# Passivity Analysis of Human as a Controller

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**Abstract**

Human controllers as pilots or drivers can be described by linear time invariant systems cascaded with time delay. Inspired by this, we 'generalize' the definition of passivity index to irrational transfer functions and use the passivity index to examine passivity of some existing human models. The results show that the human model is non-passive due to the time delay which represents inherent human limitations. We then propose a passification scheme when a human operator is in the loop. By this scheme, the closed-loop system with human in the loop as a controller can have desired positive passivity indices to address the control tasks of interest.

## 1 Introduction

Passivity and the generalized concept of dissipativity, characterize the "energy" consumption of a dynamical system and form a powerful tool in many applications. Under mild assumptions, passivity implies stability and passivity is preserved under parallel and feedback interconnections [1, 2, 3]. Thus, passivity analysis and passivity-based control (PBC) become especially useful in the area of complex large-scale systems.

Passivity and dissipativity theory for linear time invariant (LTI) systems has been well established, see e.g. [3, 4, 5]. With minimal state space realization, passivity of a LTI system described by a state variable representation is equivalent to positive realness of the corresponding 'rational' transfer function [6]. However, 'irrational' transfer functions are of particular concern in many cases. For instance, for linear systems with distributed parameters or delayed linear systems that are common in practice, the transfer function becomes irrational [7, 8, 9]. For such systems, there is no systematic passivity or dissipativity theory [10].

In this paper, we are interested in passivity analysis and PBC with regard to delayed linear systems, inspired by the fact that a human controller can be described by a delayed linear system [11, 12, 13, 14]. Human behavior modeling itself is a difficult problem, since the decision making process of a human is complicated. We focus on simple linear human models that have been studied since the 1960s [11, 12] and in particular we study passivity properties of these models. Linear systems with delayed state has been studied in e.g. [15, 16], where sufficient conditions are derived based on a Lyapunov - Krasovskii functional construction. A new supply rate dependent on delayed input is considered in [10]. Positive realness of transfer functions of delayed linear systems has been studied in [7, 8, 9]. In contrast to the previous results, we use passivity indcies to determine whether a delayed linear system is passive. The magnitude of the passivity indcies characterizes “how far a system is from being passive” and this information is quite useful in control synthesis [17, 18, 19].

Our results show that the so-called ‘quasi-linear’ human models are not passive. This may seem contradictory to the teleoperation results developed in e.g. [20], where a human is assumed to be passive when interacting with a passive environment. However, in teleoperation systems, only the human arm is modeled as a mechanical system and the reaction time of human is not considered [21, 22]. It is well-known that passivity and stability of a closed-loop system may not be preserved in the presence of communication effects, such as time delay [21]. The wave/scattering transformation is therefore introduced to ‘passify’ the two-port system [22, 23]. In the present paper, we ‘borrow’ this idea and introduce an input-output transformation through which *any* positive desired passivity index of the ‘transformed’ system can be guaranteed. In fact, this transformation can be viewed as a human/machine interface design. When the controlled element (e.g. an aircraft or a vehicle) and the human operator are interconnected through this interface, closed-loop passivity (and under mild conditions, stability) is guaranteed.

The rest of the paper is organized as follows. In Section 2, we review the background of passivity index. In Section 3, we present the “quasi-linear” human models. Passivity index of human models is studied in Section 4. A transformation is introduced in Section 5 to ensure passivity when human is in the loop as a controller. Simulation results to illustrate the effectiveness of the transformation are presented in Section 6. Section 7 concludes the paper.

## 2 Background: Passivity Index of Linear Systems

**Definition 1** ([3])**.** *The passivity index for a* stable[1] *linear system $G(s)$ is defined as*

$$\nu(G(s)) \triangleq \frac{1}{2}\min_{w\in\mathbb{R}} \underline{\lambda}(G(jw)+G^*(jw)), \tag{1}$$

*where $\underline{\lambda}$ denotes the minimum eigenvalue.*

If $\nu \geq 0$, then the stable linear system $G$ is said to be passive. If a system is non-passive, then the passivity index $\nu < 0$ and it is defined as the “minimum feedforward gain required” for a stable non-passive linear system to be passive [18, 24, 19], as shown in Fig. 1. Note that the passivity index $\nu$ is defined for stable linear systems. For such systems, we can test the passivity of a system based on its Nyquist plot. In general, if the Nyquist plot is in the closed right-hand half of the complex plane, then the system is passive.

**Remark 1.** *If a system $G(s)$ is minimum phase (does not have to be stable), then we can define another passivity index*

$$\rho(G(s)) \triangleq \frac{1}{2}\min_{w\in\mathbb{R}} \underline{\lambda}(G^{-1}(jw)+[G^{-1}(jw)]^*), \tag{2}$$

*which implies the “minimum feedback gain required” for a minimum-phase system to be passive, see Fig. 2. For a system which is unstable and non-minimum phase, we need both feedback and feedforward to ensure passivity [3].*

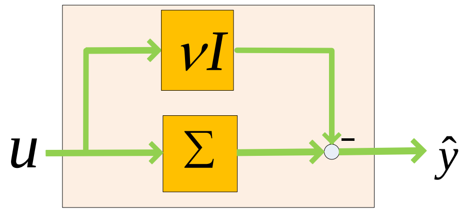


Figure 1: System $\Sigma$ has passivity index $\nu$. If $\nu < 0$, then feedforward $\nu I$ is required for closed-loop (from $u$ to $\hat{y}$) passivity.

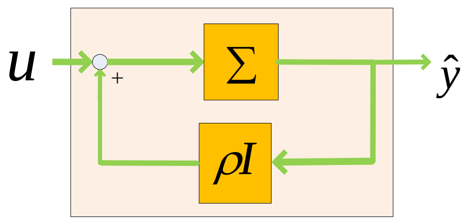


Figure 2: System $\Sigma$ has passivity index $\rho$. If $\rho < 0$, then feedback $\rho I$ is required for closed-loop (from $u$ to $\hat{y}$) passivity.

**Definition 2** ([3])**.** *The passivity index for a* stable *linear system* $G(s)$ at frequency $w$ *is given by* $\nu_F(G(s), w)$ *(or* $\nu_F(w)$ *if* $G$ *is clear from the context), where*

$$\nu_F(G(s), w) \triangleq \frac{1}{2}\underline{\lambda}(G(jw) + G^*(jw)). \tag{3}$$

It is apparent that the following relation holds for all $w \in \mathbb{R}$,

$$\nu(G(s)) \leq \nu_F(G(s), w). \tag{4}$$

From (4), we can see that the *frequency-dependent* passivity index $\nu_F(w)$ provides an upper bound on the the passivity index $\nu$. When passivity is considered over a limited range of frequency, say $[w_1, w_2]$ where $w_1, w_2 \in \mathbb{R}$, if $\nu_F(w) \geq 0$ for all frequency $w \in [w_1, w_2]$ (instead of $w \in \mathbb{R}$), then the system is called *locally passive*, see e.g. [17].

**Remark 2.** *For SISO system* $G(s)$*, the passivity index* $\nu$ *can be calculated as* $\nu(G(s)) \triangleq \min_{w\in\mathbb{R}} Re[G(jw)]$*, and the passivity index at frequency* $w$ *is given by* $\nu_F(G(s), w) \triangleq Re[G(jw)]$*.*

**Remark 3.** *An equivalent definition of passivity index for system* $\Sigma$ *(which may be nonlinear) with input* $u$ *and output* $y$ *(see e.g. [3, 25, 26]) is given as follows: if there exists a constant* $\beta \leq 0$ *such that the following inequality holds for all* $u \in \mathbb{R}^m$ *and all* $T \geq 0$*, (see Lemma 4 in the Appendix)*

$$\int_0^T u^T(t)y(t)\mathrm{d}t \geq \nu \int_0^T u^T(t)u(t)\mathrm{d}t + \beta, \tag{5}$$

[1] A function $G(s)$ is called *stable* if it is analytic in the closed right half plane of the complex plane, see e.g. [7].

