The Stable Transformation Path*

Francisco J. Buera[†] Joseph Kaboski[‡] Martí Mestieri[§]
Daniel G. O'Connor[¶]

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Abstract

Many growth models lack balanced growth paths (BGPs). Instead, the sectoral, productivity, and capital dynamics change drastically as the economy develops. We define the Stable Transformation Path (STraP), a generalization of the BGP to non-stationary models, for a wide class of models and prove its existence and uniqueness. We use the STraP to evaluate the implications of benchmark models of structural transformation. Secular structural change can account for a quarter of growth in miracle economies, but it fails to explain the growth experience in the early industrial period.

Keywords: Non-balanced Growth, Structural Change, Investment Dynamics

JEL codes: E2, O1, O4

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[†]Washington University in St. Louis. Email: fjbuera@wustl.edu
[‡]University of Notre Dame. Email: jkaboski@nd.edu
[§]UPF-BSE-CREi, Federal Reserve Bank of Chicago. Email: mestieri.marti@gmail.com
[¶]Massachusetts Institute of Technology. Email: doconn@mit.edu

1 Introduction

Economic development is a non-stationary phenomenon. The emergence of modern economic growth in Europe in the XIX century and the development of growth miracles in more recent times are prominent examples of growing economies exhibiting dramatic changes in their structure of production and consumption. As a consequence, growth theories of these phenomena do not feature balanced growth paths with a turnpike property, i.e., a balanced growth path to which transition paths quickly converge. Instead, these models only feature asymptotic balanced growth paths (ABGP) in the far future when structural transformation stops (Acemoglu and Guerrieri, 2008; Hansen and Prescott, 2002; Herrendorf et al., 2021). A natural question arises: what is a useful generalization of a balanced growth path to capture turnpike properties of non-stationary models? In particular, is there a stable, non-balanced path that describes the medium-term development dynamics independent of initial conditions? Is it possible to separate the secular implications of such models from the short-run dynamics determined by initial conditions?

In this paper, we propose a generalization of a stable balanced growth path which preserves the turnpike property, a non-stationary path to which transitional dynamics quickly converge. The Stable Transformation Path (STraP) gives the "natural" development path independent of any initial conditions. This concept allows researchers to separate the secular implications of the productivity process from the short-run implications that come from Neoclassical capital accumulation dynamics. Given its independence from initial conditions, the STraP also opens up the possibility of analyzing the implications of theories of development backward in time when information about initial conditions is unavailable.

In particular, the STraP is defined as the transitional path connecting a hypothetical initial ABGP with the long-run final ABGP.[1] The use of an initial ABGP, a hypothetical one that is approached as time is run backward towards $-\infty$, ensures that the STraP is independent of initial conditions. The final ABGP is the stationary limit

[1]The definition presumes that the model studied is asymptotically stationary as times runs backward and forward towards $\pm\infty$, i.e., the model features ABGPs in the backward and forward limits. For example, the economy converges to an initial (final) ABGP with only agriculture (service) in models of structural change driven a differential productivity growth (García-Santana et al., 2021; Gollin et al., 2004) or to an initial (final) ABGP where production features a low (high) labor intensity in models with factor intensity differences (Acemoglu and Guerrieri, 2008; Hansen and Prescott, 2002).

towards which all transitional paths converge in the long run. Importantly, transition paths quickly approach the STraP in the medium term. Therefore, the STraP summarizes the medium-term, non-stationary dynamics. We show that in a broad class of structural transformation models, one needs to make very special assumptions to obtain a BGP, but a unique STraP always exists.

To illustrate the usefulness of the STraP, we calibrate a three-sector extension of the benchmark model in Herrendorf et al. (2021). We use it to decompose the sources of growth in a miracle economy and to uncover four new dynamic implications of the model that are independent of initial capital.

In the first exercise, we examine Thailand, whose growth exceeded that of the United States by 2 percentage points in the post-1950 period. (In the US, the STraP reproduces the US growth nearly identically indicating that Neoclassical convergence was negligible.) We break down Thailand's excess growth into growth driven by structural transformation (that is, the STraP medium-term dynamics driven by the standard productivity process), excess capital accumulation (driven by initial capital conditions), and excess productivity growth relative to the US. Of the 2 percentage points of excess growth, 0.5 percentage points were the result of time-varying dynamics driven by structural transformation and the relatively large agricultural and industrial sectors. Another 0.2 percentage points were driven by Neoclassical convergence from a below-STraP initial capital stock, and the remaining 1.3 percentage points were from Thailand's higher productivity growth.

We also use the STraP to discover four new implications of the benchmark structural change model (relating to technologies, preferences, and productivity processes) that are independent of initial capital. First, and perhaps unsurprisingly, the canonical model calibrated to the recent US qualitatively matches patterns for the US over the past century. Second, the implications of this productivity process for growth and investment depend markedly on the parameter assumptions of the model. For example, seemingly innocuous parameter choices, e.g., log preferences versus a more typical calibration of the intertemporal elasticity or whether all investment is being produced in industry can impact whether growth and investment rise or fall over development. Third, the secular dynamics of the recent US productivity process can quantitatively explain the postwar development patterns of countries across the wide spectrum of development, including convergence. Fourth, this productivity process *cannot* explain the longer-run (two-century) patterns of advanced economies. Hence, fitting these historical data under stable preferences will require a different productivity process or technology. Importantly, these four evaluations of the model independent of initial capital are only possible with the STraP. This is clearest for our long-run historical analysis, for which income data are available but no capital data are available.

1.1 Related Literature

The paper builds on and relates to an existing literature on structural transformation. There are some earlier analyses of non-stationary transformation paths from stable equilibria to asymptotic BGPs. Hansen and Prescott (2002) and Gollin et al. (2004) analyze transitions from stagnant or slow-growing agricultural economies to modern growth. While these papers only study transitional dynamics given specific initial conditions, we can show that the dynamic paths in these models are dominated by the more general, medium-term dynamics along the STraP, rather than Neoclassical convergence dynamics. Extending this work, a contribution of our paper is to define the STraP in a general class of environments and use it to distinguish short-run Neoclassical dynamics from medium-term dynamics due to structural change.

Our contribution relates to the earlier literature of turnpike theorems, which can be divided into two types of contributions. The first involved optimal growth in endogenous growth models of capital accumulation, often when looking at the mixes of heterogeneous capitals, and using modeling assumptions that are now atypical, e.g., no discounting, finite time, "optimal" paths between arbitrary points A and B, and unbounded labor supply. The finite time horizon of these models emphasized that turnpikes would be reached relatively quickly and would describe the dynamics of the model for most of its intermediate time, which is consistent with our simulations of relatively quick convergence to the STraP.[2] The second subliterature simply described the convergence of competitive equilibria to BGPs from initial capital stocks using more standard models. Both turnpike concepts have the common feature that optimal paths from various initial conditions converge, and this is a shared feature of the STraP. The STraP is indeed a feature of a model with turnpike properties, but it goes beyond declaring convergence: it isolates the time-varying path to which competitive

[2]In turn, this result is reminiscent of the fast convergence of the Neoclassical growth model to the balanced growth path (King and Rebelo, 1993).

equilibria converge in nonstationary models with standard assumptions.[3]

The model in our application is motivated by the existing literature in incorporating biased productivity growth as a driving force for structural transformation (Ngai and Pissarides, 2007), investment being intensive in manufacturing (García-Santana et al., 2021), and structural transformation in investment (Herrendorf et al., 2021). Ngai and Pissarides (2007) is part of a larger literature that models balanced growth at the same time as structural transformation.[4] Relative to this literature, we analyze the dynamics of changing growth rates, show that parameter values matter for these implications, and empirically demonstrate that growth and capital dynamics are not balanced. García-Santana et al. (2021) and Herrendorf et al. (2021) emphasize the important role for investment in structural transformation.[5] We confirm their findings and show that these implications are driven by medium-term dynamics rather than initial conditions.

Finally, because the model in our application leads to a declining relative price of capital, it relates to the concept of investment-specific technical change (ISTC). *Qualitatively*, while a standard ISTC model with constant technical change (e.g., Greenwood et al., 1997) leads to growth in the ratio of real capital to real output, it also yields a BGP and therefore predicts no medium-run growth dynamics. *Quantitatively*, our analysis combines both ISTC and structural transformation, a distinct, though complementary, explanation for the decline in the price of investment. Along the STraP, the forces driving structural transformation, i.e., sectoral productivity growth, explain nearly all of the growth (as noted by Herrendorf et al., 2021) and roughly half of the growth in the capital-output ratio and decline in the relative price of investment. The recent work of Caunedo et al. (2023) and especially Caunedo and Keller (2023) makes a related but distinct point that much of the driving force behind structural transformation, which we capture as biased productivity growth, is driven by differential rates of capital-embodied technical change across sectors.

[3] McKenzie provides both verbal (McKenzie, 1998) and mathematical reviews (McKenzie, 1976) of the turnpike literature. In the 1998 review, he argued that models with balanced growth paths are "virtually impossible to believe in" but, in describing results for bounded class of nonstationary models, he anticipates the STraP writing "optimal paths from different initial stocks have a tendency to converge, whatever their shapes may be." (p. 6).

[4] Other important contributions include Kongsamut et al. (2001) and Acemoglu and Guerrieri (2008). Buera and Kaboski (2009) show that the assumptions used effectively divorce growth from structural transformation, making the two phenomena orthogonal.

[5] Ju et al. (2015) makes a related point in a different model setup.

2 Model

In this section, we present a model of structural transformation that nests many of the recent developments in the literature. This will allow us to show that, generically, balanced growth paths do not exist, but the STraP does.

For the sake of concreteness, the economy consists of three sectors: agriculture, manufacturing, and services—but the model can be extended to an arbitrary number of sectors. Sectors may differ in their productivity growth rates as in Ngai and Pissarides (2007) and their capital intensity as in Acemoglu and Guerrieri (2008). Moreover, structural transformation can take place in both investment and consumption. We follow Herrendorf et al. (2021) in modeling the investment final good as a constant elasticity of substitution (CES) aggregate over the three sectoral goods. Finally, consumers have nonhomothetic CES preferences over the three sectors following Comin et al. (2021).[6]

2.1 Environment

A representative household starting at time τ has standard preferences

$$\mathcal{W}(t) = \int_{\tau}^{\infty} e^{-\rho(t-\tau)} \frac{C(t)^{1-\theta} - 1}{1-\theta} dt,$$

where $C(t)$ is the consumption aggregate at time t, ρ is the discount rate, and θ is a parameter controlling the elasticity of intertemporal substitution.

The consumption aggregate is implicitly defined by

$$1 = \sum_{j=a,m,s} \omega_{cj}^{\frac{1}{\sigma_c}} \left[\frac{C_j(t)}{C(t)^{\varepsilon_j}} \right]^{\frac{\sigma_c - 1}{\sigma_c}},$$

where $C_j(t)$ is the value added consumption of sector j. We assume that $\omega_{cj} > 0$ and that $\varepsilon_j > 0$. We normalize the CES weights, $\sum_{j=a,m,s} \omega_{cj} = 1$. Consistent with standard structural change patterns, we further assume that sectors are gross complements,

[6] It is possible to use other nonhomothetic aggregators, e.g., Boppart (2014); Alder et al. (2019). Nonhomothetic CES preferences defined as in Comin et al. (2021) have the advantage of being well-defined for any level of expenditure and prices, which simplifies the exposition of the STraP.

i.e., the price elasticity satisfies $\sigma_c < 1$. The parameter ε_j introduces nonhomotheticity in the aggregator. If all ε_j are the same, e.g., $\varepsilon_j = 1$, we obtain the standard homothetic CES aggregator. When they differ, sectors with a higher value of ε_j are more expenditure-elastic. To match qualitative patterns of structural change, we assume that $\varepsilon_s > \varepsilon_m > \varepsilon_a$.

