. The results in Lemma 18 and Theorem 19 are similar to their counterparts for the feedback system with event-triggered sampler of plant output. However, note that the triggering condition (30) now depends on α_c, β_c and ν_p. Moreover, the matrices Q, S and R in (31) are different from those in (8).

For the passivation problem, Theorem (21) gives the conditions of rendering the interconnected system passive.

Theorem 21. *Assume $w_2 = 0$ and the triggering condition is determined by* (30)*. The interconnected system with the event-triggered sampler (Fig. 4) is passive with respect to the input w_1 and output y_p if the passivity indices satisfy the conditions*

$$\nu_p = 0 \tag{41}$$

$$\rho_c \geq \beta_c \tag{42}$$

$$\rho_p + \nu_c \geq \alpha_c. \tag{43}$$

Proof: If $w_2(t) = 0$ and $\nu_p = 0$, (36) becomes

$$\dot{V}(t) \leq w_1^T(t)y_p(t) - (\rho_c - \beta_c)y_c^T(t)y_c(t) - (\rho_p + \nu_c - \alpha_c)y_p^T(t)y_p(t)$$

Since we have $\rho_c \geq \beta_c$ and $\rho_p + \nu_c \geq \alpha_c$, we can conclude that

$$\begin{aligned}\dot{V}(t) \leq\ & w_1^T(t)y_p(t) - (\rho_c - \beta)y_c^T(t)y_c(t) - (\rho_p + \nu_c - \alpha_c)y_p^T y_p \\ \leq\ & w_1^T(t)y_p(t).\end{aligned} \tag{44}$$

■

Remark 22. The condition (41) requires the plant G_p to be a OFP system. Because of (41), the triggering condition (30) can be further simplified as $\|e_c(t)\|_2 = 2\sqrt{\alpha_c\beta_c}\,\|y_c(t)\|_2$, which shows that the triggering condition is independent of the passivity indices of the plant G_p and controller G_c. Therefore, one can first design a desired triggering condition by choosing α_c and β_c, and then design a passive controller satisfying the conditions (41)-(43), and vice versa.

Corollary 23. *Suppose that the conditions* (41)-(43) *are satisfied. If we choose* ϵ *and* δ *such that*

$$\begin{cases} \epsilon = 0 \\ 0 \leq \delta \leq \rho_p + \nu_c - \alpha_c \end{cases}, \tag{45}$$

the interconnected system with event-triggering (Fig. 4) has the passivity indices ϵ *and* δ *satisfying*

$$\dot{V}(t) \leq w_1^T(t)y_p(t) - \epsilon w_1^T(t)w_1(t) - \delta y_p^T(t)y_p(t) \tag{46}$$

Proof: If the condition (41)-(43) are satisfied, we have

$$\dot{V}(t) \leq w_1^T(t)y_p(t) - (\rho_c - \beta)y_c^T(t)y_c(t) - (\rho_p + \nu_c - \alpha)\,y_p^T y_p \leq w_1^T(t)y_p(t) - (\rho_p + \nu_c - \alpha)\,y_p^T y_p$$

If δ is chosen such that $0 \leq \delta \leq \rho_p + \nu_c - \alpha$, we can show that $(\rho_p + \nu_c - \alpha - \delta)\,y_p^T(t)y_p(t) \geq 0$, which implies that

$$\dot{V}(t) \leq w_1^T(t)y_p(t) - (\rho_p + \nu_c - \alpha)\,y_p^T y_p \leq w_1^T(t)y_p(t) - \epsilon w_1^T(t)w_1(t) - \delta y_p^T(t)y_p(t)$$

where $\epsilon = 0$. ■

Remark 24. The condition (45) implies that the closed-loop system is actually an OSP system with an OFP index $\delta \leq \rho_p + \nu_c - \alpha_c$. The ideas of passivity indices design and passivity-resource trade off discussed in Remark (17) apply likewise.

V. EXAMPLES

The examples show how to passivate a nonlinear plant with a linear feedback controller with event-triggered samplers at the plant output (Fig. 3) and at the controller output (Fig. 4), respectively. For both examples, it is assumed that $w_2 = 0$.