*then $\nu$ is the passivity index $\nu$ of system $\Sigma$. If $\nu > 0$, then the system is called* input strictly passive *(ISP). If $\nu \geq 0$ and (5) holds for finite $T$ and control input $u \in \mathcal{U}$, where $\mathcal{U}$ is a subset of $\mathbb{R}^m$, then the system $\Sigma$ is called locally passive*[2].

The passivity index of LTI systems can be found by solving linear matrix inequalities (LMI) [28]. Assume that system $G(s)$ has a minimal state-space realization, where

$$\begin{aligned} \dot{x} &= Ax + Bu, \\ y &= Cx + Du. \end{aligned} \tag{6}$$

The passivity index $\nu$ is given by the solution to the following LMI, where $P > 0$ and

$$\Pi \triangleq \begin{bmatrix} A^T P + PA & PB - \frac{1}{2}C^T \\ B^T P - \frac{1}{2}C & -\frac{1}{2}(D + D^T) + \nu I \end{bmatrix} \leq 0. \tag{7}$$

In the present paper, we consider transfer functions which may be irrational (i.e. cannot be represented by (6)). In particular, when delayed linear system is under consideration, an 'approximate' passivity index can also be found through LMI based methods when using Pade approximation of the delay term. For instance, a first-order Pade approximation of the pure delay $D(s) = e^{-\tau s}$ is given as follows,

$$D(s) \approx \frac{1 - \frac{\tau}{2}s}{1 + \frac{\tau}{2}s}. \tag{8}$$

**Proposition 1** ([3],[29]). *For systems $H$ and $\alpha H$ where $\alpha$ is a constant, if $H$ has passivity index $\nu$, then $\alpha H$ has passivity index $\alpha\nu$.*

This is called the "scaling property" of the passivity index; $H$ may not necessarily be a linear system. In the following analysis, we may 'ignore' the constant $\alpha$ and examine the passivity index of $H$ when analyzing the passivity index of $\alpha H$.

# 3 Human as a controller: Quasi-linear Models

Many different human behavior models have been proposed [30, 31, 32]. The basic block diagram of these models is given in Fig. 3, where $w$ denotes the external disturbance and $z$ denotes the output of the controlled element. In the present paper, we use quasi-linear models of the human operator. Of course, these models cannot accurately describe complicated human behavior. For instance, human may exhibit nonlinear behavior [32] and "feedforward" pursuit behavior [33, 31]. However, the quasi-linear models and the feedback compensation have been proven powerful in many applications [30, 32, 11].

Quasi-linear model describes the human operator by means of five free parameters in [11, 34, 32]. The model is given as follows,

$$H(s) = \left\{ \frac{K(as+1)}{(bs+1)} \right\} \left\{ \frac{e^{-\tau s}}{(cs+1)} \right\}. \tag{9}$$

[2]The definition of local passivity is also used in e.g. [27] (and references therein) when analyzing passivity of a nonlinear system using its linearized model around the equilibrium point.

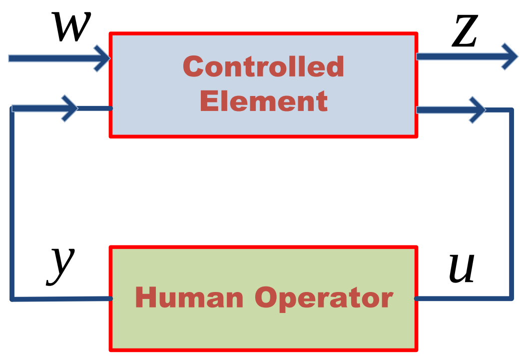


Figure 3: The basic structure of compensation models.

The right bracket represents some inherent human limitations and the left bracket depends on the controlled element and the input signal (thus called 'quasi-linear'), where

- $\tau > 0$ represents the reaction time delay;
- $c > 0$ represents the neuromuscular delay time;
- $K > 0$ denotes the static gain;
- $a > 0$ denotes the lead time constant;
- $b > 0$ denotes the lag time constant.

**Remark 4.** 1. *The quasi-linear model is evaluated as the "most general linear model of the human as a controller" [32]. However, this model is not satisfactory since the behavior of the system depends on the parameters which can only be determined approximately. Thus, "cross-over" human models have been proposed; These models view the controlled element and the human operator as one system and result in fewer parameters.*

2. *Note that the modeling of human behavior itself is a difficult problem. For instance, the models may represent "anticipative" and "hybrid" characteristics of human decision process [32, 31]. However, this is not our concern in the present paper. Our aim is to examine the passivity property of simple human models.*

To investigate whether the human model given by (9) is passive, we first look at a numerical example with parameters taken from [13], where the transfer function is given by

$$H_{tf} = \frac{1 + 0.06s}{0.05 + 0.28s + 0.001s^2} e^{-0.2s}. \tag{10}$$

The frequency dependent passivity index is given in Fig. 4. We can see that system $H_{tf}$ has passivity index $\nu = -0.48 < 0$ which implies that the system is non-passive. However, for low frequencies $w \in [-1, 1]$, we can see that the passivity index $\nu_F > 0$ over such frequencies, see Fig. 4. The reason is that the time delay induces a phase lag and when $w > 1$, the phase angle exceeds

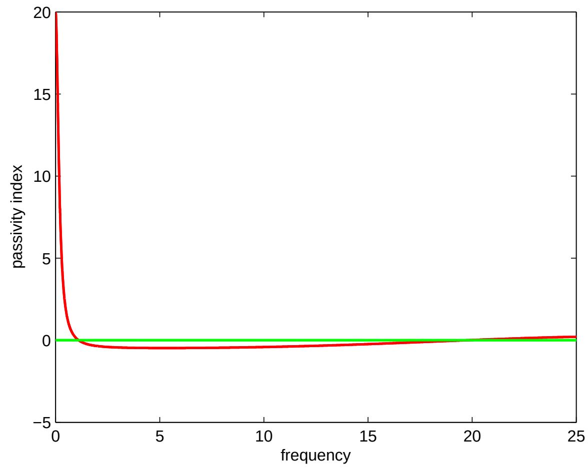


Figure 4: Passivity Index of system $H_{tf}$ given by (10) over frequencies.

$-90^o$. As a result, passivity is not preserved. However, system $H_{tf}$ is finite-gain stable with gain given by 20. From this example, we conjecture that a 'reasonable' human model, e.g. given by (9) may have the following properties:

- stable;
- passive over low frequencies;
- non-passive if all frequencies are considered.

When human is in the loop as a controller , we will further assume that the human model should perform as a 'good' controller, e.g. stabilize the system and guarantee certain performance.

# 4 Preliminary Results: Non-passivity of Quasi-linear Human Models

## 4.1 A First-Order Model

The simplest pilot describing function form in [11, 12] is given by

$$Y(s) = K\frac{as+1}{bs+1}e^{-\tau s}, \tag{11}$$

where $K$ denotes the static pilot gain, $a$ represents the lead time constant, $b$ represents the lag time constant and $\tau$ denotes the *effective delay* which includes the effects of neuromuscular delay. We assume that $a \neq b$ and $a, b \geq 0$. Note that the values of $a$, $b$ and $K$ may change according to the specific control tasks and $\tau$ represents the inherent human limitations.

For system (11), its passivity index depends on the system parameters. In the following analysis, we present results that 'approximate' the exact value.

**Lemma 1.** *Consider the system given by (11), its passivity index satisfies*

1. $-K \leq \nu < 0$ *if* $a < b$;
2. $-K\frac{a}{b} \leq \nu < 0$ *if* $a > b$.