Capital depreciates at a rate $\delta \in [0, 1]$, but can be accumulated through investment, $X(t)$:

$$\dot{K}(t) = X(t) - \delta K(t). \tag{1}$$

Investment is a homothetic CES aggregate of agriculture, $X_a(t)$, manufacturing, $X_m(t)$, and services, $X_s(t)$:

$$X(t) = A_x(t) \left[\sum_{j=a,m,s} \omega_{xj}^{\frac{1}{\sigma_x}} X_j(t)^{\frac{\sigma_x - 1}{\sigma_x}} \right]^{\frac{\sigma_x}{\sigma_x - 1}}. \tag{2}$$

We normalize the weights again to sum to one, $\sum_{j=a,m,s} \omega_{xj} = 1$, but they are specific to the investment sector. Note also that the investment aggregator experiences sector-neutral technological change through $A_x(t)$, which we assume occurs at a constant rate:

$$\dot{A}_x(t) = \gamma_x A_x(t)$$

with $\gamma_x > 0$.

Finally, we allow the elasticity of substitution in the investment aggregator, σ_x, to differ from that in the consumption aggregator. However, consistent with standard structural change patterns, we again assume that sectors are gross complements in investment, i.e., $\sigma_x < 1$.

Each sector $j \in \{a, m, s\}$ produces value added combining capital and labor using a Cobb-Douglas technology with sector-specific factor output elasticities, α_j, and productivity parameters, $A_j(t)$:

$$Y_j(t) = A_j(t) K_j(t)^{\alpha_j} L_j(t)^{1-\alpha_j}. \tag{3}$$

These productivities grow at constant rates:

$$\dot{A}_j(t) = \gamma_j A_j(t),$$

where the growth rates are also sector-specific and, consistent with standard structural change patterns, ordered as follows: $\gamma_a > \gamma_m > \gamma_s > 0$ (see, e.g., Duarte and Restuccia, 2010). Factor shares are ordered $\alpha_a > \alpha_m \geq \alpha_s$ following Valentinyi and Herrendorf (2008).

Labor is inelastically supplied. Then feasibility requires that the labor and capital used by each sector be less than the aggregate supply:

$$\sum_{j=a,m,s} L_j(t) \leq L, \tag{4}$$

and

$$\sum_{j=a,m,s} K_j(t) \leq K(t). \tag{5}$$

Likewise, for any sector $j = a, m, s$, sectoral consumption and investment cannot exceed sectoral output,

$$C_j(t) + X_j(t) \leq Y_j(t). \tag{6}$$

2.2 Market Structure and Equilibrium

In this section, we describe the market structure and define the competitive equilibrium.

Representative Household The household inelastically supplies labor and owns capital, which it can accumulate through investment. It maximizes utility taking wages $W(t)$, the rental rate of capital $R(t)$, the price of each sector $P_j(t)$, and the price of investment $P_x(t)$ as given, subject to a budget constraint:

$$\begin{aligned} \max_{C(t),C_j(t),K(t),X(t),B(t)} \int_\tau^\infty & e^{-\rho(t-\tau)} \frac{C(t)^{1-\theta}-1}{1-\theta} dt \\ s.t. \qquad & 1 = \sum_{j=a,m,s} \omega_{cj}^{\frac{1}{\sigma_c}} \left[\frac{C_j(t)}{C(t)^{\varepsilon_j}}\right]^{\frac{\sigma_c-1}{\sigma_c}}, \\ & \sum_{j=a,m,s} P_j(t)C_j(t) + P_x(t)X(t) \leq W(t)L + R(t)K(t), \\ & \dot{K}(t) = X(t) - \delta K(t). \end{aligned} \tag{7}$$

Firms Competitive representative firms in each sector maximize profits, taking prices, wages, and the rental rate of capital as given. The standard cost minimization problem for a firm with Cobb-Douglas technology implies that

$$P_j(t) = \frac{1}{A_j(t)} \left(\frac{R(t)}{\alpha_j}\right)^{\alpha_j} \left(\frac{W(t)}{1-\alpha_j}\right)^{1-\alpha_j}. \tag{8}$$

And factor demand is

$$\begin{aligned} R(t)K_j(t) &= \alpha_j P_j(t) Y_j(t) \\ W(t)L_j(t) &= (1-\alpha_j) P_j(t) Y_j(t). \end{aligned} \tag{9}$$

A competitive firm in the investment sector also maximizes profits, taking as given the price of the sectoral goods and the final investment good. CES cost minimization implies

$$P_x(t) = \frac{1}{A_x(t)} \left[\sum_{j=a,m,s} \omega_{xj} P_j(t)^{1-\sigma_x}\right]^{\frac{1}{1-\sigma_x}}. \tag{10}$$

And demand is

$$P_j(t)X_j(t) = \omega_{xj} \left(\frac{P_j(t)}{A_x(t)P_x(t)}\right)^{1-\sigma_x} P_x(t)X(t). \tag{11}$$

We are now ready to define a competitive equilibrium.

Definition 1. *Given an initial state consisting of $K(\tau)$, $A_x(\tau)$, and $\{A_j(\tau)\}_{j=a,m,s}$, a* ***competitive equilibrium*** *is:*

- *an allocation $C(t)$, $K(t)$, $X(t)$, $\{C_j(t), X_j(t), K_j(t), L_j(t)\}_{j=a,m,s}$ and*
- *prices $P_x(t)$, $W(t)$, $R(t)$, and $\{P_j(t)\}_{j=a,m,s}$,*

for $t \geq \tau$ in which

- *households maximize utility taking prices, wages, and rental rates as given, subject to the budget constraint and capital accumulation, as shown in equation (7);*
- *representative firms in the agriculture, manufacturing, services, and investment sectors maximize profits taking prices as given, as shown in equations (8), (9), (10), and (11); and*

- *all markets clear, as shown in equations (4), (5), and (6).*

2.3 (Lack of) BGP along the Development Path

We now turn to explaining why this model of structural change generically does not have a BGP. We start by giving the definition of a BGP that we will be using.

Definition 2. *Given some initial productivity vector $A_x(0)$ and $\{A_j(0)\}_{j=a,m,s}$, a* ***Balanced Growth Path*** *is*

- *an allocation, $C(t)$, $K(t)$, $X(t)$, $\{C_j(t), X_j(t), K_j(t), L_j(t)\}_{j=a,m,s}$, and*
- *prices $P_x(t)$, $W(t)$, $R(t)$, and $\{P_j(t)\}_{j=a,m,s}$,*

for $t \in (-\infty, \infty)$ in which:

- *starting at any time τ, the subsequent path is a competitive equilibrium starting with initial capital $K(\tau)$, and*
- *$K(t)$ and $\frac{P_c(t)C(t)}{P_x(t)}$ grow at a constant rate.*

This is sometimes called an aggregate balanced growth path because the aggregates are growing at a constant rate while the sectoral variables can grow at varying rates. At least theoretically, this allows for structural change to happen while the economy is growing, but it severely limits the form it can take, as we show below. Another important aspect of this definition: we impose that consumption expenditures (normalized by the price of investment) grow at a constant rate rather than consumption itself. This is standard in the literature and allows for more interesting models to have balanced growth paths.

We start by considering the capital accumulation equation. From the budget constraint, $R(t)K(t) + W(t)L = P_x(t)X(t) + P_c(t)C(t)$. Therefore,

$$\begin{aligned}\frac{\dot{K}(t)}{K(t)} &= \frac{X(t)}{K(t)} - \delta \\ &= \frac{R(t)}{P_x(t)} + \frac{W(t)L}{P_x(t)K(t)} - \frac{P_c(t)C(t)}{P_x(t)K(t)} - \delta.\end{aligned}$$

Generically, each of these terms will need to be constant in any balanced growth path. So immediately we obtain that not only are $\frac{P_c(t)C(t)}{P_x(t)}$ and $K(t)$ growing at a constant rate, but that they are growing at the same rate. Wages normalized by the price of investment and the rental rate of capital normalized by the price of investment will also have to grow at that same rate.

Next, we further characterize the solution to the household utility maximization problem (7). The problem can be broken up into two parts: an intertemporal problem and an intratemporal problem. In the intertemporal problem, the household decides how much money to spend on consumption and how much to spend on investment. Then, in an intratemporal problem, the household decides how to divide consumption expenditures across the three sectors. See Comin et al. (2021) for the details.

The implied demand is

$$P_j(t)C_j(t) = \omega_{cj}\left(\frac{P_j(t)}{P_c(t)}\right)^{1-\sigma_c} C(t)^{(\varepsilon_j-1)(1-\sigma_c)}P_c(t)C(t), \tag{12}$$

where $P_c(t)$ is the total spending on consumption divided by the amount of the aggregate consumed, i.e., $P_c(t) = \frac{\sum P_j(t)C_j(t)}{C(t)}$. It takes the form

$$P_c(t) = \left[\sum_{j=a,m,s} \omega_{cj} P_j(t)^{1-\sigma_c} C(t)^{(\varepsilon_j-1)(1-\sigma_c)}\right]^{\frac{1}{1-\sigma_c}}. \tag{13}$$

When $\varepsilon_j = 1$ for all j, the price index of the consumption good depends only on sectoral prices and taste parameters $\{\omega_{cj}\}$, but when preferences are nonhomothetic the price index also depends on the level of utility.

More important for our purposes is the Euler equation, which we will transform to be in terms of expenditures on consumption in units of investment $\tilde{C}(t) = \frac{P_c(t)C(t)}{P_x(t)}$:

$$\theta\frac{\dot{\tilde{C}}(t)}{\tilde{C}(t)} = \underbrace{\frac{R(t)}{P_x(t)}}_{(i)} - \delta - \rho + \underbrace{(1-\theta)\left(\frac{\dot{P}_x(t)}{P_x(t)} - \frac{\dot{P}_c(t)}{P_c(t)}\right)}_{(ii)} - \underbrace{\frac{\dot{\overline{\varepsilon}}(t)}{\overline{\varepsilon}(t)}}_{(iii)}, \tag{14}$$

where $\overline{\varepsilon}(t) = \sum_j \varepsilon_j P_j(t)C_j(t)/(\sum_i P_i(t)C_i(t))$ is the average of the income elasticity parameters, using consumption shares as weights. In any BGP, consumption expenditures need to grow at a constant rate, so the right-hand side must be constant.

This Euler equation differs from the usual one-sector model in three important ways. Term (i) has the rental rate of capital that typically arises in a single-sector Euler equation but is here normalized by the price of capital. The term captures the flow rate of return on investment. If this is not constant, then the household would want to change the time profile of investment and consumption growth, and it will be generically impossible to have a BGP.

