Online Appendix for
"Financing Costs and Development"
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A Data Appendix

This appendix provides a more detailed description of the data (together with summary statistics) and the definition, sources, and values of the moments targeted in our calibration.

A.1 Data Description

Data on bank loans to all formal firms in Brazil are from the Brazilian Public Credit Registry (SCR - *Sistema de Informações de Crédito*). This is a confidential loan-level database, managed by the Central Bank of Brazil. It contains information of all formal loans granted from January 2006 until December 2016. We observe the lender, borrower, loan type (investment, working capital, revolving, etc.), size of the loan, the interest rate on the loan, the loan maturity, default rates, and whether or not it was at an earmarked interest rate. This dataset allows us to construct information on the borrower-lender relationship, such as the length of a firm-bank relationship.

The other main dataset that we use in our empirical analysis is RAIS (*Relação Anual de Informações Sociais*), a matched employer-employee administrative dataset covering all formal firms in Brazil. This a mandatory annual survey maintained by the Ministry of Labor. RAIS provides information on firms, such as sector of activity and location, and information on employees, which we use to construct firm-level measures of employment and labor compensation. It is also possible to identify the date of entry and exit of firms. With this dataset, we can capture important firm dynamics for all formal firms in Brazil.

Using the unique firm tax identifier, we merge the SCR and RAIS datasets. Financial firms, public administration, non-governmental organizations and multilateral agencies were excluded in our sample. Our database therefore comprises more than 25 million observations between 2006 and 2016.

To be precise about how the dataset is used, the Brazilian Central Bank has a loan-level dataset in each year. The underlying loan-level data are aggregated to construct credit flows and stocks at the level of firm-bank-loan type-year combination. This is the dataset we were provided. Similarly, we use loan-weighted averages to construct average spreads, maturity, non-performing loans, and other measures at these levels. We have the firm identifier, so we can control for firm fixed effects or aggregate loans at the firm level weighting by the loan amount. In the latter, the loan interest rate corresponds to the weighted average of all loans a firm had in a particular year. Similar procedures can be used to construct loan characteristics at the firm level, but our main regressions are based on firm-bank-loan type observations.

Table A1 provides descriptive statistics for the main variables used in our empirical analysis. As we can see, there are almost 25 million observations and statistics for the following variables are provided: firm size (number of employees and wage bill), firm age in years, spread (difference between the loan interest rate and the deposit interest rate) rate, average maturity, non-performing loan (NPL), loan type (working capital, investment, foreign trade), earmarked loan, duration (in months) of the firm-bank relationship and number of banks a firm got loans in a particular year. We also have indicator variables for the location (state) of the firm, firm legal nature and economic sector.

Table A1: Summary Statistics: Credit Data (SCR)

	N	mean	sd	p10	p50	p90	min	max
Firm wagebill	25,441,646	66547.98	1827883.21	675.86	4989.79	49174.29	0.00	1.10e+09
Firm age	25,441,646	15.67	20.63	2.56	9.93	29.59	0.00	118.62
Spread	25,441,646	77.10	93.26	3.08	31.51	218.27	-14.32	607.78
NPL	25,441,646	0.05	0.43	0.00	0.00	0.15	0.00	1996.88
Default	25,441,646	0.04	0.20	0.00	0.00	0.00	0.00	1.00
Maturity	25,441,646	13.79	32.62	1.27	3.87	35.29	0.00	651.83
Earmarked loan	25,441,646	0.48	0.50	0.00	0.00	1.00	0.00	1.00
BNDES loan	25,441,646	0.00	0.04	0.00	0.00	0.00	0.00	1.00
Firm-Bank relat.	25,441,646	81.84	141.92	2.35	36.99	185.55	0.00	1407.20
Number of banks	25,441,646	2.89	1.99	1.00	2.00	5.00	1.00	36.00

A.2 Moments

Here we provide the definition, source and value of the moments used in Section 4 of the paper to internally calibrate the model parameters.