*Proof.* See Appendix. □

The lower bound of $\nu$, e.g. $K$ when $a < b$, implies the requirement of a feed-forward controller to render the model (11) passive. For example, if the controller has a constant gain that is greater than $K$, then it renders the system passive, see also Fig. 1. This result is especially useful when the reaction time delay of human models is taken to be time-varying due to the human status (alert, tired, distracted, etc.) or when the exact time delay is not accessible. The minimum value of the passivity index can be determined before hand.

**Remark 5.** *From the proof, we can see that there must exist at least one frequency* $w \in [0, \frac{\pi}{\tau}]$ *(and denote the first one as* $w_0$*) such that* $\nu_F(w) = 0$ *due to the continuity of* $\nu_F(w)$*. System model given by (11) is passive over frequencies* $[-w_0, w_0]$*.*

### 4.2 A Second-Order Model

Now, let us consider the quasi-linear model (9) introduced in Section 3. The next result shows that this model is non-passive for all possible parameters.

**Lemma 2.** *Consider human model given by (9), the system has passivity index* $\nu < 0$*.*

*Proof.* See Appendix. □

Similar to the previous case, the model given by (9) can be shown passive over frequency band $[-w_0, w_0]$, where $w_0 > 0$ denotes the smallest frequency such that $\nu_F(w_0) = 0$. Again, the precise passivity index can be computed when the time delay $\tau$ is given. It is worthwhile to mention that in general it does not make sense to describe human behavior by means of higher-order (order greater than two) differential equations [32].

**Remark 6.** *For the first-order human model* $Y(s)$[3] *given by (11) and the second-order human model* $H(s)$ *given by (9), there exists a frequency* $w_0 > 0$ *such that* $\nu_F(w_0) = 0$ *and* $\nu_F(w) \geq 0$ *for all* $w \in [-w_0, w_0]$*. This fact is consistent with the findings in [35, 36] where passivity of the system is often violated at high frequencies.*

## 5 Passification Schemes for Stable Systems

In this section, we focus on single-input-single-output (SISO) systems with zero initial conditions. However, our results can be extended to the cases of MIMO systems or non-zero initial conditions

[3] $Y(s)$ given by (11) can be seen as an 'approximation' of $H(s)$ given by (9), where the factor $\frac{1}{cs+1}$ is included in the effective delay $\tau$ in (11).

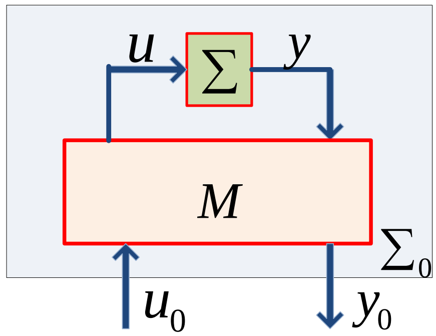


Figure 5: Passification Scheme used in this paper. $\Sigma$ with input $u$ and output $y$ is stable and $\Sigma_0$ with input $u_0$ and output $y_0$ has passivity index $\rho > 0$ (or $\nu > 0$) based on the transformation matrix $M$.

as well. The scheme we will introduce for passification is suitable for any 'stable' linear or nonlinear systems. Further, this scheme can be used for passification of negative feedback interconnection of two systems.

### 5.1 Input-Output Transformation

The transformation matrix $M$ as shown in Fig. 5 is used as a passification scheme for stable non-passive systems. It can also be viewed as a scheme that aims for a passive system to have "any" desired passivity index. As shown in Fig. 5, $\begin{bmatrix} u_0 \\ y_0 \end{bmatrix} = M \begin{bmatrix} u \\ y \end{bmatrix}$, where the matrix $M$ is constrained to be invertible and is defined as

$$M \triangleq \begin{bmatrix} m_{11}I & m_{12}I \\ m_{21}I & m_{22}I \end{bmatrix}.$$

We have the following results.

**Theorem 1.** *Consider a system $\Sigma$ which is finite gain stable with gain $\gamma$ and a passivation matrix $M$ as shown in Fig. 5. Then the system $\Sigma_0 : \mathbf{u}_0 \to \mathbf{y}_0$ is*

1. *passive, if $M$ is chosen such that*

$$m_{11} = m_{21}, \quad m_{22} = -m_{12}, \quad m_{11} \geq m_{22}\gamma > 0. \tag{12}$$

2. *OSP with OFP level $\rho_0 = \frac{1}{2}\left(\frac{m_{11}}{m_{21}} + \frac{m_{12}}{m_{22}}\right) > 0$, if $M$ is chosen such that*

$$m_{21} \geq m_{22}\gamma > 0, \quad m_{11}m_{22} > m_{12}m_{21} > 0. \tag{13}$$

3. *ISP with IFP level $\nu_0 = \frac{1}{2}(\frac{m_{21}}{m_{11}} + \frac{m_{22}}{m_{12}}) > 0$, if $M$ is chosen such that*

$$m_{11} \geq m_{12}\gamma > 0, \quad m_{12}m_{21} > m_{11}m_{22} > 0. \tag{14}$$

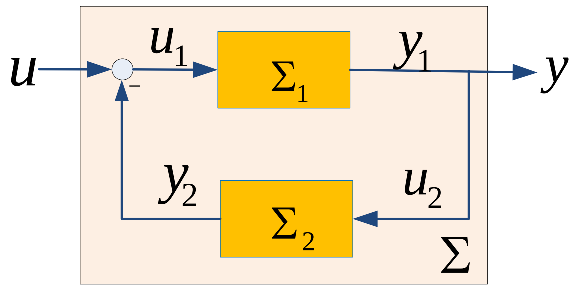


Figure 6: The negative feedback interconnection of two systems: $\Sigma_1$ and $\Sigma_2$.

4. *VSP with passivity levels* $\delta_0 = \frac{1}{2}\frac{m_{11}}{m_{21}} > 0$ *and* $\epsilon_0 = \frac{a}{2}\frac{m_{21}}{m_{11}} > 0$, *if* $M$ *is chosen such that*

$$m_{11} > 0, \quad m_{12} = 0, \quad m_{21} \geq \frac{m_{22}\gamma}{\sqrt{1-a}} > 0, \tag{15}$$

*where* $0 < a < 1$ *is an arbitrary real number.*

*Proof.* See Appendix. □

This result is applicable to the case when $\Sigma$ represents the human model given by (9) and (11). These models are $\mathcal{L}_2$ stable since the linear model has stable poles and the delay does not change the magnitude (or the gain) of the system. Thus, the transformation $M$ can be used to 'passify' the human model such that the 'transformed' system $\Sigma_0$ has a positive passivity index.

## 5.2 Passivity with Human in the Loop

Consider the negative feedback interconnection of a plant model $\Sigma_1$ (e.g. a vehicle) and a controller $\Sigma_2$, see Fig. 6. If $\Sigma_1$ and $\Sigma_2$ are passive, then their interconnection (i.e. system $\Sigma$ with input $u$ and output $y$) is also passive. However, in practice, the plant $\Sigma_1$ may not be stable (which of course implies that it will be non-passive) such that a stabilizing controller $\Sigma_2$ is needed. In the present paper, our aim is to ensure that the closed-loop system $\Sigma$ is passive which in general is stronger than being stable. One advantage of $\Sigma$ being passive is that when $\Sigma$ is part of a large scale system (for instance, it is part of an automobile network with vehicle-to-vehicle communication), the nice compositionality of passivity can be used to facilitate the analysis and synthesis of the large scale system.

The following results state that the "shortage" of passivity in one system (e.g. system $\Sigma_2$) can be compensated by the "excess" of passivity in another system (e.g. system $\Sigma_1$).

**Lemma 3** ([23])**.** *Consider negative feedback interconnection of two systems:* $\Sigma_1$ *and* $\Sigma_2$*.*

1. *Assume that system* $\Sigma_1$ *has OFP($\rho$) and system* $\Sigma_2$ *has IFP($\nu$). Then, system* $\Sigma$ *is OSP if* $\rho + \nu > 0$.