Example 25. We first consider the case that the event-triggered sampler is implemented at the plant output (as shown in Fig. 3). Assume that the plant G_p is a nonlinear system (an adapted model of H_3 used in [24]), given by

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -0.5x_1^3 + 0.1x_2 + u_p \\ y_p &= x_2 \end{aligned}$$

G_p admits a storage function given by

$$V(x) = \frac{1}{8}x_1^4 + \frac{1}{2}x_2^2,$$

and

$$\dot{V}(x) = u_p y_p + 0.1y_p^2.$$

Therefore the OFP level for G_p are $\rho_p = -0.1$.

The feedback controller G_c is a 2nd-order system with

$$A = \begin{bmatrix} -2 & -1 \\ -3 & -5 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 \\ 2 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 1 \end{bmatrix},\ D = 1.$$

We can determine the passivity levels of G_c to be $\nu_c = 0.3$ and $\rho_c = 0.5$.

It can be seen that the triggering condition (7) depends on two non-negative scalars α_p and β_p, in addition to the passivity levels of G_p and G_c. In order to guarantee passivity of the closed-loop system, α_p and β_p need to satisfy the conditions (42)-(43), proposed in Theorem 21. Therefore, we choose $\alpha_p = 0.3 < \rho_c$ and $\beta_p = \rho_p + \nu_c = 0.2$ so that the obtained triggering condition is

$$\|e_p(t)\|_2 > 0.2497\,\|y_p(t)\|_2\,. \tag{47}$$

It is noted the closed system is now a passive system with the passivity levels $\epsilon = 0$ and $\delta = 0$, given by (45).

The simulation results are shown in Fig. 5-7. Fig. 5 verifies that the trajectory of the function $\int w_1 y_p dt$ is always above 0 along the time of simulation. Fig. 6 shows the event-triggered sampler only samples the plant output at certain time instants determined by the triggering condition. Fig. 7 presents the evolutions of $\|e_p\|_2$ and $0.2497\,\|y_p(t)\|_2$, illustrating the time instants when the triggering condition is satisfied.

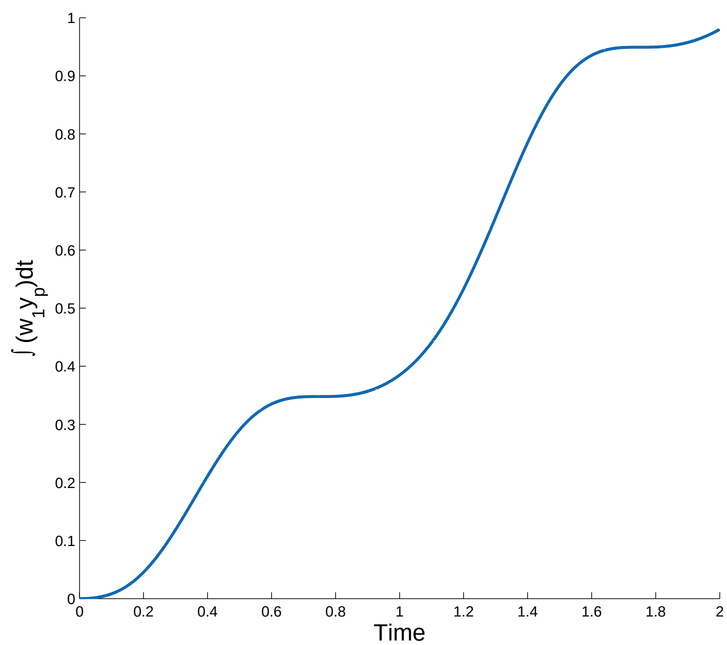

Figure 5. The trajectory of the function $\int w_1 y_p dt$ over time t for $w_1(t) = \sin(2\pi t) + 1$, under the triggering condition (47)

As discussed in Remark 15 and 17, it is possible to increase the passivity levels of the closed-loop systems at the cost of a "tighter" bound on the triggering condition. We can choose $\beta_p = 0.05$ so that the triggering condition becomes

$$\|e_p(t)\|_2 > 0.0754\,\|y_p(t)\|_2\,. \tag{48}$$

Although the new triggering condition results in more frequent sampling of the plant output, based on Corollary 23 the closed system is an OSP system with OFP level $\delta = 0.15$.

Similarly, the simulation results under the triggering condition (48) are shown Fig. 8-10. Note that Fig. 8 verifies that the trajectory of the function$\int w_1 y_p - 0.15 y_p^2 dt$ is always above 0 along the time of simulation. Compared to Fig. 6 and 7, Fig. 9 and 10 show that the sampler is triggered more frequently due to the tighter triggering condition.