Term (ii) captures the capital gain (or loss) on investment that comes from the changing price of investment goods relative to consumption goods. The 1 in $(1-\theta)$ reflects this return. The $-\theta$ arises because we measure the consumption growth on the left-side in units of investment. If the intertemporal elasticity of substitution $(1/\theta)$ is greater than 1, the qualitative effect of changes in relative prices on expenditure growth in units of investment is dominated by the change in real consumption growth rather than the direct effect of relative prices, and the reverse is true when $1/\theta < 1$. Only in the knife-edge case of $\theta = 1$, when expenditure growth does not respond to relative price changes, will a BGP be possible when the growth rate of relative prices is time-varying.

The final term (iii) captures how heterogeneity in income elasticities across goods alters the effective intertemporal elasticity of substitution and the effective discount factor. Intuitively, nonhomotheticicities imply that the optimal composition of the household consumption basket depends on the level of expenditures, and the household internalizes how this impacts the growth rate of the price of marginal consumption when choosing its growth rate of expenditures. Mathematically, these impacts depend on the distribution of income elasticities of goods in the consumption basket, weighted by the consumption share of the sectors and are, in general, time varying.[7]

[7] In particular, rearranging the nonhomothetic correction term, the Euler equation can be written as

$$\theta\left(t\right)\frac{\dot{\tilde{C}}}{\tilde{C}} = \frac{R}{P_x} + \left(1-\theta\left(t\right)\right)\left(\frac{\dot{P}_x}{P_x} - \frac{\dot{P}_c}{P_c}\right) - \delta - \rho\left(t\right)$$

where $\theta(t) = \theta + (1-\sigma)\,\bar{\varepsilon}\ \mathrm{var}_\chi\left(\frac{\varepsilon_i}{\bar{\varepsilon}}\right)$ and $\rho(t) = \rho + (1-\sigma)\,\mathrm{cov}_\chi\left(\frac{\varepsilon_i}{\bar{\varepsilon}}, \frac{\dot{P}_i}{P_i}\right)$ where the variance and covariance terms are computed using expenditure shares χ as weights. Notice that, provided that the consumption of individual sectors are complements, $\sigma_c < 1$, the effective intertemporal elasticity of substitution is decreasing in the dispersion of income elasticities across sectors, and the effective discount factor is increasing in the covariance between the income elasticity and the (log) change in price of the sector.

If a good has a high income elasticity, households continue wanting more of it even if they are rich. Therefore, other things being equal, a household allows more substitution across time. If this is changing through the development process, there cannot be a balanced growth path.[8]

In what follows, we show precisely that the realistic features of structural change that we incorporate into our model will make at least one of these three terms vary. We do so constructively, starting from the Ngai and Pissarides (2007) model, which features a BGP and structural change.

2.3.1 The Balanced Growth Path Benchmark

Ngai and Pissarides (2007) developed a special case of the model that can deliver the BGP. They assume that factor shares are homogeneous $\alpha = \alpha_j$ for all j; preferences are homothetic, $\varepsilon_j = 1$ for all j; the investment aggregator only uses manufacturing as an input, $\omega_{xm} = 1$; and the intertemporal preferences are logarithmic, $\theta = 1$.

Proposition 1 (Ngai-Pissarides)**.** *If $\alpha = \alpha_j$, $\varepsilon_j = 1$, $\omega_{mx} = 1$, and $\theta = 1$, there is a BGP.*

Proof. Under these assumptions, the proof follows from Ngai and Pissarides (2007). □

We give an informal discussion of the argument for the existence of BGP here. As we have discussed, to have a BGP, the right-hand side of the Euler equation, (14), must be constant, and investment needs to grow at the same rate as capital so that the capital accumulation equation, (1), is satisfied.

The $\theta = 1$ restriction immediately implies that term (ii) in (14), $(1-\theta)\left(\frac{\dot{P}_x(t)}{P_x(t)} - \frac{\dot{P}_c(t)}{P_c(t)}\right)$, is zero. Furthermore, assuming that utility is homothetic implies that term (iii), $\frac{\dot{\bar{\varepsilon}}(t)}{\bar{\varepsilon}(t)}$, is zero as well.

Verifying that term (i), $\frac{R(t)}{P_x(t)}$, can be constant and investment can grow at a constant rate is the only non-trivial part. Intuitively, because investment only uses one sector, its productivity grows at a constant rate. Furthermore, because all sectors have the

[8]There are nonhomothetic demand systems that admit a balanced growth path absent other mechanisms (see Kongsamut et al., 2001; Boppart, 2014; Alder et al., 2019, including nonhomothetic CES with a continuum of sectors, Bohr et al., 2023).

same capital intensity, the capital used in investment will also grow at a constant rate so that the real return to capital will remain constant.

2.3.2 Breaking the BGP

Much of the recent literature has moved away from the restrictive Ngai-Pissarides assumptions because they are inconsistent with empirical reality. In this section, we summarize how these changes preclude a BGP.

Multiple Sectors in the Investment Sector Suppose we depart from the Ngai-Pissarides benchmark by allowing investment in services to match the structural change in investment shown in Herrendorf et al. (2021). They show how one can solve for an aggregate production function for the investment sector in terms of the total capital and labor embodied in the value-added aggregated into investment:

$$X(t) = \mathcal{A}_x(t) K_x(t)^{\alpha} L_x(t)^{1-\alpha},$$

where

$$\mathcal{A}_x(t) = A_x(t) \left[\sum_{j=a,m,s} \omega_{xj} A_j(t)^{\sigma_x - 1} \right]^{\frac{1}{\sigma_x - 1}}. \tag{15}$$

Herrendorf et al. (2021) refer to $\mathcal{A}_x(t)$ as *effective* productivity because it includes not only the direct productivity of the aggregator, but also the productivities in producing the different sector j value-added components from labor. From this, one can calculate the rental rate of capital in units of the investment good:

$$\frac{R(t)}{P_x(t)} = \alpha \mathcal{A}_x(t) \left(\frac{K(t)}{L} \right)^{\alpha - 1}.$$

Recall from our discussion of the Euler equation (14) that term (i), $\frac{R(t)}{P_x(t)}$, must be constant for a BGP to exist. Since $K(t)$ must grow at a constant rate in any BGP, it follows that $\mathcal{A}_x(t)$ must also grow at a constant rate. However, equation (15) shows that this effective productivity of investment, like the price of investment, is subject to the changing composition of value added, so it will not generically grow at a constant rate—let alone under our assumption of constant sectoral total factor productivity

(TFP) growth. Thus, the presence of structural change in the investment aggregator leads to the lack of a BGP (even in the case of log intertemporal preferences).

Non-unitary Intertemporal Elasticity of Substitution, θ Consider now departing from the Ngai-Pissarides BGP benchmark by allowing an intertemporal elasticity of substitution different from one, $\theta \neq 1$. In this case, term (ii) in the Euler equation (14) does not disappear. From (10) and (13), we have that the relative price of investment to consumption is

$$\frac{P_x(t)}{P_c(t)} = \frac{1}{A_x(t)} \frac{\left[\sum_j \omega_{xj} P_j(t)^{1-\sigma_x}\right]^{\frac{1}{1-\sigma_x}}}{\left[\sum_j \omega_{cj} P_j(t)^{1-\sigma_c}\right]^{\frac{1}{1-\sigma_c}}}.$$

Since we are allowing for different elasticities of substitution in the two aggregators, σ_x and σ_c, and also different weights, ω_{cj} and ω_{xj}, structural change takes place at different rates in these two sectors. These different rates of structural change imply that the relative price of investment to consumption does not grow at a constant rate (unless all sector prices are growing at the same rate), precluding the existence of a BGP.

Heterogeneous Factor Shares, α_i Suppose instead we depart from the Ngai-Pissarides BGP benchmark by allowing heterogeneous factor intensities in the value-added production functions, as in Acemoglu and Guerrieri (2008). Acemoglu and Guerrieri have already shown that the setup does not have a BGP. To see this in the context of our model, recall that $R(t)/P_x(t)$ must be constant. Capital demand in the manufacturing sector implies that $\alpha_m A_x(t) A_m(t) = \frac{R(t)}{P_x(t)} K_m(t)^{1-\alpha_m}$. Therefore, $K_m(t)^{1-\alpha_m}$ needs to grow at the same rate as $A_x(t)A_m(t)$. That is to say, capital used in manufacturing needs to grow at a constant rate. At the same time, in any BGP, aggregate capital needs to grow at a constant rate. This is not generically possible unless capital grows at the same rate in all sectors. However, capital growing at the same rate in all sectors violates producers' first-order conditions because sectors have different factor shares.

Nonhomotheticity, ε_i Finally, suppose we want to include nonhomothetic preferences to the Ngai-Pissarides BGP benchmark following Comin et al. (2021). As the economy grows and consumption is reallocated, e.g., from agriculture towards manufacturing and services, we have that expenditure shares in consumption change over time. As a result, the nonhomotheticity correction $\frac{\dot{\bar{\varepsilon}}(t)}{\bar{\varepsilon}(t)}$ appearing in the Euler equation (14) is going to be non-zero as long as the ε_j are different across sectors. In particular, the aggregate Euler equation would feature time varying effective intertemporal elasticity of substitution and discount rate. Therefore, a BGP cannot exist.

2.3.3 Why Do We Care?

Why do we care that the structural transformation models that can capture important development facts do not have a balanced growth path? As discussed in the introduction, without a balanced growth path, there is no clear definition of what a natural level of capital looks like. Although we can simulate the model forward from any initial capital stock to obtain predictions, we cannot know the role played by the initial capital stock relative to other model elements. In particular, we cannot separate the Neoclassical capital convergence dynamics from the dynamics implied by the rich elements of structural transformation models that we have discussed above.

This leaves us incapable of asking certain questions. For instance, what are the implications of our models for the far past when we do not have capital data? Or what is the contribution of structural change versus excess capital accumulation for growth? It also leaves us with an incomplete understanding of the implications of our model assumptions, since the dynamics obtained from a forward simulation intertwine the model structure with the choice of the initial capital stock. The STraP that we introduce next is key to providing answers to these questions.

3 The STraP

In this section, we define the Stable Transformation Path (STraP), which generalizes the BGP to a broader class of structural change models. We give the conditions for existence and uniqueness and sketch out an intuition for the proof. The actual proof itself is technical and relegated to the Online Appendix. Finally, we close this section with an example of a STraP that is not a BGP that can be solved in closed form to give some intuition.

3.1 Definition of the Strap

Most structural change models feature an asymptotic balanced growth path in the far future when the entire economy is taken over by the asymptotically dominant sector, e.g., the service sector in our model application or the most capital-intensive sector in Hansen and Prescott (2002) and Acemoglu and Guerrieri (2008). This feature plays an important role in computational and analytic analysis. Analytically, one can linearize the economy around that balanced growth path to see how the economy would change as it completes its structural transformation. Furthermore, that balanced growth path serves as the end value of any shooting algorithm when computing the equilibrium path.

Less emphasized is the existence of an asymptotic balanced growth path in the far past, which in our model application corresponds to the entire economy being in agriculture. In that limit, there is no structural change so that the relative price of consumption and investment grows at a constant rate, the rental rate of capital is proportional to the cost of investments, and the non-homothetic preferences do not matter. Therefore, the Euler equation (14) and the law of motion for capital (1) admit a balanced growth path asymptotically.