2. *Assume that system* $\Sigma_1$ *has IFP($\nu$) and system* $\Sigma_2$ *has OFP($\rho$). Then, system* $\Sigma$ *is ISP if* $\rho + \nu > 0$ *and* $\nu > 0$.

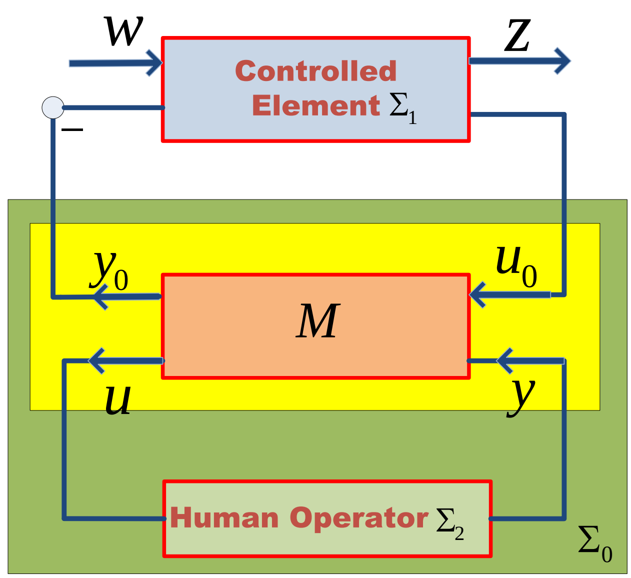


Figure 7: The negative feedback interconnection of the controlled element denoted by $\Sigma_1$ and the human operator denoted by $\Sigma_2$. The system with input $u_0$ and output $y_0$ based on the transformation $M$ is denoted by $\Sigma_0$. The two-port transformation $M$ can viewed as a human/machine interface.

We can view system $\Sigma_1$ as the vehicle that is being controlled by the human operator $\Sigma_2$. Based on this lemma, we can obtain that the feedback system $\Sigma$ is passive if

1. the vehicle has OSP $\rho > -\nu > 0$ where $\nu < 0$ is the IFP of the human operator.
2. the vehicle has ISP $\nu > -\rho > 0$ where $\rho < 0$ is the OFP of the human operator.

However, this may not be the case in practice. For instance, the vehicle model $\Sigma_1$ may not have an excess of passivity as required. The transformation we introduced in Theorem 1 can be applied to ensure closed-loop passivity for such cases. We have the following result.

**Theorem 2.** *Consider negative feedback interconnection of two systems: $\Sigma_1$ and $\Sigma_0$ as in Fig. 7. Assume that the human operator denoted by $\Sigma_2$ is $\mathcal{L}_2$ stable with finite gain $\gamma > 0$. Then the feedback system with input $w$ and output $z$ is passive if one of the following statements holds,*

1. *the controlled element $\Sigma_1$ has OFP $\rho < 0$, and $M$ is chosen so that (14) is satisfied and*
$$\frac{1}{2}\left(\frac{m_{21}}{m_{11}} + \frac{m_{22}}{m_{12}}\right) > -\rho > 0.$$
2. *the controlled element $\Sigma_1$ has OFP $\rho > 0$, and $M$ is chosen so that (14) is satisfied.*
3. *the controlled element $\Sigma_1$ has IFP $\nu > 0$, $M$ is chosen so that (13) is satisfied.*

*Proof.* This result is immediate from the Lemma 3 and Theorem 1. More precisely, for the first two cases, the feedback system is OSP with passivity index given by $\rho(\Sigma) = \frac{1}{2}\left(\frac{m_{21}}{m_{11}} + \frac{m_{22}}{m_{12}}\right) + \rho$; for the second case, the feedback system is ISP with passivity index $\nu(\Sigma) = \frac{1}{2}\left(\frac{m_{11}}{m_{21}} + \frac{m_{12}}{m_{22}}\right) + \nu$. $\square$

**Remark 7.** *This result is useful when the the "shortage" of passivity index of one system cannot be compensated by the "excess" of passivity of the other system. If this is the case,*

1. *We can use the input-output transformation $M$ to obtain "desired" passivity index, through which the closed-loop passivity can be ensured.*

2. *Another possible way is to design local controllers for the system that lacks passivity. For instance, if the vehicle has OFP $\rho < 0$, then a positive feedback controller with OFP $K \geq -\rho + \tilde{\rho} > 0$ can render the system have OFP $\tilde{\rho} > 0$.*

## 6 Numerical Examples

**Example 1:** Consider a controlled element ($\Sigma_1$ in Fig. 7) $G_1 = \frac{s+1}{s-1}$ that has passivity index $\rho = -1$. The human operator ($\Sigma_2$ in Fig. 7) $G_2$ is given by (10) that has passivity index $\nu = -0.48$. It can be verified that $G_2$ is stable with gain 20 and thus $\gamma = 400$. According to Theorem 2, we choose $M$ such that

$$\frac{m_{21}}{m_{11}} + \frac{m_{22}}{m_{12}} > 2, \quad \frac{m_{11}^2}{m_{12}^2} \geq \gamma, \quad \frac{m_{21}}{m_{11}} > \frac{m_{22}}{m_{12}} > 0.$$

For instance, let $m_{22} = 1, m_{21} = 33, m_{12} = 1, m_{11} = 22$. With such $M$, we obtain that $G_0$ given by

$$G_0 \triangleq \frac{m_{21} + m_{22}G_2}{m_{11} + m_{12}G_2} \tag{16}$$

has passivity index $\nu = 1.25$. Then based on Lemma 3, the closed-loop system $G$ is OSP. The Nyquist plot of the system $G_0$ is given in Fig. 8. These two plots indicate that the system $G_0$ is passive with index $\nu = 1.25$ and the closed-loop system $G$ is passive with $\rho = 0.25$ (see Fig. 9), which are consistent with our results.

**Remark 8.** *For this example, the transformation $M$ can be viewed as a phase lead compensator and is discussed in Appendix.*

**Example 2:** Tracking Performance. The basic block diagram for tracking tasks is shown in Fig. 10, where $w$ is the reference input and $z$ is the output of the controlled element. In this example, we implement the transformation $M$ introduced in Theorem 1 in Simulink and test the tracking performance by varying the parameters of $M$.

**Remark 9.** *Here, the transformation $M$ can be viewed as a filter which 'pre-filers' the input and also 'post-filters' the output of the human operator.*

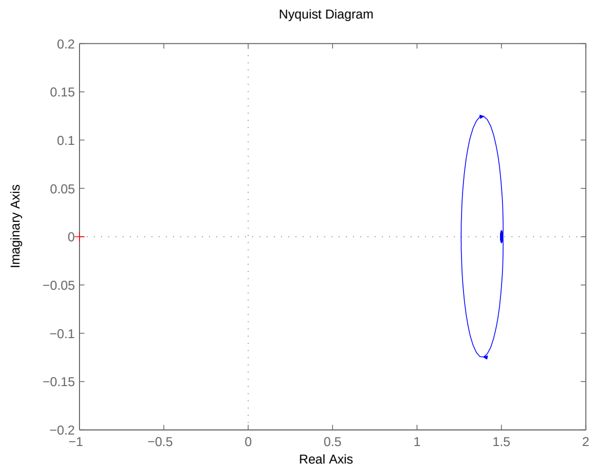


Figure 8: Nyquist plot of $G_0$ in Example 1 to indicate passivity index $\nu = 1.25$.