Example 26. We then consider the case that the event-triggered sampler is implemented at the controller output (as shown in Fig. 4). We still use the same plant G_p and controller G_c as defined in Example 25.

In this case, the triggering condition (30) depends on two non-negative scalars α_c and β_c, other than the passivity levels of G_p and G_c. We can choose $\alpha_c = 0.2$ and $\beta_c = 0.5$ so that the conditions (41)-(43) in Theorem (21) are

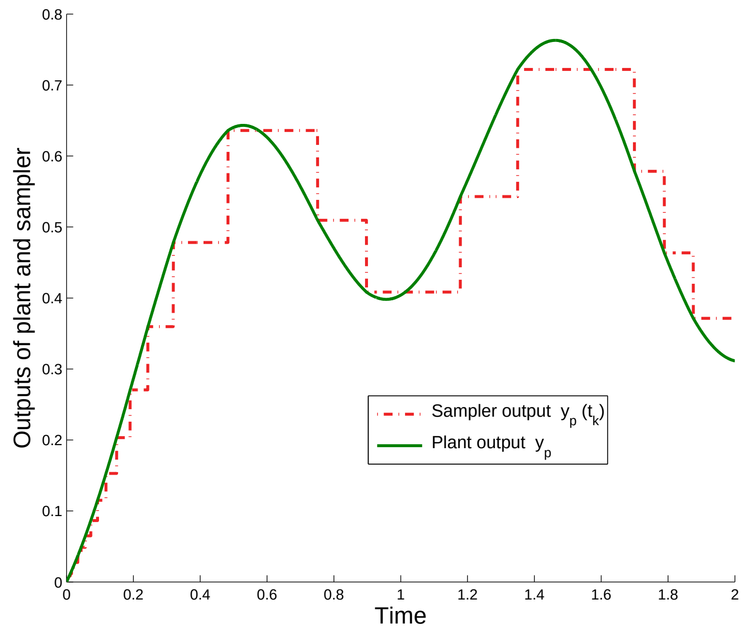

Figure 6. The trajectories of the event-triggered sampler output $y_p(t_k)$ and the plant output y_p over time t, under the triggering condition (47)

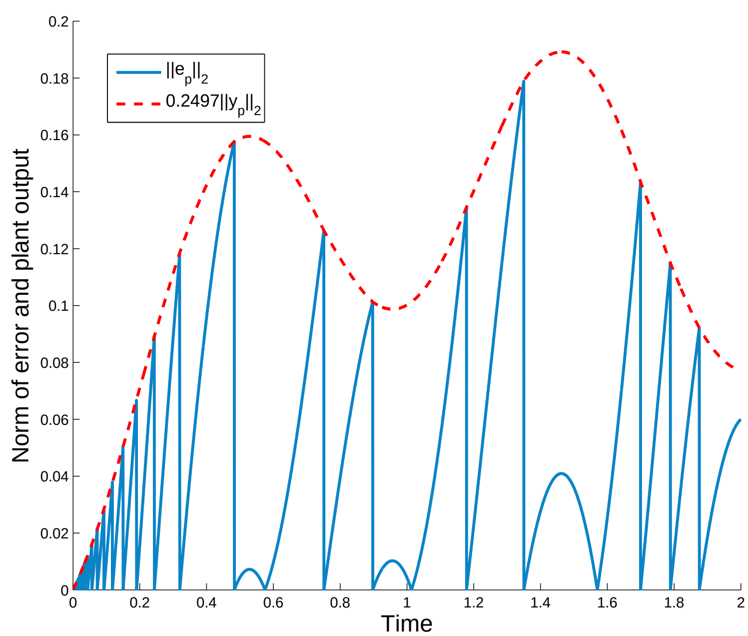

Figure 7. The trajectories of the error $\|e_p\|_2$ and $0.2497\,\|y_p(t)\|_2$ in the triggering condition (47)