The STraP is the path that connects these two asymptotic balanced growth paths while satisfying the equilibrium conditions.

Definition 3. *Given a benchmark productivity vector $A_x(0)$ and $\{A_j(0)\}_{j=a,m,s}$, a* ***Stable Transformation Path*** *for the model is*

- *an allocation, $C(t)$, $K(t)$, $X(t)$, $\{C_j(t), X_j(t), K_j(t), L_j(t)\}_{j=a,m,s}$, and*
- *prices $P_x(t)$, $W(t)$, $R(t)$, and $\{P_j(t)\}_{j=a,m,s}$,*

for $t \in (-\infty, \infty)$ in which:

- *starting at any time τ, the subsequent path is a competitive equilibrium starting with initial capital $K(\tau)$, and*
- *in the limit as $t \to \pm\infty$, $K(t)$ and $\frac{P_C(t)C(t)}{P_x(t)}$ grow at a constant rate.*

One important feature of the STraP is that it does not depend on an arbitrary initial capital like a competitive equilibrium. Instead, for each time t, the STraP goes through a particular capital level, which we will call the natural level of capital. In this way, the STraP is a strict generalization of a balanced growth path. The balanced growth path also gives the natural level of capital for the economy. And starting from any initial conditions, the economy will converge to the BGP. The only difference is that the STraP level of capital could be time varying.

The STraP also generalizes the BGP mathematically. Instead of requiring that $K(t)$ and $\frac{P_c(t)C(t)}{P_x(t)}$ grow at a constant rate for all time, we only require that they grow at a constant rate asymptotically. This observation leads immediately to our first result about the STraP.

Proposition 2. *Any balanced growth path is a STraP.*

This result trivially establishes the existence of the STraP for models that feature a true BGP. The next section analyzes the existence and uniqueness of the STraP for more general models.

3.2 Existence and Uniqueness of the STraP

In this subsection we turn to proving that the STraP exists and is unique in the model presented above. The first step is similar to the first step in finding a balanced growth path: we need to find a function $\mathcal{A}_k(t)$ that can serve to de-trend capital and consumption expenditures. The main difference here is that when a model has a balanced growth path, the economy grows at a constant rate. By contrast, the $-\infty$ agriculture balanced growth path could have a very different growth rate than the $+\infty$ services balanced growth path. Therefore, $\mathcal{A}_k(t)$ must have a time-varying growth rate for $\frac{K(t)}{\mathcal{A}_k(t)}$ and $\frac{\tilde{C}(t)}{\mathcal{A}_k(t)}$ to remain bounded away from 0 and infinity. Define γ_{-k} as the growth rate of $K(t)$ in the balanced growth path of the all-agriculture economy. Similarly define γ_{+k} as the growth rate of $K(t)$ in the balanced growth of the all-services economy.

Lemma 1. *There exists a function $\mathcal{A}_k(t)$ such that $\gamma_k(t) \equiv \frac{\dot{\mathcal{A}}_k(t)}{\mathcal{A}_k(t)}$ exists for all t, is continuous, converges to $\gamma_{+k} \geq 0$ as $t \to \infty$, and converges to $\gamma_{-k} \geq 0$ as $t \to -\infty$.*

In our model, as the economy comes to be completely dominated by services, we need that $\gamma_k(t) \to \frac{\gamma_x+\gamma_s}{1-\alpha_s}$. Similarly, when the economy is dominated by agriculture as

$t \to -\infty$, we need that $\gamma_k(t) \to \frac{\gamma_x + \gamma_a}{1-\alpha_a}$. In between, we can leave things relatively uninhibited as long as the function does not behave too erratically.

We can rewrite the laws of motion with the transformed variables to be

$$\theta \frac{\dot{\tilde{c}}(t)}{\tilde{c}(t)} = R(t) - \delta - \rho - (1-\theta)\left(\frac{\dot{P}_c(t)}{P_c(t)} - \frac{\dot{P}_x(t)}{P_x(t)}\right) - \frac{\dot{\bar{\epsilon}}(t)}{\bar{\epsilon}(t)} - \theta\gamma_k(t) \tag{16}$$

and

$$\frac{\dot{k}(t)}{k(t)} = \frac{x(t)}{k(t)} - \delta - \gamma_k(t), \tag{17}$$

where $x(t) \equiv \frac{X(t)}{\mathcal{A}_k(t)}$ is the de-trended investment. We can then state the theorem.

Theorem 1. *The STraP exists and is unique.*

We provide a brief overview of the proof here and leave the formal proof to Online Appendix A. Given the setup, the Hamiltonian conditions and transversality condition are sufficient to yield a unique equilibrium path forward from any $k(t) > 0$. We denote the unique optimal investment level $x(k,t)$. This simplifies the system to a one-dimensional non-autonomous system in $k(t)$. Proving the existence and uniquness of the STraP therefore comes down to proving that there exists one unique path that has $k(t) \to k_{-\infty}$ as $t \to -\infty$.

We apply an existing theorem (Theorem 4.7.5 in Hubbard and West, 1991) for the existence and uniqueness of time paths in an antifunnel, which requires several conditions: 1) a single differential equation; 2) a Lipschitz condition within the antifunnel, 3) narrowing upper and lower fences that define the antifunnel; and 4) a condition on the derivative of the right-hand side of the differential equation that bounds it away from $-\infty$ in a particular sense.[9] Verifying these conditions requires characterizing $x(k,t)$ to some extent. We start by transforming our original non-autonomous two-dimensional system into a more easily analyzed three-dimensional autonomous system by including time as a variable and reparametrizing it onto the compact interval $[0,1]$. This requires that the system is well behaved in the limit as $t \to \pm\infty$, which we confirm in the Online Appendix.

[9]Heuristically, a fence is a one-way gate for a dynamic path. A lower fence pushes solutions up; an upper fence pushes them down. An antifunnel is a region defined by upper and lower fences as lower and upper limits, respectively, which a path, starting from the outside, cannot enter. Narrowing is the property that the size of this region shrinks to 0. For a deeper explanation of antifunnels and fences, see Hubbard and West (1991). In particular, Chapter 1 gives a nice intuitive introduction.

That reparametrization allows us to show that $x(k,t)$ gets arbitrarily close to the investment function in the agriculture asymptotic growth path as $t \to -\infty$. With that, we can construct the upper and lower narrowing fences that define the antifunnel and verify that the other conditions hold.

This basically confirms that if one starts with capital higher than the STraP, going forward, capital will converge towards the STraP. However, simulating backwards, the amount of capital will explode towards infinity. Similarly, if one starts with capital below the STraP, one will converge towards the STraP. But if you simulate backwards capital will collapse towards zero. Only one path remains bounded away from 0 and ∞ simulating forward and backwards: the STraP.

3.3 A Simple STraP

We now derive the model's dynamics for an easily characterized special case where the STraP is not a BGP. This special case allows us to illustrate in closed form how the medium-term dynamics of the economy are characterized by a path that is independent from the initial conditions of the economy.

We focus on the Herrendorf et al. (2021) special case where consumers have homothetic preferences ($\varepsilon_j = 1$) and the sectors have the same capital intensity ($\alpha_j = \alpha$), but there is still structural change within investment, preventing a balanced growth path. We describe the dynamics of normalized variables using the natural normalizing factor $\mathcal{A}_x(t)^{1/(1-\alpha)}$ described in (15).

Proposition 3. *Suppose that $\alpha = \theta$. Let k_0 denote the initial capital stock at time τ. The solution to the competitive equilibrium is*

$$k(t,k_0) = \left\{ \left[k_0^{1-\alpha} - k^*(\tau)^{1-\alpha}\right] \frac{\mu(\tau)}{\mu(t)} + k^*(t)^{1-\alpha} \right\}^{\frac{1}{1-\alpha}} \tag{18}$$

and

$$\tilde{c}(t) = M(t)k(t,k_0), \tag{19}$$

where $M(t)$, $\mu(t)$, and $k^*(t)$ are continuous, positively-valued functions satisfying

$$\lim_{t\to\pm\infty} M(t) = \frac{\delta + \rho + (1-\alpha)\gamma_x}{\alpha} - \delta, \tag{20}$$

$$\frac{d\mu(t)}{dt} > 0, \qquad \lim_{t\to\infty} \mu(t) = \infty, \tag{21}$$

$$\lim_{t\to\pm\infty} k^*(t) = \overline{k}_{\pm\infty}. \tag{22}$$

We relegate the details of the derivation to Online Appendix D. Here, we briefly outline the derivation of Proposition 3 and discuss its relevance. The assumption that $\alpha = \theta$ allows us to decouple the dynamic system given by the Euler equation (14) and the law of motion for capital (1) into a differential equation for $\tilde{c}/k$ and another for k. These two equations for $\tilde{c}/k$ and k depend explicitly on time, implying that a balanced growth path cannot be a solution to the competitive equilibrium for any finite value of t. To solve these two differential equations, we need two boundary conditions. One is given by the initial capital level, k_0. The other comes from requiring that as $t \to \infty$, the competitive equilibrium converges to an economy dominated by the slowest-growing sector (services) featuring a balanced growth path with steady-state capital $\bar{k}_{+\infty}$.[10]

Next, we turn to the equilibrium time path of capital and rewrite equation (18) as

$$k\left(t, k_0\right)^{1-\alpha} = \underbrace{\left[k_0^{1-\alpha} - k^*\left(\tau\right)^{1-\alpha}\right]}_{\text{Initial Condition}} \underbrace{\frac{\mu\left(\tau\right)}{\mu\left(t\right)}}_{\longrightarrow 0} + \underbrace{k^*\left(t\right)^{1-\alpha}}_{\text{Medium-run}}. \tag{23}$$

Equation (23) shows that the equilibrium trajectory of capital, $k(t, k_0)$ can be usefully decomposed into two parts. We start with the $k^*(t)^{1-\alpha}$ term. Notice that if $k(\tau, k_0) = k^*(\tau)$, then $k(t, k_0) = k^*(t)$ for all $t > \tau$. That is to say, starting on $k^*(t)$, equilibrium capital stays on $k^*(t)$. Furthermore, $k^*(t)$ converges toward the asymptotic balanced growth paths going in either direction. Therefore, $k^*(t)$ is a STraP, and thanks to our theorem, we know that it is the only STraP.

The capital dynamics can then be thought of as the STraP dynamics (summarized by

[10] This result comes from the assumption that sectors enter consumption and investment aggregators as gross complements, i.e., $\sigma_c, \sigma_x < 1$. Thus, as services overtake the entire economy, the competitive equilibrium converges to a balanced growth path "as if" we had a one-sector model (services). We compute the normalized steady-state capital level $\bar{k}_\infty$ of this "one-sector" model and require that the economy converges towards this steady state.

$k^*(t)^{1-\alpha}$) plus the Neoclassical capital dynamics summarized by the first term $[k_0^{1-\alpha} - k^*(\tau)]\frac{\mu(t_0)}{\mu(t)}$. That term takes the initial distance between capital k_0 and the STraP capital, $k^*(\tau)$, and weights it by $\mu(\tau)/\mu(t)$. As $t \to \infty$, we know $\mu(t) \to \infty$ so that $\mu(\tau)/\mu(t)$ converges to 0. The growth rate of $\mu(t)$ then determines the speed of capital convergence to the STraP. Without the concept of the STraP, it would be impossible to separate out Neoclassical capital dynamics from natural changes in the optimal capital level.