1. Exit rate: 11%. Source: RAIS. We calculate the exit rate using the RAIS dataset from 2006 to 2016.

2. Average firm size (number of employees): 13. Source: RAIS.

3. Average firm growth rate (growth in number of employees): 3.4%. Source: RAIS.

4. Top 10-percentile employment share: 77%. Source: RAIS.

5. Top 10-percentile earnings share: 55.6%. Source: Morgan (2017).

6. Capital-output ratio: The capital-output ratio is calculated from Feenstra et al. (2015), the Penn World Tables (PWT) 10.01. We use the share of gross capital formation at current PPPs and the perpetual inventory method to construct the capital to output ratio, assuming that the Brazilian economy is on the balanced growth path from 1995 to 2005. The capital to output ratio is then 2.55 in 2016.

7. Real risk free interest rate: 2%. For the real interest rate in Brazil, we took the average rate from 2005.1 to 2016.12 of the monthly Over/Selic interest rate (Brazilian Central Bank rate) minus the inflation rate measured by the IGP-DI (General price index from Vargas Foundation). We then annualized the monthly average real interest rate, and we get 5.87%. We also deducted country default risks, measured by the sovereign default spreads. Damodaran (2020) shows that default spreads varied from 2 to 4% in the period (see Figure 13 of this paper) and the value in April. Therefore, we set the interest rate in the small open economy to 2%.

8. Credit to output ratio: we target a value of 48.7%. Following Buera, Kaboski, and Shin (2011), we define this by the sum of the credit to non-financial corporations, private bond market capitalization, and stock market capitalization.

9. The average spreads once some factors are washed out: 64%. We calculate this by using the predicted value of the Regression (8) in Table 1. We set maturity at 1 year (as in our model) and non-performing loan (lag, contemporaneous and lead) at 0 (there is no default in our model). We then calculate the average predicted spread and the resulting value is 64%, which is about 13 percentage points lower than the unconditional average spread. Source: Our dataset based on the SCR and RAIS.

10. Weighted average spreads: From the predicted spread above, we calculate the credit-weighted spread of 45%. Source: Our dataset based on the SCR and RAIS.

11. Standard deviation of spreads: 40%. Following the same procedure as above, we calculate the standard deviation of spreads once some loan effects are washed out. The value we target is: 40%, which is about 53 percentage points lower than the unconditional standard deviation of spreads. Source: Our dataset based on the SCR and RAIS.

12. Entrepreneurs with credit: 26%. We use an extensive margin measure of credit. Roughly 26% of entrepreneurs have credit. Source: Our dataset based on the SCR and RAIS.

B Theory Appendix

We provide proofs of Propositions 1 and 2 together with an analytical derivation of Figure 1.

B.1 Proof of Proposition 1

Notice that $x_k = r + \tau(a,z)$ and we have that

$$\tilde{r}(a,z) = r + \tau(a,z) + \frac{\chi}{k^b(a,z) - a}\left(y^b(a,z) + \tau(a,z)a - \kappa - \tilde{w}(a,z)\right).$$

The proof when $\chi = 0$ is trivial.

When $\chi = 1$ and $\tau(a,z) = \tau_0$, then by the implicit function theorem, we have that

$$\frac{\partial \tilde{r}(a,z)}{\partial z} = \frac{\frac{\partial y^b(a,z)}{\partial z} - (\tilde{r}(a,z) - (r+\tau_0))\frac{\partial k^b(a,z)}{\partial z} - \frac{\partial \tilde{w}(a,z)}{\partial z}}{k^b(a,z) - a}.$$

This can be rewritten as:

$$\frac{\partial \tilde{r}(a,z)}{\partial z} = \frac{y^b(a,z) - (\tilde{r}(a,z) - (r+\tau_0))k^b(a,z) - (1-\alpha-\theta)\left(za^{\alpha}\left(\frac{\theta}{w}\right)^{\theta}\right)^{\frac{1}{1-\theta}} I_{w<\tilde{\pi}}}{z(1-\alpha-\theta)(k^b(a,z) - a)},$$

where $I_{w<\tilde{\pi}}$ is an indicator function which takes value equal to one when $w < \tilde{\pi}(a,z)$ and zero otherwise. The denominator of this expression is clearly positive. The numerator is positive for any $S^e \geq 0$.