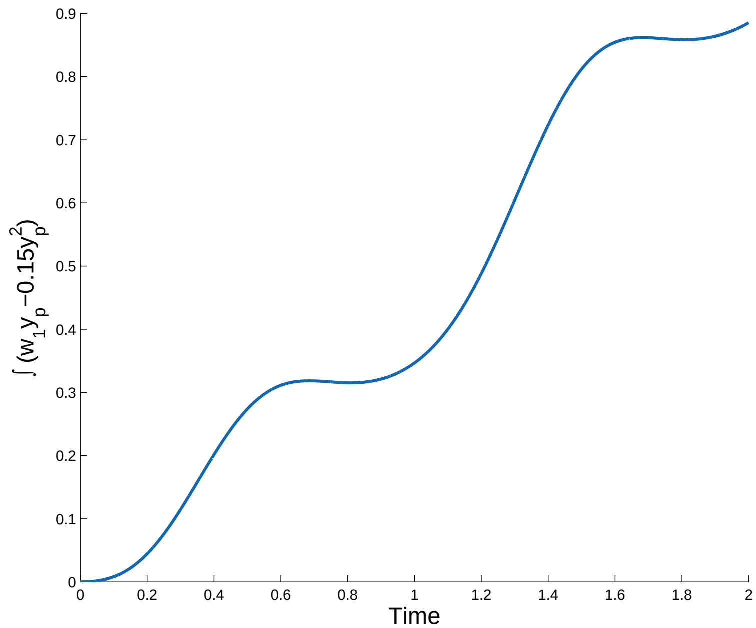

Figure 8. The trajectory of the function $\int w_1 y_p - 0.15y_p^2 dt$ over time t for $w_1(t) = \sin(2\pi t) + 1$, under the triggering condition (48)

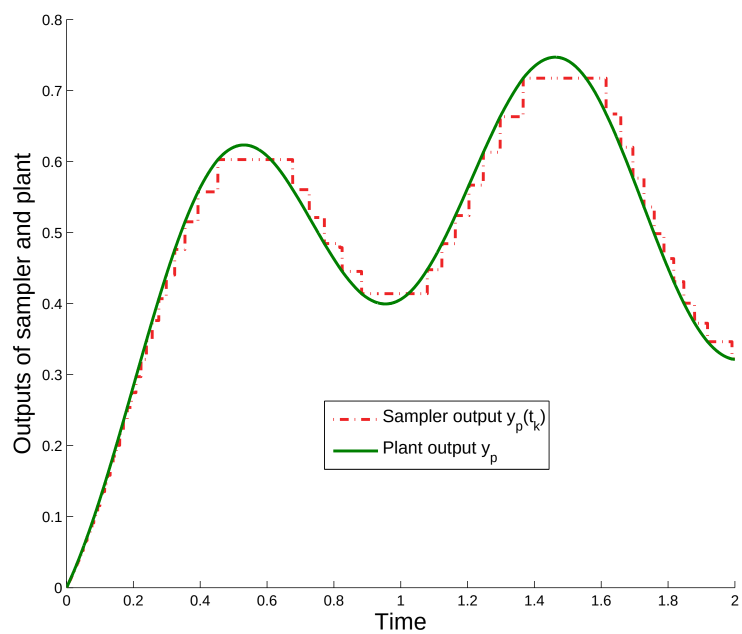

Figure 9. The trajectories of the event-triggered sampler output $y_p(t_k)$ and the plant output y_p over time t, under the triggering condition (48)

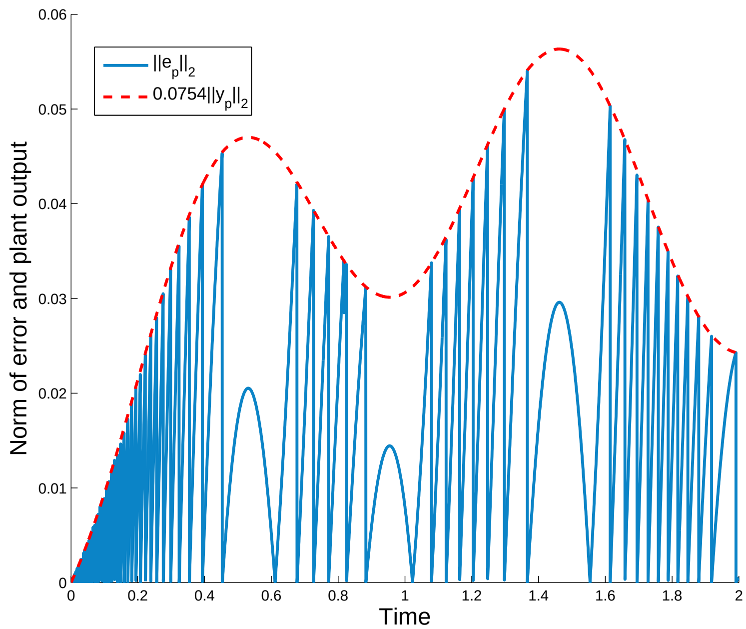

Figure 10. The trajectories of the error $\|e_p\|_2$ and $0.0754\,\|y_p(t)\|_2$ in the triggering condition (48)