4 Simulations of STraP Dynamics and Convergence

In this section, we calibrate and simulate a particular version of the model developed in Section 2 to illustrate the behavior of the STraP and to show how to differentiate STraP dynamics from Neoclassical convergence. We start by calibrating the model, then we describe the dynamic properties both on and off of the STraP, and finally, we show these properties for the empirically observed cross-section of initial capital stocks in the Penn World Tables (PWT).

4.1 Calibration

To illustrate and compute the STraP, we use a simple benchmark model. Specifically, following Ngai and Pissarides (2007) and Herrendorf et al. (2021), we model homothetic preferences ($\varepsilon_j = 1$) and uniform capital share across sectors ($\alpha_j = \alpha$), but differential productivity-driven structural transformation in both consumption and investment (as in Herrendorf et al., 2021).

For its computation we move to discrete time. We maintain the same growth notation, but we use the discrete analogs, e.g., $A_{x,t+1}/A_{x,t} = 1 + \gamma_{x,t}$, and the discount rate, ρ, is replaced by the discount factor, β.[11] In discrete time, the unnormalized Euler equation for the homothetic model is

$$\left(\frac{C_{t+1}}{C_t}\right)^{\theta} = \beta\left(1 + r_t\right) = \beta\left[1 - \delta + \frac{R_{t+1}}{P_{x,t+1}}\right]\left[\frac{P_{x,t+1}/P_{c,t+1}}{P_{x,t}/P_{c,t}}\right].$$

[11] Although Theorem 1 ensures that a unique STraP exists, an issue of practical relevance is how to solve for the STraP. In the Online Appendix we provide an algorithm to compute the STraP of the model as well as a calibrated extension with non-homothetic preferences.

where r_t is the interest rate on a bond that pays off in units of aggregate consumption. The analogous law of motion is

$$\frac{K_{t+1}}{K_t} = \frac{X_t}{K_t} + (1 - \delta) \,.$$

We start by assigning parameters: a standard value for depreciation, $\delta = 0.05$, and an intertemporal elasticity parameter of $\theta = 2$, which diverges from the log utility in the Ngai and Pissarides (2007) and Herrendorf et al. (2021) but is a more common value in the broader macro literature.[12]

Calibration then involves two steps. First, we obtain relative productivity parameters and preference parameters that drive structural transformation by calibrating strictly to postwar US data in an internally consistent way.[13] Second, because we will compare the growth simulations to cross-country panel data on growth and capital accumulation, we calibrate the absolute productivity growth rate, discount rate, and capital elasticity to wealthy countries in the income range of the post-1950 US.

The relative sectoral productivity growth rates are calibrated using US time series data on relative prices.[14] Absent non-homotheticities, Leontieff substitution between sectors provides a best fit to long-run data (Buera and Kaboski, 2009). We therefore assign a common elasticity of substitution for consumption and investment that approaches Leontieff, $\sigma_c = \sigma_x = 0.01$. We then calibrate the aggregator weights to match the average shares over the time series, where we use the input-output tables to yield the sectoral composition of investment. These values are $\omega_{c,a} = 0.013$, $\omega_{x,a} = 0.015$, $\omega_{c,m} = 0.231$, $\omega_{x,m} = 0.502$, $\omega_{c,s} = 0.756$, and $\omega_{x,s} = 0.483$.[15]

[12] Havránek (2015) provides a meta-analysis of 2,735 estimates of the elasticity of intertemporal substitution in consumption from 169 published studies covering 104 countries, finding a mean of 0.5, which implies $\theta = 2$.

[13] We combine the historical GDP by Industry data over the period 1947–1997 together with such data over the period 1997–2018 to yield sectoral prices and value added. We use aggregate real (chain-weighted) GDP, real personal consumption expenditures (PCE), and real private investment from BEA Table 1.1.6, and the prices of PCE and real private investment from BEA Table 1.1.4.

[14] Matching growth and capital accumulation in the US requires US-specific parameters values. Specifically, we use a capital elasticity of 0.33 and a discount rate of $\beta = 0.99$, to match the average interest rate, which we take from Gomme et al. (2011). They calculate the after-tax return to business capital to average 6.1% over the period 1950-2000. Finally, given relative productivities we scale absolute productivity growth rates to match US growth in income per capita of 2.74% annually.

[15] We solve for the relative weights using the relationship $\left(\frac{P_{j,t}}{P_{m,t}}\right)^{\sigma_c} = \frac{\omega_{c,j}}{\omega_{c,m}} \frac{C_{m,t}}{C_{j,t}}$, with $j = a, m$. We then use the normalization that weights sum to one to obtain values for $\omega_{c,j}$, with $j = a, m, s$.

Given these relative productivity and preference parameters, we calibrate the remaining parameters to match growth and capital moments in the PWT for advanced economies, i.e., those with income inside the range of the postwar US.[16] Specifically, we target these countries' average growth in income per capita (1.55% annually), average capital's share (0.40), and average capital-output ratio (3.99).[17] These imply $\alpha = 0.4$, $\beta = 0.9782$, and productivity growth parameters of $\gamma_a = 0.028$, $\gamma_m = 0.012$, $\gamma_s = 0.068$, and $\gamma_x = 0.0015$.

4.2 Simulations

We begin by presenting simulations of convergence toward the STraP for the calibrated model. The solid blue lines in Figure 1 represent the STraP for normalized capital over time, and the chain-weighted growth rate and interest rate along the STraP. The dashed line in the top panel of Figure 1 shows a transition path from a low level of normalized capital starting in 1900. The initial level of capital is low relative to the value of capital of the STraP in that year, given the level of the profile of productivities.[18]

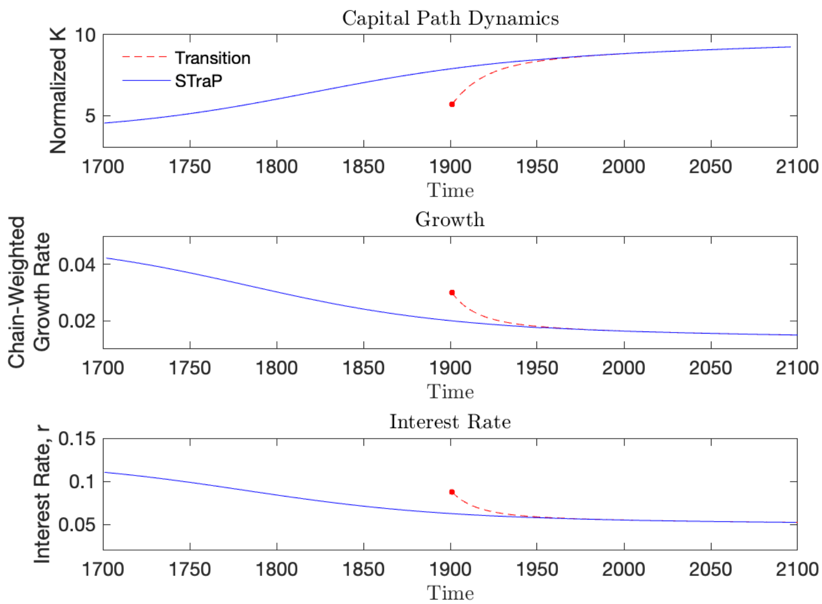
We describe the STraP as stable because capital quickly converges from the relatively lower initial level to the STraP. Initially, there is also a relatively large growth rate (see the middle panel of Figure 1), as the marginal product of capital is relatively large and capital is accumulated at a higher rate. Thus, the mechanics are akin to Neoclassical dynamics toward a BGP, except that the path for stable (normalized) capital is itself time-varying. This example illustrates not only the stability of the STraP but also the speed of convergence. The half-life in this simulated case is just eight years. This stands in contrast to the relatively slow capital-deepening dynamics along the STraP. Thus, Neoclassical forces lead to rapid convergence of capital, whereas structural transformation itself leads to slower time-varying capital deepening. We complement this global numerical counterpart with an asymptotic analytical

[16]See Online Appendix F for details on the construction of the sample and all data variables.

[17]Relative to the US data, the PWT data for countries with a similar level of real income per capita have slightly slower per capita income growth, slightly higher capital's share, and substantially higher capital-output ratios.

[18]The level of capital could be initially low because there was a negative shock, e.g., a war, destroying part of the capital stock. Alternatively, the initial level of capital could correspond to the value of a capital in an alternative STraP with a lower level of productivity. In this second interpretation, the transition is triggered by an unanticipated productivity shock, and the new STraP is a scaled-up version of the original one.

Figure 1: Transitional Dynamics



characterization of the systems' eigenvalues (see Online Appendix C). Reasonable values in these formulas confirm the faster dynamics for convergence relative to those for the structural transformation patterns captured by the STraP. Indeed, the benchmark half-life of structural transformation dynamics is 70 years, an order of magnitude larger than that for convergence dynamics.

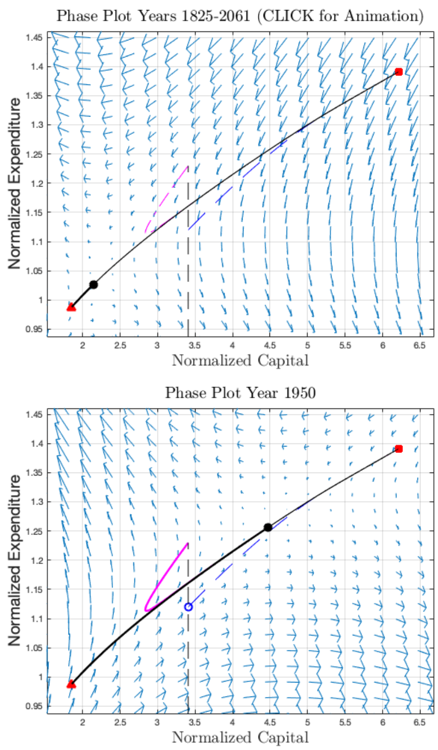
Figure 2 illustrates the multidimensional aspect of convergence by plotting the dynamic paths in the normalized capital–normalized consumption expenditure space. The top left panel of the figure adds the remaining dimension of time, which controls productivity — this is shown with an animation of the movement of the economy (the black dot) along the STraP (black line), from the agricultural BGP (the red triangle) through the transformation to the services' BGP (the red square). (The animation starts when clicked on.) The vector field of time-varying arrows is a phase diagram showing the system's instantaneous trajectories for arbitrary expenditure-capital combinations. To further illustrate convergence properties, we start two economies from the identical normalized capital stock, but at two different points in time. The upper pink trajectory is an economy that starts at an earlier time with a higher-than-STraP level of capital given its productivity at that time, while the lower blue trajectory starts at a later date with a lower-than-STraP level of capital. The animation shows the rapid convergence to the STraP over time for both initial values. Note that the distances of the convergence paths in normalized expenditure-capital space are *not* reflective of the time required to converge. (Indeed, for more extreme levels of productivity — either extremely early or extremely late time periods — the convergence paths could go directly toward the BGPs.)