It can also be shown that

$$\frac{\partial \tilde{r}(a,z)}{\partial a} = \frac{\tilde{r}(a,z) - r - \frac{\partial \tilde{w}(a,z)}{\partial a}}{k^b(a,z) - a}.$$

The denominator for this expression is always positive for entrepreneurs (a,z) with $a < k^u(a,z)$ and $S^e \geq 0$. If $w > \tilde{\pi}(a,z)$, then $\frac{\partial \tilde{w}(a,z)}{\partial a} = 0$ and

$$\frac{\partial \tilde{r}(a,z)}{\partial a} = \frac{\tilde{r}(a,z) - r}{k^b(a,z) - a} > 0,$$

which is positive, otherwise financial intermediaries surplus would be negative. If $w < \tilde{\pi}(a,z)$, then

$$\frac{\partial \tilde{r}(a,z)}{\partial a} = \frac{\tilde{r}(a,z) - \alpha \left(z a^{\alpha} \left(\frac{\theta}{w}\right)^{\theta} \right)^{\frac{1}{1-\theta}} \frac{1}{a}}{k^b(a,z) - a} < 0.$$

The numerator for this case has to be negative for an agent (a,z) with $a < k^u(a,z)$ and $S^e \geq 0$. In order to see this, notice that $\alpha \left(z a^{\alpha} \left(\frac{\theta}{w}\right)^{\theta} \right)^{\frac{1}{1-\theta}} \frac{1}{a}$ corresponds to the marginal productivity net of labor cost of an additional capital at the collateral of an entrepreneur who is borrowing. This object has to be larger than her cost of capital, $\tilde{r}(a,z)$, otherwise she will not borrow.

If $\tau(a,z) = \tau_0$, $S^e \geq 0$ and $a < k^u(a,z)$ but the incentive compatible constraint is not binding, then clearly $\frac{\partial k^b(a,z)}{\partial a} = 0$. It can also be shown that

$$\frac{\partial k^b(a,z)}{\partial z} = \frac{y^b(a,z) - (\tilde{r}(a,z) - (r+\tau_0))k^b(a,z) - (1-\alpha-\theta)\left(z a^{\alpha} \left(\frac{\theta}{w}\right)^{\theta} \right)^{\frac{1}{1-\theta}} I_{w<\tilde{\pi}}}{(1-\alpha-\theta) z (\tilde{r}(a,z) - (r+\tau_0))}.$$

This is clearly positive for any $S^e \geq 0$, $a < k^u(a,z)$ and when the incentive compatible constraint does not bind.

When $\chi \in (0,1)$, we have that in equilibrium $x_k = r + \tau(a,z)$, which is independent of χ. Therefore $\frac{\partial k^b(a,z)}{\partial \chi} = 0$. In addition,

$$\frac{\partial \tilde{r}(a,z)}{\partial \chi} = \frac{y^b(a,z) + \tau(a,z)a - \kappa - \tilde{w}(a,z)}{k^b(a,z) - a} > 0.$$

B.2 Proof of Proposition 2

When $\chi = 0$ and entrepreneurs' bargaining power is equal to one, then it is optimal to set $S^b = 0$. Then equation (7) implies

$$\phi x(k^b(a,z)) = (r+\tau) k^b(a,z),$$

and equation (8) requires:

$$\tilde{r}(a,z) = r + \tau(a,z).$$

Then the interest rate on loan will be negatively related with the asset value of each entrepreneur, a, and with her managerial productivity, z. Using the incentive compatible constraint we can show that $\frac{\partial k^b(a,z)}{\partial a} > 0$ and $\frac{\partial k^b(a,z)}{\partial z} > 0$, as required.