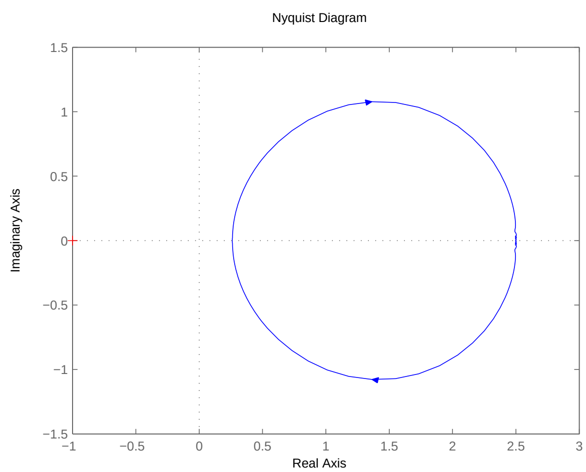


Figure 9: Nyquist plot of the inverse of $G$ in Example 1 to indicate passivity index $\rho = 0.25$.

Consider a controlled element ($\Sigma_1$ in Fig. 10) given by $P = \frac{s+1}{s+2}$ which has passivity index $\nu > 0$ (also $\rho > 0$). A 'human'-like controller ($\Sigma_2$ in Fig. 10) is given by $C = \frac{s+10}{0.2s+1}e^{-0.1s}$ that has finite gain 10 such that $\gamma = 100$. According to Theorem 2, we choose a transformation $M$ such that $m_{11} = 6, m_{12} = 0.5, m_{21} = 11, m_{22} = 1$. The simulink model is given in Fig. 11 and tracking performance for an step input with magnitude 1 is given in Fig. 12. As shown in Fig. 12, by using the transformation $M$, the tracking performance of the system is quite good.

**Remark 10.** *1. If plant is unstable, without transformation $M$ (or equivalently when $M = I$, where $I$ is the identity matrix), our simulation results show that the output of the system goes unstable.*

*2. It can also be verified that the choice of transformation is not unique and the conditions given*

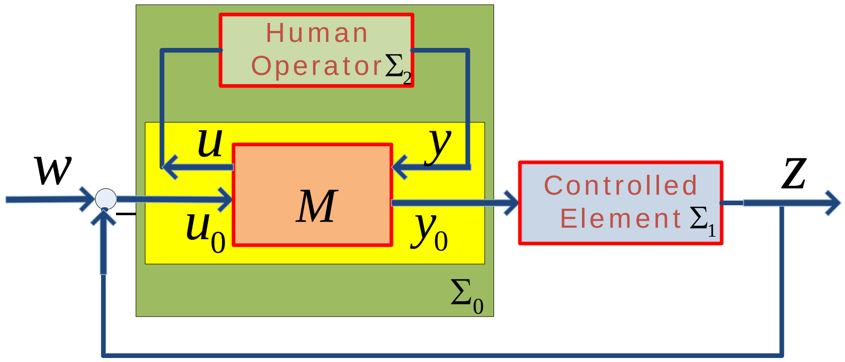


Figure 10: The basic structure for tracking compensation tasks with transformation $M$ of Example 2.

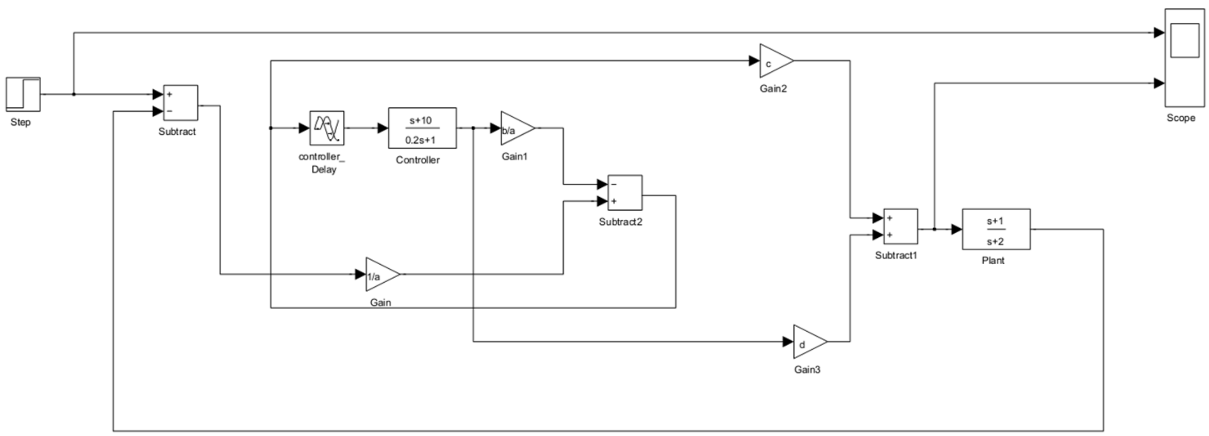


Figure 11: Simulink Model of Example 2.

*in this paper are merely sufficient but not necessary.*

# 7 Conclusion and Future Works

In this paper, we analyze passivity properties of some existing linear human models by means of calculating the passivity index. The main result shows that these models are non-passive because of the human reaction time delay. We propose a passification scheme by designing a human/machine interface to ensure that the interconnected system is passive with positive passivity indices. This result is especially useful when human is in the loop interacting with passive or non-passive environment (e.g. driving a car). The simulation results show that our proposed scheme can guarantee stability of the closed-loop system and also 'good' performance such as tracking. It is interesting to examine the effectiveness of the proposed passification scheme for more complicated scenarios, for instance, steering or lateral control of a vehicle when a human operator is in the loop.

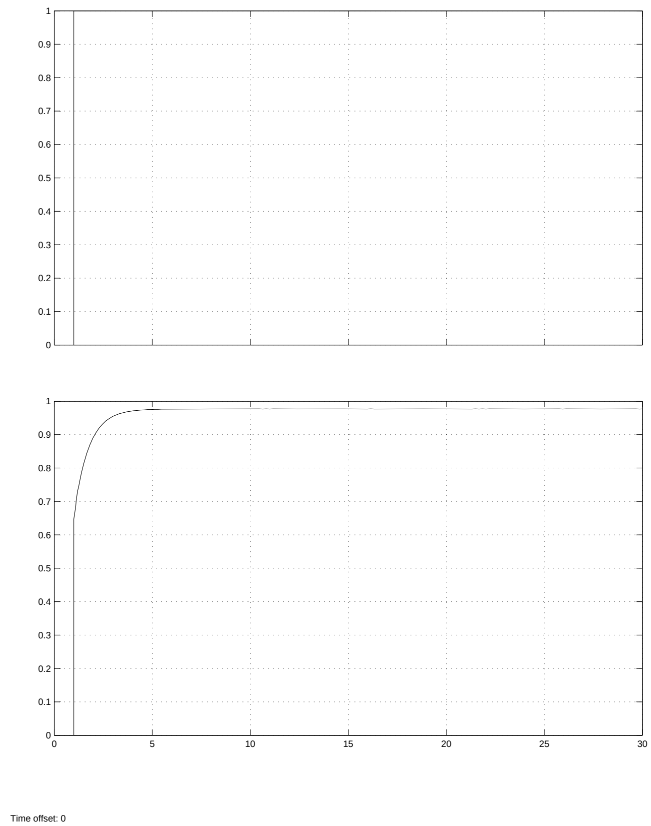


Figure 12: Tracking in Example 2: the top plot is the step input and the bottom plot is the output of the closed-loop system.

# Acknowledgment


The support of the National Science Foundation under Grant No. CNS-1035655 is gratefully acknowledged. The first author would like to thank Dr. Michael McCourt for valuable discussions.