The other panels of Figure 2 illustrate snapshots from this animation. The different panels are discrete jumps in time, which serve to emphasize how the vector field — and the point toward which an economy moves — varies with time. The top right panel illustrates this starting point of the upper pink trajectory in the year 1850. Given high levels of capital, initial expenditures also exceed those of the STraP economy as the open pink circle indicates. The higher expenditures of the pink economy drive down the (normalized) capital stock over time. The bottom left panel brings the economy forward to the year 1950, where the full, completed convergence path of the pink economy can be seen. Interestingly, although convergence of the pink economy is from above, it involves a period of both decreasing and increasing normalized capital stocks. The same panel also shows the starting point of the blue dashed trajectory. Given the later date, the same capital stock is now to the left of the STraP and the vector field has rotated. Hence, the economy chooses a lower-than-STraP level of expenditures and will accumulate capital. The bottom right panel shows the economy in the year 1990, where both economies have converged to the STraP (here we show the instance where the blue hollow dot has almost converged so that it can be compared with the black dot).

Finally, we show the empirical implications of the stability properties by simulating convergence dynamics for countries in the PWT 9.1 from the date of the earliest reliable capital stock. (As we do later in our empirical analysis, we focus on the middle 25-75th percentiles of these data.) We then simulate the time-series for the same productivity process and initial conditions of capital-output and real income per capita. We simulate these in a "US equivalent time", where the US level of income in 2000 corresponds to the year 2000 and each countries initial level of income is lined up with the year of US income in this simulated STraP.

The top panel of Figure 3 shows the convergence of capital-output ratios, some from above but most from below, toward the capital-output ratio of the STraP, which in-

Figure 2: STraP Convergence in the Capital-Expenditure Space

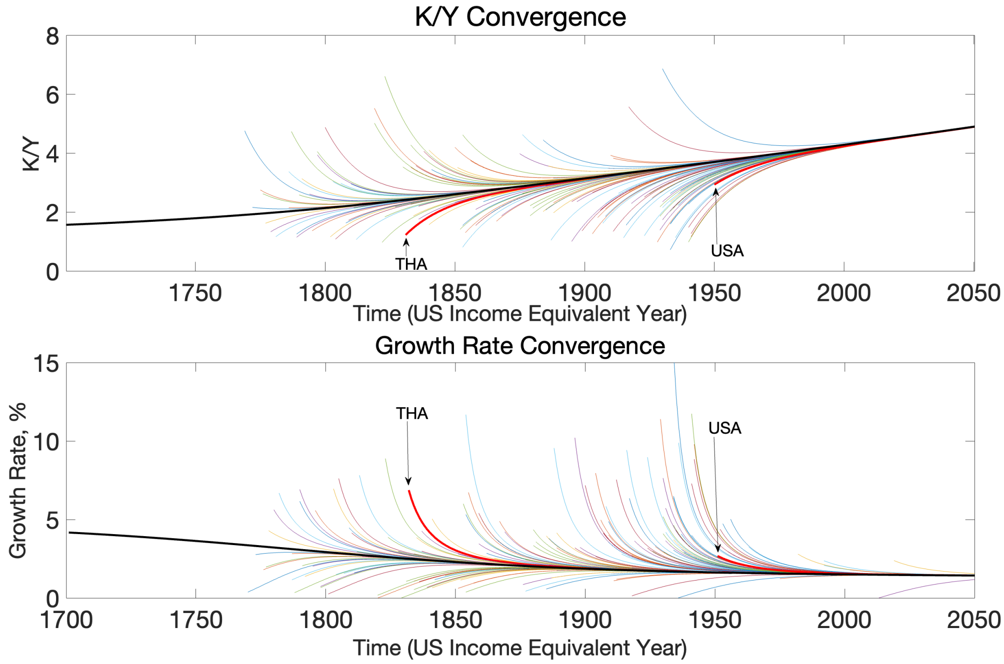


creases over time. The bottom panel shows the variation in growth rates, where the high initial growth rates correspond to below-STraP initial conditions. Much as in the Neoclassical growth model, the half-life of initial condition growth dynamics die out rapidly, while the medium-term growth dynamics of the STraP are more prolonged over development. (We have labeled Thailand, as THA, and the USA for reference, since we will decompose their growth in Section 5.2.)

5 STraP Implications for Structural Change and Growth

This section presents three quantitative exercises to highlight how the STraP concept can be successfully used to analyze the rich implications of dynamic models of structural change. First, we show that the medium-term implications of the model, as captured by the dynamics along the STraP, are very sensitive to small deviations of key model parameters. In particular, not only do deviations from knife-edge assumptions that are

Figure 3: Convergence to the STraP from Cross-Country Initial K/Y Data



required to obtain a BGP matter quantitatively for structural transformation and interest rates, but they also influence the *qualitative* patterns of growth and investment. Second, we demonstrate how the STraP can be used to decompose growth into growth driven by structural transformation (i.e., medium-term dynamics driven by the standard productivity process), excess capital accumulation (driven by initial conditions), and excess productivity growth. We start by demonstrating this for Thailand, where the STraP alone can explain a fair portion of its fast growth because Thailand specializes in sectors that grow fast. We then turn to growth in the cross-section, where the STraP can rationalize the average differences in growth rates by development level.

Finally, we use the STraP to analyze the implications of the benchmark model for growth over long historical periods and contrast them with data from the Maddison Project database. This last exercise points to important counterfactual implications of the benchmark model, when confronted with data from the 19th and early 20th centuries. While the theory predicts the largest growth to have occurred early in this period, as is well understood, the data shows an acceleration at the start of the period of modern growth. We conclude by discussing extensions of the benchmark model that can bring the model closer to the historical data.

5.1 Sensitivity to Alternative Calibrations

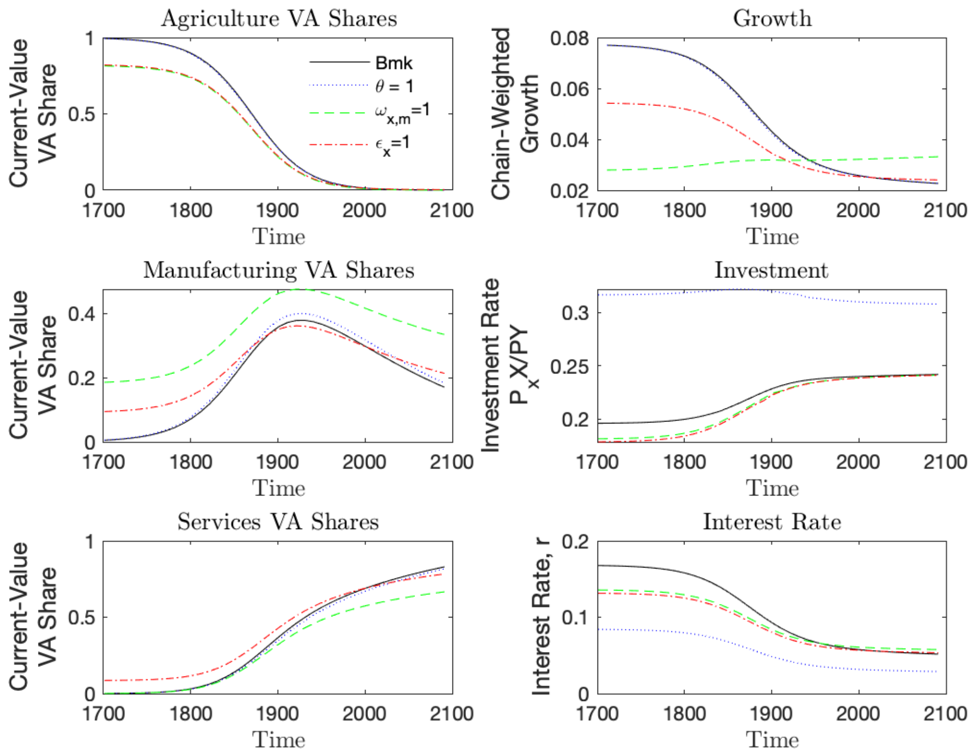
Our benchmark model relaxed assumptions that were necessary for a BGP but are no longer necessary in the STraP. In this section, we examine the importance of these assumptions by simulating alternative models with different parameter choices. That is, we perform simulated comparative statics changing one assumption at a time. Ultimately, not only do these parameters matter quantitatively for structural transformation and interest rates, but they also influence the qualitative patterns of growth and investment. Indeed, we show that relatively small changes in parameters can lead to surprisingly different qualitative implications for growth trajectories along the STraP.

The alternative models we present are as follows. The first two adopt the assumptions in Ngai and Pissarides (2007), one at a time. The first alternative is their assumption of $\theta = 1$, i.e., log intertemporal preferences (relative to our benchmark of $\theta = 2$). The second alternative is their assumption that investment consists only of manufacturing value added, i.e., $\omega_{x,m} = 1$, (relative to the benchmark where it is heavy in manufacturing, but still a mix that undergoes structural transformation). Finally, we consider an alternative in which the elasticity of substitution in the investment sector is unitary. This captures the idea of an investment sector with a mixed but stable composition, as in García-Santana et al. (2021), and contrasts it with an investment sector that undergoes structural transformation itself.

Figure 4 plots these alternatives versus the benchmark to demonstrate their impact on sectoral shares (left panels), growth (top right), investment (middle right), and interest rates (bottom right).

The impact of the first alternative, log preferences (the dotted blue lines), shows up most strikingly in the investment rate and interest rate. On the one hand, the level of the investment rate starts out much higher than in the benchmark (roughly, 0.31 versus 0.20) and falls slightly over time rather than rising as in the benchmark (and other alternatives). On the other hand, the interest rate starts substantially lower (8% versus 17%) and declines over time as in the benchmark, though by substantially less (5 percentage points relative to 11 percentage points). Focusing on the structural transformation patterns, one can see the impact of the higher investment rate; the peak in the manufacturing hump is slightly higher, since investment is relatively intensive in manufacturing value added.

Figure 4: Alternative Calibrations



The second alternative, only manufacturing in investment (the dashed green line), leads to different sectoral distributions, growth rates, and interest rates. The fact that all investment comes from the manufacturing sector changes the sectoral distributions. Manufacturing naturally constitutes a higher share of output, but the impact can be most easily seen in the asymptotic sectoral compositions; agriculture is less than one initially, while the economy never fully becomes exclusively services as it grows. Compared with the growth rate in the benchmark economy, the growth rate is lower and varies less over time.[19] Moreover, it rises with structural transformation rather than declining steeply as the economy leaves agriculture. Finally, we see that the interest rate starts lower (roughly 14.5% versus 17%), but it is relatively stable and indeed 1 percentage point higher by the end of the sample than the benchmark rate, which falls much more. (The impact on the investment rate is more subtle. It starts half a percentage point lower than the benchmark rate, but rises a bit faster and is

[19]It is not absolutely constant as in Ngai and Pissarides (2007) because we report a chain-weighted growth rate rather than using manufacturing as the numeraire.

comparable to the benchmark rate by the end of the sample.)

The third alternative, a unitary elasticity of substitution in investment (the dash-dotted red line), tends to dampen the patterns relative to our benchmark. The decline in agriculture, hump shape in manufacturing, and increase in services are all less pronounced that in the benchmark simulations. Looking at the panels on the right, we see that the decline in growth is also less pronounced, as is the decline in the interest rate. The intuition is clear: structural transformation is weaker because it is only occurring within the consumption sector. However, the investment rate rises somewhat more without structural transformation as agriculture is not as important to investment early on, so the relative price of investment does not fall as rapidly.