When $\chi = 1$ and $\tau(a,z) = \tau_0$, then it can be shown that

$$\frac{\partial \tilde{r}(a,z)}{\partial z} = \frac{(\phi x_k - \tilde{r})\tilde{w}_z(a,z) + \tilde{r}(1-\phi)x_z}{(1-\phi)x_k(k-a)}.$$

If $w > \tilde{\pi}(a,z)$, then $\tilde{w}_z(a,z) = 0$ and this expression is clearly positive. In addition, for this case $\frac{\partial k^b(a,z)}{\partial z} = -\frac{x_z}{x_k} < 0$. When $w < \tilde{\pi}(a,z)$, then $\tilde{w}_z(a,z) = x_z(a) > 0$ and we cannot determine the sign of $\frac{\partial \tilde{r}(a,z)}{\partial z}$. The first term of the numerator of the above equation is negative ($\phi x_k - \tilde{r} = \frac{\tilde{r}(\alpha+\theta-1)k - \alpha\tilde{r}a}{(1-\theta)k} < 0$.) while the second is positive. Moreover, $\frac{\partial k^b(a,z)}{\partial z} = \frac{x_z(a)-(1-\phi)x_z}{(1-\phi)x_k} < 0$, which can be positive or negative.

Notice also that:

$$\frac{\partial \tilde{r}(a,z)}{\partial a} = \frac{(\phi x_k - \tilde{r})(r + \tilde{w}_a(a,z)) + \tilde{r}(1-\phi)x_k}{(1-\phi)x_k(k-a)},$$

and

$$\frac{\partial k^b(a,z)}{\partial a} = \frac{r + \tilde{w}_a(a,z)}{(1-\phi)x_k}.$$

If $w > \tilde{\pi}(a,z)$, then $\tilde{w}_a(a,z) = 0$ and $\frac{\partial k^b(a,z)}{\partial a} = \frac{r}{(1-\phi)x_k} > 0$ and $\frac{\partial \tilde{r}(a,z)}{\partial a} = \frac{(\phi x_k - \tilde{r})r + \tilde{r}(1-\phi)x_k}{(1-\phi)x_k(k-a)}$. It can be shown that the first term of the numerator is negative while the second is positive. Therefore, it is not clear how the loan interest rate varies with assets a.

If $w < \tilde{\pi}(a,z)$, then $\tilde{w}_a(a,z) = x_k(a) - r$ and $\frac{\partial k^b(a,z)}{\partial a} = \frac{x_k(a)}{(1-\phi)x_k(k)} = \frac{1}{1-\phi}\left(\frac{k}{a}\right)^{\frac{1-\alpha-\theta}{1-\theta}} > 1$. In addition, $\frac{\partial \tilde{r}(a,z)}{\partial a} = \frac{(\phi x_k - \tilde{r})x_k(a) + \tilde{r}(1-\phi)x_k}{(1-\phi)x_k(k-a)}$. Once more it is not possible to sign the numerator of the above expression and therefore it is not clear how the loan interest rate varies with assets a.

For the case in which $\chi \in (0,1)$ and $\tau(a,z) = \tau_0$, we can easily show that

$$\frac{\partial k^b(a,z)}{\partial \chi} < 0,$$

while

$$\frac{\partial \tilde{r}(a,z)}{\partial \chi} = -\frac{((1-\theta-\alpha)\tilde{r}k + \alpha\tilde{r}a)\frac{\partial k^b(a,z)}{\partial \chi}}{k^b(a,z) - a} > 0.$$

B.3 Analytical Derivation of Figure 3

Consider the case in which $\chi = 0$. Let $d \leq a$ be the amount of assets entrepreneurs use in their business, and let l be loans, such that $k = d + l$. Clearly, since $\tilde{r} = r + \tau(a, z) \geq r$ for all finite a, then if $l > 0$, then $d = a$. The problem of the entrepreneur can be rewritten as

$$\pi(a, z) = \max_{n,d,l \geq 0} z(d+l)^{\alpha} n^{\theta} - wn - \tilde{r}l - rd - \kappa, \tag{16}$$

subject to

$$l \leq \frac{\phi(z(d+l)^{\alpha} n^{\theta} - wn)}{\tilde{r}}, \text{ with } \tilde{r} = r + \tau_0 + \frac{\tau_a}{1+a} + \frac{\tau_z}{1+z}, \tag{17}$$

$$d \leq a. \tag{18}$$

The Lagrangean associated with this problem is:

$$L = z(d+l)^{\alpha} n^{\theta} - wn - \tilde{r}l - rd - \chi + \lambda \left[\frac{\phi(z(d+l)^{\alpha} n^{\theta} - wn)}{\tilde{r}} - l \right] + \mu [a - d]$$