# 8 Appendix

## 8.1 Lemma 4 and a Proof

**Lemma 4.** *Consider a stable linear system $G(s)$ that maps the input $\mathbf{u}$ to the output $\mathbf{y}$. Then, $\nu \triangleq \nu(G(s))$ defined in (1) is the largest IFP level of system $G(s)$.*

*Proof.* For notational convenience, we denote $\langle u, y\rangle_T = \int_0^T u^T y \, \mathrm{d}t$. Define $M(w) \triangleq \frac{1}{2}(G(jw) + G^*(jw))$. It is apparent that $M(w) = M^*(w)$, thus $M(w)$ is Hermitian, see e.g. [37]. Denote the minimum eigenvalue of $M$ as $\nu_F(w)$, where

$$\nu_F(G(s), w) \triangleq \frac{1}{2}\underline{\lambda}(G(jw) + G^*(jw)). \tag{17}$$

Thus, we can obtain the following relation for all $w \in \mathbb{R}$ and for all $x \in \mathbb{C}^n$ ([37, p 176]),

$$\nu x^* x \leq \nu_F(w) x^* x \leq x^* M(w) x. \tag{18}$$

We prove the theorem in two steps. First, we show that (5) holds for $\nu$ defined in (1). Consider the truncated input function $u_T(t) \in \mathcal{L}_2$ for all $T \geq 0$, where

$$u_T(t) = \begin{cases} u(t), & \text{if } 0 \leq t \leq T \\ 0. & \text{otherwise} \end{cases}$$

It follows from Parseval's Theorem (see e.g. [5, p 363] and [4, p 210]) that

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} U_T^*(jw) U_T(jw) \mathrm{d}w = \int_0^{\infty} u_T^T(t) u_T(t) \mathrm{d}t,$$

where $U_T(jw)$ is the Fourier transform for the causal signal $u_T(t)$. Then we obtain

$$\begin{aligned}\langle u, u\rangle_T &= \int_0^{\infty} u_T^T(t) u_T(t) \mathrm{d}t \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} U_T^*(jw) U_T(jw) \mathrm{d}w. \end{aligned} \tag{19}$$

If $G(s)$ is stable, then with input $u_T(t)$, the output $y(t) \in \mathcal{L}_2$. Thus, we also have

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} U_T^*(jw) Y(jw) \mathrm{d}w = \int_0^{\infty} u_T^T(t) y(t) \mathrm{d}t,$$

where $Y(jw)$ is the Fourier transform for $y(t)$. Therefore, for any $T \geq 0$, we obtain

$$\begin{aligned}\langle u, y\rangle_T &= \int_0^{\infty} u_T^T(t) y(t) \mathrm{d}t \\ &= \frac{1}{4\pi}\int_{-\infty}^{\infty} U_T^*(jw) \left[G(jw) + G^*(jw)\right] U_T(jw) \mathrm{d}w \\ &\triangleq \frac{1}{2\pi}\int_{-\infty}^{\infty} U_T^*(jw) M(w) U_T(jw) \mathrm{d}w, \end{aligned} \tag{20}$$

From (18), we obtain $U_T^*(jw) M(w) U_T(jw) \geq \nu U_T^*(jw) U_T(jw)$ for all $w \in \mathbb{R}$, thus

$$\begin{aligned}&\frac{1}{2\pi}\int_{-\infty}^{\infty} U_T^*(jw) M(w) U_T(jw) \mathrm{d}w \\ &\geq \nu \frac{1}{2\pi}\int_{-\infty}^{\infty} U_T^*(jw) U_T(jw) \mathrm{d}w.\end{aligned}$$

Then, from equations (19) and (20), we have

$$\langle u, y\rangle_T \geq \ \nu \langle u, u\rangle_T.$$

This implies that $\nu$ defined as (1) is an IFP level of $G(s)$, i.e. (5) is satisfied. Next, we show that $\nu$ defined in (1) is the largest IFP such that (5) holds. We prove this by contradiction. Suppose $\tilde{\nu} > \nu$ is an IFP level of $G(s)$ such that for all $u(t)$ and all $T \geq 0$,

$$\langle u, y \rangle_T \geq \tilde{\nu} \langle u, u \rangle_T. \tag{21}$$

Since $\tilde{\nu} > \nu$, then there exists at least one frequency $w_0 \in \mathbb{R}$ so that $\tilde{\nu} > \nu_F(w_0)$. Consider the control input to be $\bar{u}(t) = \cos(w_0 t)$ and denote the corresponding output as $\bar{y}(t)$. The Fourier transform for $\bar{u}_T(t)$ is given by $\bar{U}_T(jw)$. Then, we have the following relation

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{U}_T^*(jw) \left( M(w) - \tilde{\nu} I \right) \bar{U}_T(jw) \mathrm{d}w < \ 0.$$

From equations (19) and (20), we obtain that

$$\langle \bar{u}, \bar{y} \rangle_T < \tilde{\nu} \langle \bar{u}, \bar{u} \rangle_T,$$

which is a contradiction in view of (21). Thus, we have $\tilde{\nu} \leq \nu$. This completes the proof. $\square$

**Proof of Lemma 1**

*Proof.* The following analysis focus on the case when $K = 1$ and the result is immediate from the 'scaling property'. The frequency response of system (11) is given by

$$Y(jw) = \frac{ajw + 1}{bjw + 1} e^{-\tau jw}.$$

The passivity index $\nu_F(w)$ at frequency $w$ can be calculated as

$$\begin{aligned} \nu_F(w) &= \frac{(abw^2 + 1)\cos(\tau w) + (a - b) w \sin(\tau w)}{b^2 w^2 + 1} \\ &= \frac{\sqrt{(abw^2 + 1)^2 + (a - b)^2 w^2}}{b^2 w^2 + 1} \cos(\tau w - \alpha(w)), \end{aligned} \tag{22}$$

where $\alpha(w) \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ is given by

$$\tan(\alpha(w)) = \frac{(a - b) w}{abw^2 + 1}.$$

Thus, we obtain for all frequencies $w \in \mathbb{R}$,

$$|\nu_F(w)| \leq \frac{\sqrt{(abw^2 + 1)^2 + (a - b)^2 w^2}}{b^2 w^2 + 1} \triangleq \sqrt{f(w)}.$$

Next, we examine the maximum value of function $f(w) = \frac{A(w)}{B(w)}$ over frequencies $w$, where $A(w) = (abw^2 + 1)^2 + (a - b)^2 w^2 > 0$ and $B(w) = (b^2 w^2 + 1)^2 > 0$. Two cases are possible.

1. If $a < b$, then we have

$$A(w) - B(w) = (a^2 - b^2)w^2(b^2w^2 + 1) \leq 0,$$

and thus $f(w) = \frac{A(w)}{B(w)} \leq 1$. Therefore, we have $|\nu_F(w)| \leq 1$ for all $w$. This implies that for all $w$, we have $\nu_F(w) \geq -1$ and thus $\nu \geq -1$.

2. If $a > b$, then we obtain

$$b^2A(w) - a^2B(w) = (b^2 - a^2)(b^2w^2 + 1) < 0,$$

and thus $f(w) = \frac{A(w)}{B(w)} \leq \frac{a^2}{b^2}$. Therefore, we have $|\nu_F(w)| \leq \frac{a}{b}$ for all $w$. This implies for all $w$, we have $\nu_F(w) \geq -\frac{a}{b}$ and thus $\nu \geq -\frac{a}{b}$.