satisfied. Therefore the obtained triggering condition is

$$\|e_c(t)\|_2 > 0.6325\,\|y_c(t)\|_2\,. \tag{49}$$

Moreover, Corollary (23) shows that the closed system is a passive system with the passivity levels $\epsilon = 0$ and $\delta = 0$. The simulation results are shown in Fig. 11-13. Fig. 11 shows the evolution of the function $\int w_1 y_p dt$. Fig. 12 shows the evolution of the outputs of the controller and sampler. Fig. 13 shows the evolution of the signals in the triggering condition (49).

Analogously to the previous example, we can also enhance the passivity levels of the closed-loop system by choosing an event-triggering condition which leads to more frequent sampling. Therefore we can change α_c to 0.05 so that the same OFP index ($\delta = 0.15$) is obtained, as in Example 25. The resulting triggering condition is

$$\|e_c(t)\|_2 > 0.3162\,\|y_c(t)\|_2\,. \tag{50}$$

The simulation results under the triggering condition (50) are shown in Fig. 14-16. It can be seen that the OFP index has been increased to 0.15 by increasing sampling frequency of the sampler.

VI. Conclusion

In this paper, we considered the problems in passivity analysis and passivation using passivity indices for event-triggered feedback interconnected systems, which extended our previous work in [15] for feedback interconnected

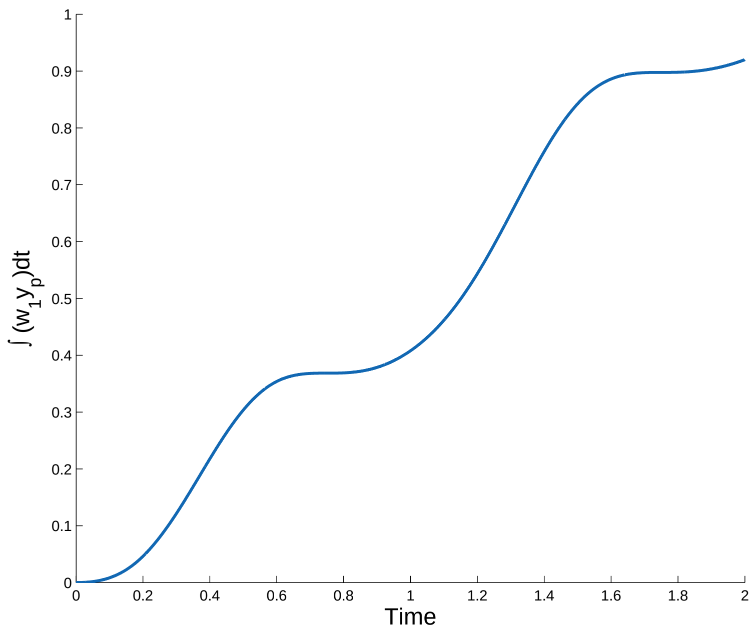

Figure 11. The trajectory of the function $\int w_1 y_p dt$ over time t for $w_1(t) = \sin(2\pi t) + 1$, under the triggering condition (49)