5.2 Growth Decompositions

In this section, we demonstrate how the STraP can be used to decompose growth into growth driven by structural transformation (i.e., medium-term dynamics driven by the standard productivity process), excess capital accumulation (driven by initial conditions), and excess productivity growth. We start by demonstrating this for Thailand. The STraP alone can explain a large portion of its fast growth because Thailand is specialized in fast-growing sectors.

We then turn to growth in the cross section, where the STraP can rationalize the average differences in growth rates by development level. For these cross-country exercises, we use the Penn World Tables (PWT) 9.1 cross-country panel (Feenstra et al., 2015) for aggregates and the Groningen 10-Sector Database for sectoral data.

5.2.1 Illustrative Example: Thailand

A comparison of the United States and Thailand is a nice illustrative example. Empirically, post-1950, real income per capita in the United States grew at 1.6% annually, 2 percentage points lower than Thailand's annual growth of 3.6%.[20] For the US, the data, transition path from an initial condition, and STraP each yield roughly the same 1.6% contribution, indicating that initial conditions played essentially no role. Thus, the US was not appreciably away from its STraP and is well approximated by the

[20]We use the periods available in the Groningen 10-sector database. Thai data start in 1951.

benchmark productivity process. For Thailand, however, the STraP yields a growth rate of 2.1%, indicating that 0.5 (i.e., 2.1% - 1.6%) percentage points of its additional growth (one-quarter of the 2 percentage point difference) came from STraP dynamics. The transition path to the STraP from initial capital yielded an average annual growth of 2.3%, indicating that initial conditions accounted for an additional 0.2 (i.e., 2.3% - 2.1%) percentage points, somewhat smaller than the contribution of medium-term dynamics.[21] The remaining 1.3 percentage points come from Thailand's higher productivity growth.

Much of Thailand's additional growth along the STraP is driven by structural transformation, where the share of agriculture declines by 12 percentage points in Thailand and only 2 percentage points in the US. (Although this difference is substantial, it is much smaller than the 38 and 6 percentage point declines implied, respectively, in the data.) Again, structural transformation leads to both Baumol's disease and capital accumulation. Capital per capita increases 1.8% in the US but 3.0% in Thailand annually in the simulations from observed capital levels. Of this 1.2 (i.e., 3.0% - 1.8%) percentage point difference in capital accumulation in the model, roughly half (i.e., 2.5%-1.8%, where the difference is rounding) comes along the STraP, and the other half is due to a low initial stock of capital. (Empirically, capital grew at 4.4% annually in Thailand, which is 1.2 percentage points higher than in the Thai simulation and again reflects faster-than-benchmark technical change.)

5.2.2 Cross-Country Panel

We now present the implications of medium-term versus short-term dynamics over a wide range of development that spans well beyond the calibrated range.[22] With 126 countries and over 4,000 observations, the data themselves are dense and have wide variance. To make the data clearer, we use three lines to characterize them: non-parametric fits (using 100 income bins) of the 25th and 75th percentile of the data at

[21]When we compute a country-specific STraP using country-specific technological progress, we can account for the full growth, but the contribution of initial conditions remains nearly the same, at 0.4 percentage points. The country-specific model can fully account for the difference in growth rates.

[22]We present the results over a wide range of development outcomes that span well beyond the calibrated range, from a log real income per capita of 7 (roughly $1,100) up to the US income per capita in 2000, 10.75 (roughly $46,500).(Although limited data are available above and below these ranges, the set of countries thins out quickly, and patterns can be easily driven by country-specific effects and the changing sample.)

each income level and a linear fit.[23] For comparison's sake, we include a simulated, calibrated Cass-Koopmans-Ramsey growth model — the benchmark one-sector Neoclassical growth model. The comparison is insightful, since this benchmark exhibits no medium-term dynamics and therefore isolates the standard short-term Neoclassical dynamics, while the STraP exhibits no short-term dynamics and isolates the medium-term dynamics. We start the Neoclassical simulation off at log real income per capita of 7 with the average capital-output ratio in the data bin at that level of income.[24]

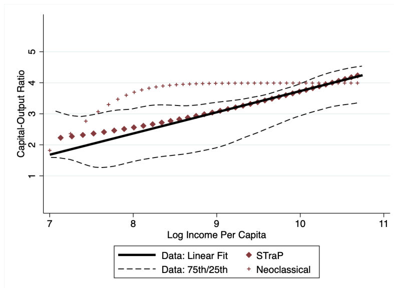
In the figures that follow, we stress that everything is out-of-sample except for the *average* capital-output ratios and annual growth rates at very high incomes (log incomes above 9.59) and the initial (i.e., log income of 7) capital-output ratio in the Neoclassical model. Looking back at lower incomes is a test of whether the stability of the productivity process and the structural transformation forces modeled can add insight into broader development patterns.

We start by examining the patterns and determinants of the capital-output ratio, since, at the aggregate level, structural transformation provides a theory for a time-varying capital-output ratio over the medium term, and the STraP characterizes these dynamics. Figure 5 presents these results. The top left panel plots the capital-output ratio over development. The black lines show the data, which have wide variance but clearly trend up and in an economically important way. In the linear fit, the capital-output ratio rises from roughly 1.8 to 4.2. The capital-output ratios in the STraP (diamonds) structural transformation model mirror this increase fairly well, though not fully: the STraP can explain an increase in the capital-output ratio from roughly 2.2 to 4. By construction, the Neoclassical growth model (the '+' symbols) explains the full increase in K/Y, but the path displays its well-known short-lived dynamics, rising quickly and then remaining flat. The key point here is over the broad range of development, the assumption of a constant, balanced growth capital-output ratio does not hold in the data. The structural transformation models yield persistent medium-run capital accumulation dynamics that (i) go beyond the rapid convergence dynamics in the Neoclassical growth model that stem from initial levels of capital and (ii) can partially explain the overall idea of a rising capital-output ratio over development.

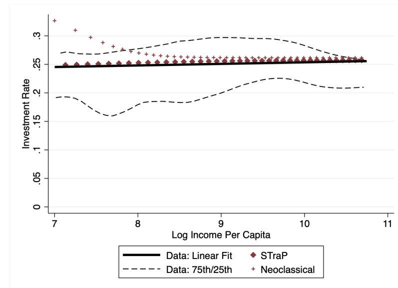
[23]In all the figures that we present, the coefficient in the linear fit is significant at the 5% level.

[24]The calibrated Neoclassical growth model uses the same Cobb-Douglas parameter and depreciation rate, and productivity growth and the discount factor are again chosen to match the identical calibration targets.

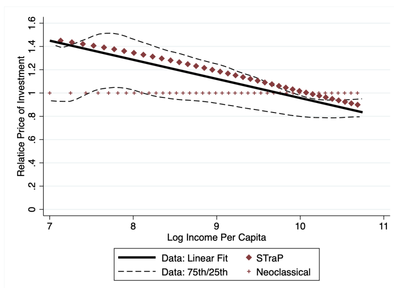
Figure 5: Capital Accumulation and its Determinants over Development



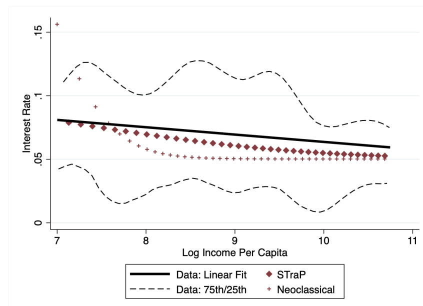
(a) Capital-Output Ratio



(b) Investment Rate



(c) Relative Price of Investment

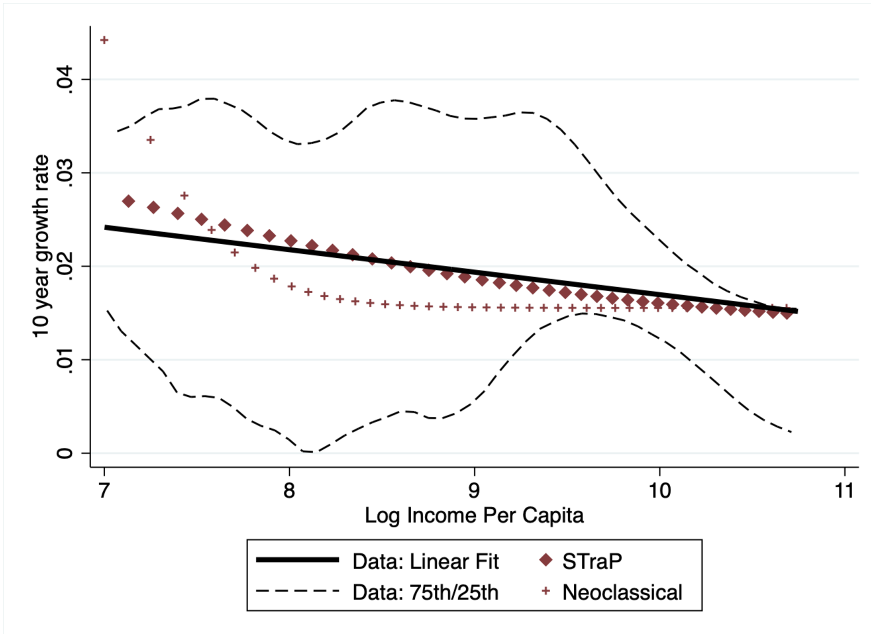


(d) Interest Rate

The remaining panels of Figure 5 explore the determinants of these capital accumulation dynamics. The top right panel shows the (current value) investment rate. The linear fit of the data shows a very mild, though statistically significant, increase. The relative flatness of this pattern was emphasized by Hsieh and Klenow (2007), and the statistical significance of the increase was noted by Inklaar et al. (2019). The STraP shows an increase in the investment rate, roughly identical to that of the linear fit of the data. Again, the Neoclassical growth model, in contrast, has a very high initial investment rate and a rapid stabilization consistent with the well-known short-run dynamics.

The bottom left panel shows the relative price of investment over development. The marked decline in the data, from over 1.4 to less than 0.9, is another pattern emphasized by Hsieh and Klenow (2007) as important for understanding development: rich countries get more real investment bang for their current-value investment rate buck. Here the structural transformation in the STraP leads to variation over development that is consistent with a declining relative price of investment that is again remarkably similar to the declining relative price given by the linear fit in the data. Again, in

Figure 6: Ten-Year Growth Over Development



contrast, the Neoclassical growth model delivers a flat prediction for relative prices.

The bottom right panel plots interest rates over development.[25] The data show a mild decline, with the linear fit dropping from roughly 8% to 6%. The STraP interest rate is only slightly lower, dropping from 7.5% to 5%. In contrast, the initial interest rate is exceedingly high in the Neoclassical growth model, with the decline in the model transition immediate, intense, and short-lived.

Figure 6 shows what is really the key finding of this analysis. It focuses on the models' implications for (ten-year) average annual per capita income growth (and implicitly convergence) over development. Again, the ten-year growth rate is used because our focus is on medium-term growth dynamics. In the data, ten-year annual growth rates, on average, fall with development from 2.5% to under 1.5%, or a decline of 40%.[26] The Neoclassical growth model displays a version of the well-known convergence puzzle.[27]

[25]We measure interest rates in the PWT data to be consistent with our consumption-based definition of interest rates.