The Kuhn-Tucker conditions are:

$$\frac{\partial L}{\partial n} = \left(\theta \frac{y}{n} - w\right)\left(1 + \lambda \frac{\phi}{\tilde{r}}\right) \leq 0,\ n \geq 0,\ \frac{\partial L}{\partial n} n = 0, \tag{19}$$

$$\frac{\partial L}{\partial d} = \alpha \frac{y}{k}\left(1 + \lambda \frac{\phi}{\tilde{r}}\right) - r - \mu \leq 0,\ d \geq 0,\ \frac{\partial L}{\partial d} d = 0, \tag{20}$$

$$\frac{\partial L}{\partial l} = \alpha \frac{y}{k}\left(1 + \lambda \frac{\phi}{\tilde{r}}\right) - \tilde{r} - \lambda \leq 0,\ l \geq 0,\ \frac{\partial L}{\partial l} l = 0, \tag{21}$$

$$\mu [a - d] = 0, \tag{22}$$

$$\lambda \left[\frac{\phi(z(d+l)^{\alpha} n^{\theta} - wn)}{\tilde{r}} - l \right] = 0. \tag{23}$$

Case 1: If $0 < d < a$, then $\mu = 0$ and $\lambda = 0$. Therefore:

$$\theta \frac{y}{n} = w,\ \alpha \frac{y}{k} = r, \text{ and } \alpha \frac{y}{k} < \tilde{r}.$$

It can be shown that

$$k^u(z) = \left(z \left(\frac{\alpha}{r}\right)^{(1-\theta)} \left(\frac{\theta}{w}\right)^{\theta} \right)^{\frac{1}{1-\alpha-\theta}},\ n^u(z) = \left(z \left(\frac{\alpha}{r}\right)^{\alpha} \left(\frac{\theta}{w}\right)^{1-\alpha} \right)^{\frac{1}{1-\alpha-\theta}}.$$

And

$$y^u(z) = \left(z \left(\frac{\alpha}{r}\right)^{\alpha} \left(\frac{\theta}{w}\right)^{\theta} \right)^{\frac{1}{1-\alpha-\theta}},$$

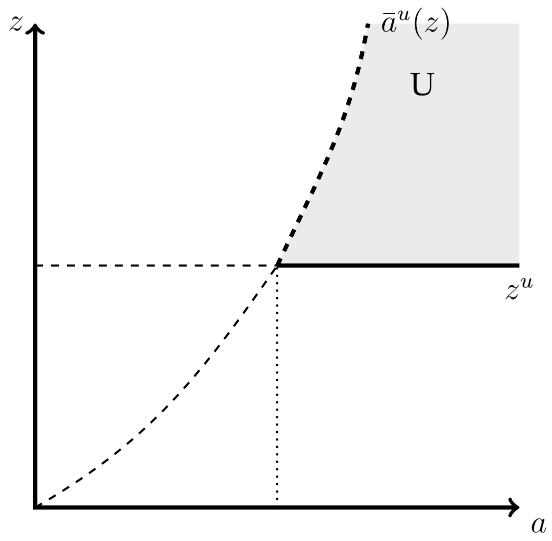


Figure B1: Case 1: $k^u(z) \leq a$. Light gray shaded area shows the measure of agents who are unconstrained entrepreneurs.

with

$$\pi^u(z) = (1 - \alpha - \theta)y^u(z) - \kappa.$$

Therefore, $\pi^u(z) \geq w$ defines a threshold ability level z^u given by

$$z^u = \left(\frac{w + \kappa}{1 - \alpha - \theta}\right)^{1-\alpha-\theta} \left(\frac{r}{\alpha}\right)^{\alpha} \left(\frac{w}{\theta}\right)^{\theta},$$

such that for all (a, z) with $a > k^u(z)$, and $z \geq z^u$ agents are entrepreneurs. Notice that z^u is independent of a. Since $k^u(z)$ is increasing with z, we can define a threshold of assets $\bar{a}^u(z)$, such that all agents with $z > z^u$ and $a > \bar{a}^u(z)$ are unconstrained entrepreneurs. Figure B1 shows the region in which entrepreneurs are unconstrained and do not borrow.