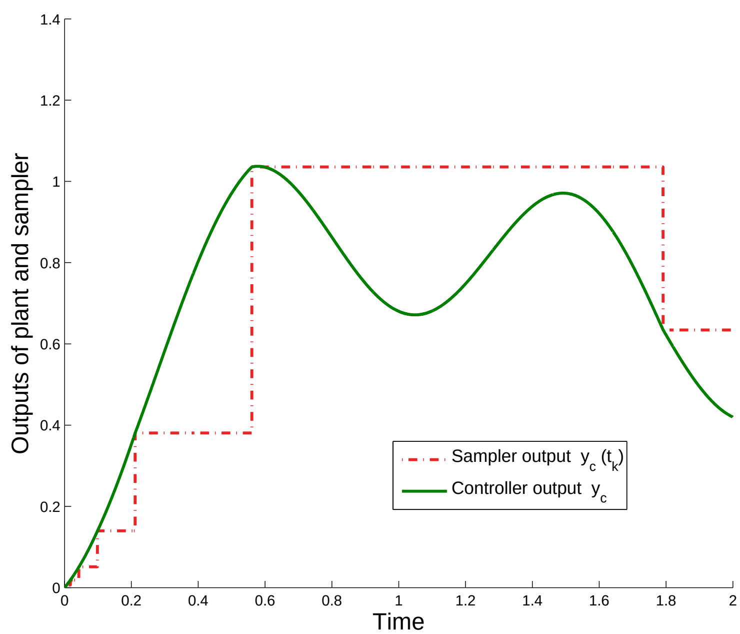

Figure 12. The trajectories of the event-triggered sampler output $y_c(t_k)$ and the controller output y_c over time t, under the triggering condition (49)

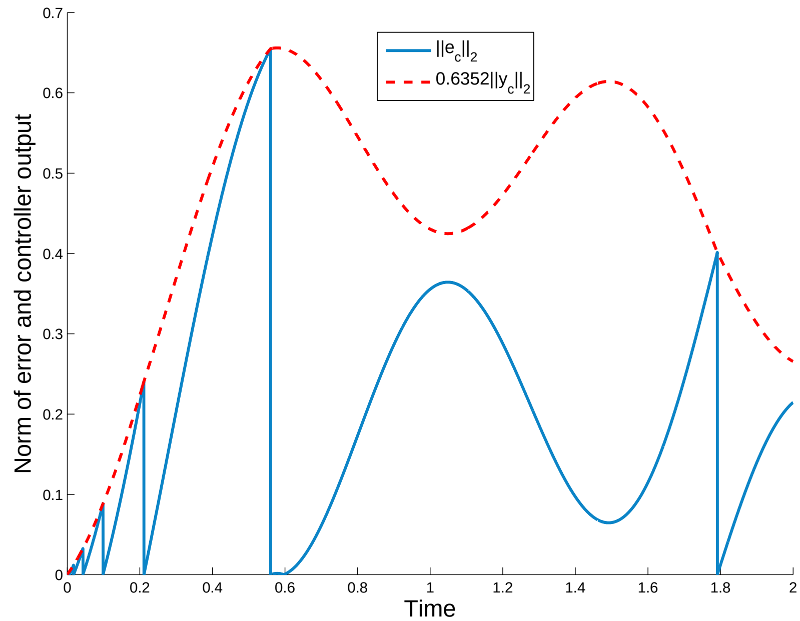

Figure 13. The trajectories of the error $\|e_c\|_2$ and $0.6325\,\|y_c(t)\|_2$ in the triggering condition (49)

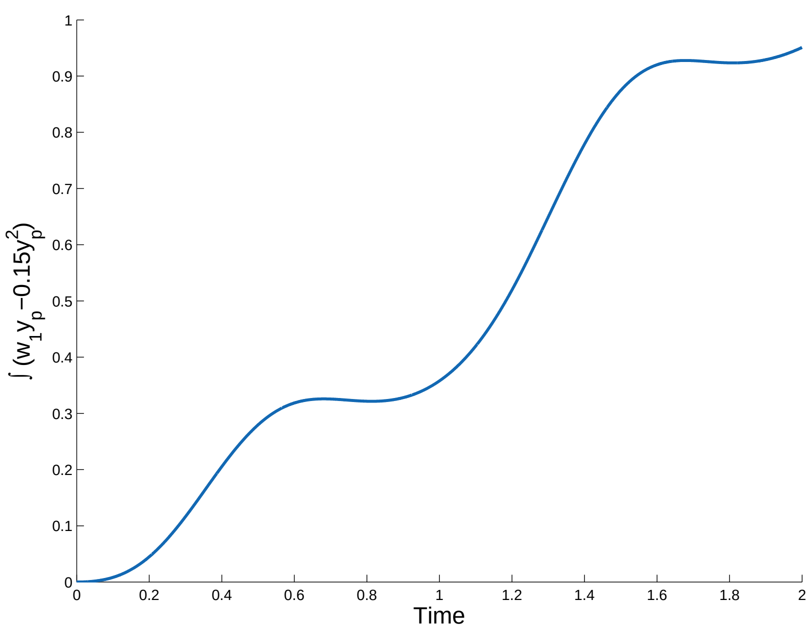

Figure 14. The trajectory of the function $\int w_1 y_p - 0.15 y_p^2 dt$ over time t for $w_1(t) = \sin(2\pi t) + 1$, under the triggering condition (50)