When $w = 0$, we obtain $\nu_F(0) = 1 > 0$. When $w = \frac{\pi}{\tau}$, we obtain

$$\nu_F(\frac{\pi}{\tau}) = -\frac{ab\pi^2 + \tau^2}{\tau^2 + b^2\pi^2} < 0,$$

and thus the passivity index $\nu \leq \nu_F(\frac{\pi}{\tau}) < 0$. Therefore, for both cases, $\nu < 0$. □

**Proof of Lemma 2**

*Proof.* Consider the case when $K = 1$. The frequency response of (9) is given by

$$H(jw) = \frac{ajw + 1}{(bjw + 1)(cjw + 1)}e^{-\tau jw}.$$

The passivity index $\nu_F(w)$ at frequency $w$ can be calculated as

$$\begin{aligned}\nu_F(w) &= \frac{(ab + ac - bc)w^2 + 1}{(1 + c^2w^2)(1 + b^2w^2)}\cos(\tau w) + \frac{(a - b - c)w - abcw^3}{(1 + c^2w^2)(1 + b^2w^2)}\sin(\tau w) \\ &\triangleq A(w)\cos(\tau w) + B(w)\sin(\tau w) \\ &= \sqrt{A^2(w) + B^2(w)}\cos(\tau w - \beta(w)),\end{aligned}$$

where $\beta(w) \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ is given by

$$\tan\beta(w) = \frac{B(w)}{A(w)} = \frac{(a - b - c)w - abcw^3}{(ab + ac - bc)w^2 + 1}.$$

It is apparent that $\nu_F(0) = 1 > 0$. Next, we show that $\nu_F(w) < 0$ for some $w \in \mathbb{R}$. Two cases are possible.

1. If $a < b$, then $B(w) < 0$ for $w > 0$. Further, we obtain that at frequency $w = \frac{\pi}{2\tau}$, the passivity index is given by

$$\nu(\frac{\pi}{2\tau}) = B(\frac{\pi}{2\tau}) < 0.$$

2. If $a > b$, then $A(w) > 0$ for $w > 0$. Further, we obtain that at frequency $w = \frac{\pi}{\tau}$, the passivity index is given by

$$\nu(\frac{\pi}{\tau}) = -A(\frac{\pi}{\tau}) < 0.$$

Therefore, there exists $w$ such that $\nu_F(w) < 0$ for both cases. Thus, we obtain that

$$\nu \leq \nu_F(w) < 0.$$

This implies that human model given by (9) is non-passive. $\square$

**Proof of Theorem 1**

*Proof.* For notational convenience, we denote $\langle u, y\rangle_T = \int_0^T u^T y \; \mathrm{d}t$. Since $u_0 = m_{11}u + m_{12}y$ and $y_0 = m_{21}u + m_{22}y$, it can be easily shown that

$$\begin{aligned}\langle u_0, y_0\rangle_T \;&= m_{11}m_{21}\langle u, u\rangle_T + m_{12}m_{22}\langle y, y\rangle_T \\ &+ \;(m_{11}m_{22} + m_{12}m_{21})\langle u, y\rangle_T,\end{aligned} \tag{23}$$

$$\langle u_0, u_0\rangle_T \;=\; m_{11}^2\langle u, u\rangle_T + 2m_{11}m_{12}\langle u, y\rangle_T + m_{12}^2\langle y, y\rangle_T, \tag{24}$$

$$\langle y_0, y_0\rangle_T \;=\; m_{21}^2\langle u, u\rangle_T + 2m_{21}m_{22}\langle u, y\rangle_T + m_{22}^2\langle y, y\rangle_T. \tag{25}$$

Since system $G$ is finite gain stable with gain $\gamma$, we have

$$\langle y, y\rangle_T \leq \gamma^2\langle u, u\rangle_T. \tag{26}$$

(i). If $M$ is chosen such that (12) is satisfied, then according to (23) and (26), we have

$$\begin{aligned}\langle u_0, y_0\rangle_T \;&=\; m_{11}^2\langle u, u\rangle_T - m_{22}^2\langle y, y\rangle_T \\ &\geq\; (m_{11}^2 - m_{22}^2\gamma^2)\langle u, u\rangle_T \\ &\geq\; 0.\end{aligned}$$

Therefore, the system $\mathbf{u}_0 \to \mathbf{y}_0$ is passive.

(ii). To find an OFP level of the system $\mathbf{u}_0 \to \mathbf{y}_0$, we use (23), (25) and the following relation

$$\begin{aligned}&\langle u_0, y_0\rangle_T - \rho_0\langle y_0, y_0\rangle_T \\ &= (m_{11}m_{21} - \rho_0 m_{21}^2)\langle u, u\rangle_T + (m_{12}m_{22} - \rho_0 m_{22}^2)\langle y, y\rangle_T \\ &\quad + (m_{11}m_{22} + m_{12}m_{21} - 2\rho_0 m_{21}m_{22})\langle u, y\rangle_T.\end{aligned}$$

Since $2\rho_0 m_{21}m_{22} = m_{11}m_{22} + m_{12}m_{21}$, we have

$$\begin{aligned}&\langle u_0, y_0\rangle_T - \rho_0\langle y_0, y_0\rangle_T \\ &= (m_{11}m_{21} - \rho_0 m_{21}^2)\langle u, u\rangle_T + (m_{12}m_{22} - \rho_0 m_{22}^2)\langle y, y\rangle_T.\end{aligned}$$

If $M$ is chosen such that (13) is satisfied, then we have

$$\begin{aligned} m_{12}m_{22} - \rho_0 m_{22}^2 &= m_{22}^2\left(\frac{m_{12}}{m_{22}} - \rho_0\right) \\ &= \frac{1}{2}m_{22}^2\left(\frac{m_{12}}{m_{22}} - \frac{m_{11}}{m_{21}}\right) \\ &= \frac{1}{2}\frac{m_{22}}{m_{21}}\left(m_{12}m_{21} - m_{11}m_{22}\right) \\ &< 0. \end{aligned}$$

Further, we can derive that $\frac{m_{12}}{m_{22}} - \rho_0 < 0$, then based on the fact that $\frac{m_{11}}{m_{21}} - \rho_0 = \rho_0 - \frac{m_{12}}{m_{22}}$, we have $\frac{m_{11}}{m_{21}} - \rho_0 > 0$. Then, from (26), we can obtain that

$$\begin{aligned} &\langle u_0, y_0\rangle_T - \rho_0\langle y_0, y_0\rangle_T \\ &\geq (m_{11}m_{21} - \rho_0 m_{21}^2)\langle u, u\rangle_T \\ &\quad - \gamma^2(\rho_0 m_{22}^2 - m_{12}m_{22})\langle u, u\rangle_T \\ &= \left[m_{21}^2\left(\frac{m_{11}}{m_{21}} - \rho_0\right) - m_{22}^2\gamma^2(\rho_0 - \frac{m_{12}}{m_{22}})\right]\langle u, u\rangle_T \\ &= \left(\frac{m_{11}}{m_{21}} - \rho_0\right)\left(m_{21}^2 - m_{22}^2\gamma^2\right)\langle u, u\rangle_T. \end{aligned}$$

Then, from the condition $m_{21} \geq m_{22}\gamma > 0$, we can obtain

$$\langle u_0, y_0\rangle_T - \rho_0\langle y_0, y_0\rangle_T \geq 0.$$

Therefore, the system $\mathbf{u}_0 \to \mathbf{y}_0$ has OFP($\rho_0 > 0$).