[26]For a poor but miracle growth country growing at 5.4% annually per capita, this additional 1 percentage point would again constitute one-quarter of the excess growth over the US, comparable to the result of our Thai miracle decomposition.

[27]The more extreme version of the convergence puzzle in Barro and Sala-i Martin (1992) assumes that all countries have the same technology (at a point in time). We combine time and cross-sectional variation and allow poorer countries at any point in time to be further behind in the productivity

Growth declines very quickly over development, as the economy rapidly converges to its BGP.

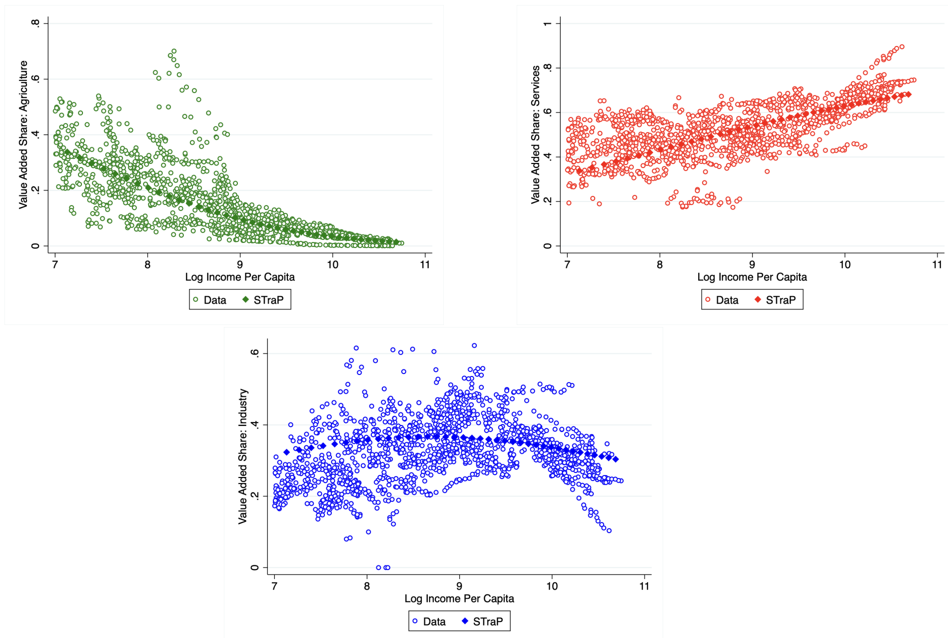
In contrast, the medium-term dynamics of the STraP exhibit slowly and persistently declining growth rates that are quantitatively important. This is not simply Baumol's disease. While the movement of resources from faster to slower productivity growth sectors dominates the dynamics, the increasing capital accumulation over development in response to a falling relative price of investment and the productivity growth slowdown partially counteracts this. Moreover, one can see that the growth slowdown is much stronger at incomes below \$13,000 PPP (log of 9.5), a range of development well below the range emphasized by Baumol. The point here is that even without an initially low capital stock and despite increasing investment rates over development, a country with an income per capita of \$1,100 (i.e., log of 7) but with the productivity growth rates of the richer countries could expect its growth rate to fall by more than one-third (from 2.8% to 1.8%) as it grows to an income per capita of \$13,000.

Our analysis combines structural transformation dynamics with the better-understood forces of investment-specific technical change (ISTC) — a potential alternative to explain the declining relative price of investment shown in Figure 5. To distinguish these forces, we compare our results with a model in which we turn off the other sources of exogenous technical change (i.e., $\gamma_a = \gamma_m = \gamma_s = 0$), leaving only investment-specific technical change (γ_x). We note four things. First, ISTC explains almost no GDP growth in the model (an annual growth rate of less than 0.1%). Second, ISTC explains only 58% of the decline in relative prices. Although this pattern is often solely attributed to ISTC, structural transformation is responsible for 42% of the decline. Third, ISTC explains only 42% of the real increase in (log) K/Y. (These two numbers matching is merely incidental.) Fourth, ISTC alone delivers a balanced growth path (Greenwood et al., 1997), and so, unsurprisingly, ISTC leads to no change in the growth rate over development. The remainder of these two last patterns are instead the result of the time-varying nature of the rate of change in relative prices and Baumol's disease — both endogenous processes.

The importance of the structural transformation mechanism for aggregate dynamics leads to the natural question of whether the structural transformations that underlie the aggregate behavior in the model also align with the data. Sectoral data are not

process.

Figure 7: Structural Transformation over Development



available in the PWT data, but we turn to the Groningen Growth and Development Centre (GGDC) 10-Sector Database (Timmer et al., 2015), which provides data on sectoral shares for 39 countries of varying levels of development over the years 1950–2010. Given the smaller sample, we can plot the STraP against the actual data. (We omit the Neoclassical model because it has no sectoral implications.)

Figure 7 shows the fit of the STraP simulation relative to the GGDC data (empty circles) for agriculture (top left panel), industry (lower panel), and services (top right panel). Despite the fact that when viewed together the figures make a sad face, the model actually follows the overall patterns of structural change in the data quite well. Again, we emphasize that these patterns are out of sample: none of these sectoral patterns have been used to calibrate the model. In sum, the structural transformation mechanisms driving the growth dynamics in the model have supporting evidence in the data as well.

5.3 Implications for Growth over Long Historical Periods

We now contrast the implications of the benchmark model with historical data from the Maddison Project Database since the beginning of the 18th century. This exercise provides a particularly powerful illustration of the usefulness of the STraP, as we are considering a period for which initial capital stock data are unavailable. Therefore, the model's implications cannot be tested by solving the transitional dynamics from an initial capital stock. By contrast, the STraP is defined both forward and backward in time. Thus, researchers can fit a structural change model for a time period for which they do not have data. The implied STraP gives predictions about growth rates and structural change far in the past along with the future.[28]

For this part of the analysis, we will focus on data from the 18th century until 1950 and on the set of countries that by 1900 had attained the level of per capita income that the UK enjoyed in 1800.[29] By this year, the Industrial Revolution and structural transformation were already occurring. To organize the historical data on the growth rate of per capita income of countries over time, we shift the time for each country so that it is measured relative to the year each country attained the level of per capita income that the UK economy had in 1800. We refer to this year as the "initial year." This could be interpreted as a simple measure of time relative to each country's STraP.

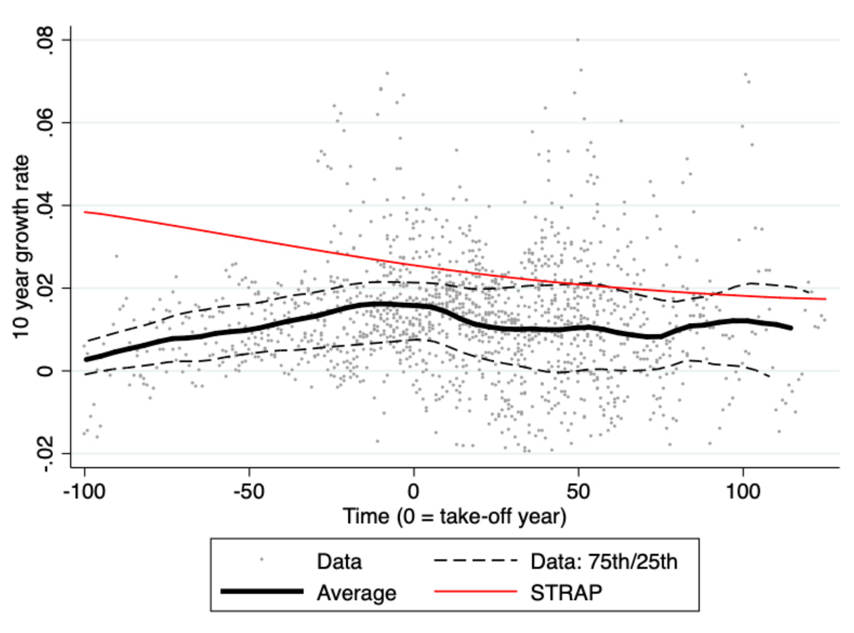
Figure 8 shows the evolution of the growth rate of per capita income of the 14 countries against the years since each country attained the per capita income the UK enjoyed in 1800. Thus, for each country, year 0 corresponds to the first year each country attained the level of per capita income that the UK enjoyed in 1800. We also show a local regression together with the 25th and 75th percentiles of the data. Consistent with modern growth being a relatively recent phenomenon, the growth rates start close to zero, but rise towards the initial year.

In Figure 8, we also plot the STraP for the benchmark US calibration. As in the data, the year zero corresponds to the one where the per capita income in the model reaches the value the UK enjoyed in 1800 (the year 1832 for the case of the US). In sharp contrast with the data, the model predicts that growth should be maximal in the very beginning of the sample, when most of the economy is in agriculture, the sector exhibiting the highest TFP growth according to our calibration.

[28]This is only possible from a capital stock along the STraP since shooting backward from arbitrary initial values will lead capital to go to zero or blow up to infinity, unless the capital stock consistent with the STraP is chosen.

[29]Given these restrictions, we are left with 14 countries: Argentina (attaining the 1800 UK per capita income in 1883), Australia (1849), Austria (1878), Belgium (1855), Canada (1883), Switzerland (1874), Chile (1896), Germany (1878), Denmark (1874), France (1878), UK (1800), New Zealand (1860), Uruguay (1882), US (1832).

Figure 8: Growth over Long Historical Periods



As shown, the STraP clearly enables an evaluation of the long-term implications of the benchmark model, highlighting a striking shortcoming of the theory. This calls for extensions to the model to explain the onset of growth and industrialization, which is a key motivation for the research on structural transformation. One natural possibility is to posit a richer productivity process. Indeed, one could use the STraP concept to calibrate a time-varying productivity process that explains the observed time-varying growth in per capita income over the long run.[30] Another, more interesting possibility is to consider more general technologies. For example, in Online Appendix G, we develop a simple model mingling elements of Acemoglu and Guerrieri (2008) and Hansen and Prescott (2002) by allowing for a choice of technologies with different factor intensities in each sector, and show that along the STraP, growth is naturally slower early in the STraP as low-capital-intensive technologies are used, with a lower capital multiplier of productivity growth.

[30]This is akin to calibrating the productivity growth rate in a BGP to match the average GDP growth rate over a given period. The main difference is that the STraP allows us to calibrate a time-varying path for growth by solving a fixed point problem over sequences of TFPs.

6 Conclusion

This paper introduces the Stable Transformation Path, which we also refer to as the STraP, and shows its existence and uniqueness for a broad class of structural change models. The STraP gives the "natural" level of capital and industrial structure at any level of development. Thus, it can be used to separate medium-run implications of a model from the usual Neoclassical capital dynamics. This opens up new questions for researchers to answer in a theory-consistent way. We give a few simple examples but leave open more serious quantitative studies to future work.

While this paper focuses on structural change, an STraP will exist in almost any setting where an asymptotic balanced growth path exists at ∞ and $-\infty$. In any such setting, the STraP could prove a useful tool for understanding the long-run implications of the model and classifying trends into long-run versus short-run phenomena. Models with changing time use, levels of urbanization, and labor share could all benefit from this analytic concept.

Link to Online Appendix (Click Here)

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