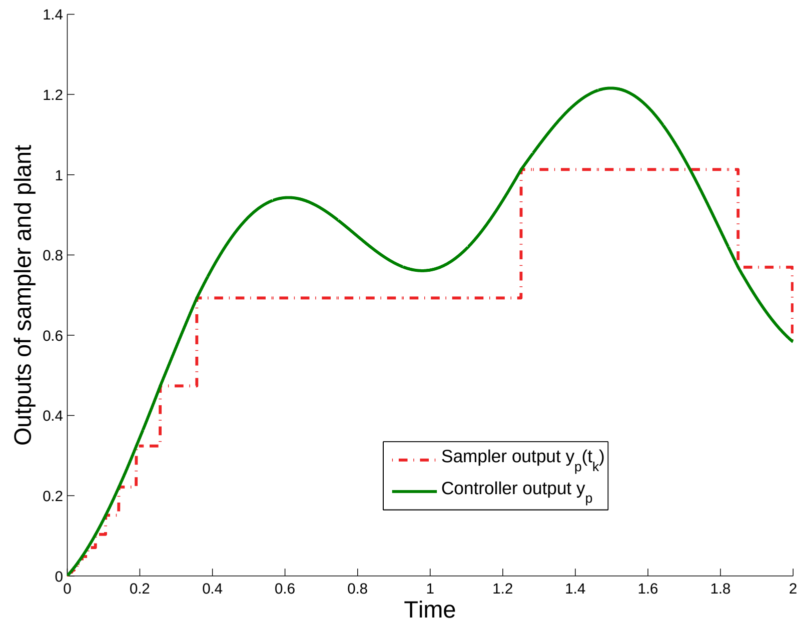

Figure 15. The trajectories of the event-triggered sampler output $y_c(t_k)$ and the controller output y_c over time t, under the triggering condition (50)

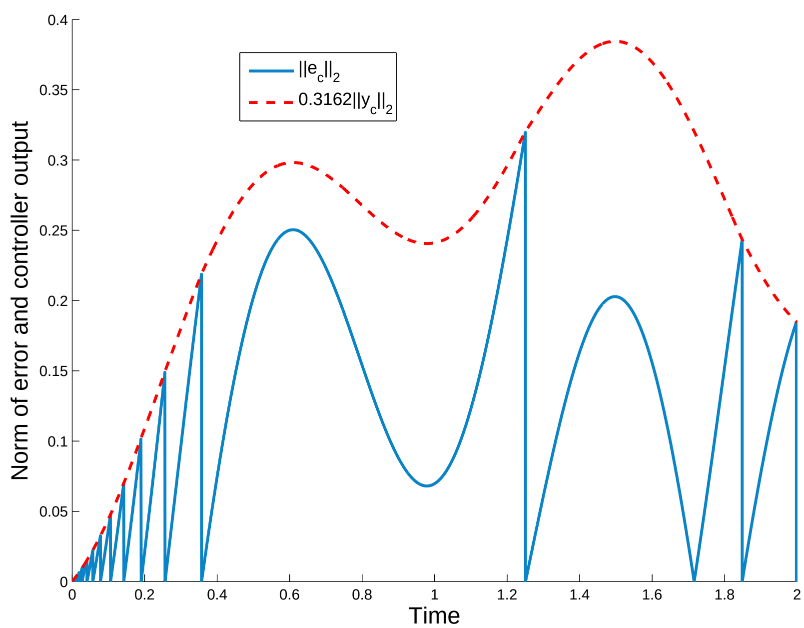

Figure 16. The trajectories of the error $\|e_c\|_2$ and $0.3162\,\|y_c(t)\|$ in the triggering condition (50)

systems assuming continuous communication in the feedback loop. We consider two event-triggered control schemes respectively: event-triggered sampler of the plant output and event-triggered sampler of the controller output. By knowing passivity indices of the plant and controller, the conditions to determine the passivity indices of interconnected systems were given, with the proposed event-triggering condition. We also showed the passivation conditions in terms of the passivity indices of the plant and controller and the triggering condition. The trade off between passivity and communication resources utilization was also discussed.

Acknowledgments

The support of the National Science Foundation under the CPS Large Grant No. CNS-1035655 is gratefully acknowledged.

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