(iii). To find an IFP level of the system $\mathbf{u}_0 \to \mathbf{y}_0$, we use (23), (24) and the following relation

$$\begin{aligned} &\langle u_0, y_0\rangle_T - \nu_0\langle u_0, u_0\rangle_T \\ &= (m_{11}m_{21} - \nu_0 m_{11}^2)\langle u, u\rangle_T + (m_{12}m_{22} - \nu_0 m_{12}^2)\langle y, y\rangle_T \\ &\quad + (m_{11}m_{22} + m_{12}m_{21} - 2\nu_0 m_{11}m_{12})\langle u, y\rangle_T \end{aligned}$$

Since $2\nu_0 m_{11}m_{12} = m_{12}m_{21} + m_{11}m_{22}$, we have

$$\begin{aligned} &\langle u_0, y_0\rangle_T - \nu_0\langle u_0, u_0\rangle_T \\ &= (m_{11}m_{21} - \nu_0 m_{11}^2)\langle u, u\rangle_T \\ &\quad + (m_{12}m_{22} - \nu_0 m_{12}^2)\langle y, y\rangle_T. \end{aligned}$$

If $M$ is chosen such that (14) is satisfied, then we have

$$\begin{aligned} m_{12}m_{22} - \nu_0 m_{12}^2 &= m_{12}^2\left(\frac{m_{22}}{m_{12}} - \nu_0\right) \\ &= \frac{1}{2}m_{12}^2\left(\frac{m_{22}}{m_{12}} - \frac{m_{21}}{m_{11}}\right) \\ &= \frac{1}{2}\frac{m_{12}}{m_{11}}\left(m_{11}m_{22} - m_{12}m_{21}\right) \\ &< 0. \end{aligned}$$

Further, we can derive that $\frac{m_{22}}{m_{12}} - \nu_0 < 0$, then based on the fact that $\frac{m_{21}}{m_{11}} - \nu_0 = \nu_0 - \frac{m_{22}}{m_{12}}$, we have $\frac{m_{21}}{m_{11}} - \nu_0 > 0$. Then, from (26), we can obtain that

$$\begin{aligned}
&\langle u_0, y_0\rangle_T - \nu_0\langle u_0, u_0\rangle_T \\
&\geq (m_{11}m_{21} - \nu_0 m_{11}^2)\langle u, u\rangle_T \\
&\quad + (m_{12}m_{22} - \nu_0 m_{12}^2)\gamma^2\langle u, u\rangle_T \\
&= \left[m_{11}^2\left(\frac{m_{21}}{m_{11}} - \nu_0\right) - m_{12}^2\gamma^2(\nu_0 - \frac{m_{22}}{m_{12}})\right]\langle u, u\rangle_T \\
&= \left(\frac{m_{21}}{m_{11}} - \nu_0\right)\left(m_{11}^2 - m_{12}^2\gamma^2\right)\langle u, u\rangle_T.
\end{aligned}$$

Then, from the condition $m_{11} \geq m_{12}\gamma > 0$, we can obtain

$$\langle u_0, y_0\rangle_T - \nu_0\langle u_0, u_0\rangle_T \geq 0.$$

Therefore, the system $\mathbf{u}_0 \to \mathbf{y}_0$ has IFP($\nu_0 > 0$).

(iv). We use (23-25) and the following relation

$$\begin{aligned}
&\langle u_0, y_0\rangle_T - \epsilon_0\langle u_0, u_0\rangle_T - \delta_0\langle y_0, y_0\rangle_T \\
&= (m_{11}m_{22} + m_{12}m_{21} - 2\epsilon_0 m_{11}m_{12} \\
&\quad - 2\delta_0 m_{21}m_{22})\langle u, y\rangle_T \\
&\quad + \ (m_{12}m_{22} - \epsilon_0 m_{12}^2 - \delta_0 m_{22}^2)\langle y, y\rangle_T \\
&\quad + \ (m_{11}m_{21} - \epsilon_0 m_{11}^2 - \delta_0 m_{21}^2)\langle u, u\rangle_T.
\end{aligned}$$

Since $m_{12} = 0$ and $\delta_0 = \frac{1}{2}\frac{m_{11}}{m_{21}}$, we have

$$m_{11}m_{22} + m_{12}m_{21} - 2\epsilon_0 m_{11}m_{12} - 2\delta_0 m_{21}m_{22} = 0,$$

so that the following relation holds,

$$\begin{aligned}
&\langle u_0, y_0\rangle_T - \epsilon_0\langle u_0, u_0\rangle_T - \delta_0\langle y_0, y_0\rangle_T \\
&= (\frac{1}{2}m_{11}m_{21} - \epsilon_0 m_{11}^2)\langle u, u\rangle_T - \delta_0 m_{22}^2\langle y, y\rangle_T.
\end{aligned}$$

Then, from (26), we have

$$\begin{aligned}
&\langle u_0, y_0\rangle_T - \epsilon_0\langle u_0, u_0\rangle_T - \delta_0\langle y_0, y_0\rangle_T \\
&\geq (\frac{1}{2}m_{11}m_{21} - \epsilon_0 m_{11}^2 - \delta_0 m_{22}^2\gamma^2)\langle u, u\rangle_T.
\end{aligned}$$

If $\epsilon_0 = \frac{a}{2}\frac{m_{21}}{m_{11}}$, $\delta_0 = \frac{1}{2}\frac{m_{11}}{m_{21}}$ and $(1-a)m_{21}^2 \geq m_{22}^2\gamma^2$, then we have

$$\begin{aligned}
&\langle u_0, y_0\rangle_T - \epsilon_0\langle u_0, u_0\rangle_T - \delta_0\langle y_0, y_0\rangle_T \\
&\geq (\frac{1-a}{2}m_{11}m_{21} - \delta_0(1-a)m_{21}^2)\langle u, u\rangle_T \\
&\geq 0.
\end{aligned}$$

Therefore, the system $\mathbf{u}_0 \to \mathbf{y}_0$ has IF-OFP $\epsilon_0 > 0$ and $\delta_0 > 0$. This completes the proof. $\square$

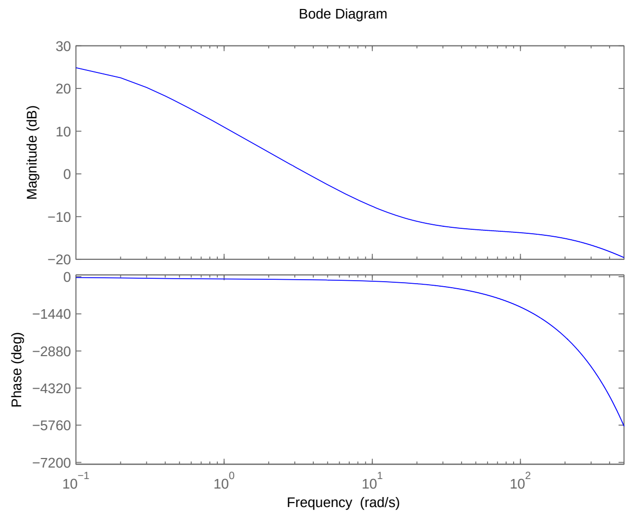


Figure 13: Bode plot of the human model in Example 1.

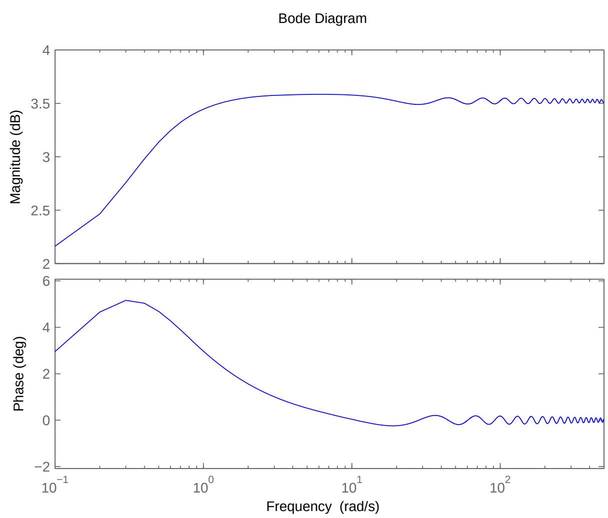


Figure 14: Bode plot of the human model with transformation $M$ in Example 1.

**The Role of the Transformation $M$ in Example 1**

Consider the human model $G_2$ given by (10) and the transformation $M$ with $m_{11} = 22, m_{12} = 1, m_{21} = 33, m_{22} = 1$ that we use in Example 1. The Bode plot of $G_2$ is given in Fig. 13, from which we can say that $G_2$ has a large phase lag when frequency is high. With transformation $M$ given above, the phase lag caused by delay is compensated, as shown in Fig. 14. Thus, the 'transformation' $M$ can be seen as a 'phase lead compensator'.

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