Case 2: If $d = a > 0$ and $l = 0$, then $\mu > 0$ and $\lambda = 0$. Consequently:

$$\theta\frac{y}{n} = w, \ \alpha\frac{y}{k} = r + \mu, \ \text{ and } \ \alpha\frac{y}{k} < \tilde{r}.$$

It can be shown that

$$k^{nb}(a, z) = a,$$

$$n^{nb}(a, z) = \left(za^{\alpha}\left(\frac{\theta}{w}\right)\right)^{\frac{1}{1-\theta}},$$

$$y^{nb}(a, z) = \left(za^{\alpha}\left(\frac{\theta}{w}\right)^{\theta}\right)^{\frac{1}{1-\theta}},$$

and

$$\pi^{nb}(a,z) = (1-\theta)y^{nb}(a,z) - ra - \kappa.$$

Condition $\pi^{nb}(a,z) \geq w$ defines a treshold ability level $z^{nb}(a,z)$ given by

$$z^{nb}(a,z) = \left(\frac{w+\kappa+ra}{1-\theta}\right)^{1-\theta} \left(\frac{w}{\theta}\right)^{\theta} \frac{1}{a^{\alpha}},$$

such that for all agents with $k^{nb}(a,z) = a$, and $z \geq z^{nb}(a)$ agents are entrepreneurs. Observe that since $1-\alpha-\theta > 0$, then $\lim_{a\to 0} z^{nb}(a) = \infty$. It can be shown that

$$sign\left(\frac{\partial z^{nb}(a)}{\partial a}\right) = sign\left((1-\alpha-\theta)ra - \alpha(w+\kappa)\right).$$

Notice that since $\alpha\frac{y}{a} = r + \mu$ at $z^{nb}(a)$, such that $(1-\theta)y^{nb}(a,z) - ra - \kappa = w$, we have

$$sign\left(\frac{\partial z^{nb}(a)}{\partial a}\right) = sign\left(-\mu a\right).$$

This is clearly negative, as long as $\mu > 0$. Therefore as $a \to \bar{a}^u$, then $z^{nb}(a) \to z^u$.

Case 3: If $d = a > 0$ and $0 < l < \frac{\phi(z(d+l)^{\alpha}n^{\theta}-wn)}{\tilde{r}}$, then $\mu > 0$ and $\lambda = 0$. Consequently:

$$\theta\frac{y}{n} = w, \ \alpha\frac{y}{k} = r + \mu, \ \text{ and } \ \alpha\frac{y}{k} = \tilde{r}.$$

It can be shown that

$$k^b(a,z) = \left(z\left(\frac{\alpha}{\tilde{r}}\right)^{(1-\theta)} \left(\frac{\theta}{w}\right)^{\theta}\right)^{\frac{1}{1-\alpha-\theta}},$$

$$n^b(a,z) = \left(z\left(\frac{\alpha}{\tilde{r}}\right)^{\alpha} \left(\frac{\theta}{w}\right)^{1-\alpha}\right)^{\frac{1}{1-\alpha-\theta}},$$

$$y^b(a,z) = \left(z\left(\frac{\alpha}{\tilde{r}}\right)^{\alpha} \left(\frac{\theta}{w}\right)^{\theta}\right)^{\frac{1}{1-\alpha-\theta}},$$

and

$$\pi^b(a,z) = (1-\alpha-\theta)y^b(a,z) + (\tilde{r}-r)a - \kappa.$$

Therefore, given that $\tilde{r} = r + \tau_0 + \frac{\tau_a}{1+a} + \frac{\tau_z}{1+z}$, the inequality $\pi^b(a,z) \geq w$ defines an ability level $z^b(a,z)$ given by

$$z^b(a,z) = \left(\frac{w+\kappa-(\tilde{r}-r)a}{1-\alpha-\theta}\right)^{1-\alpha-\theta} \left(\frac{r}{\alpha}\right)^{\alpha} \left(\frac{w}{\theta}\right)^{\theta},$$

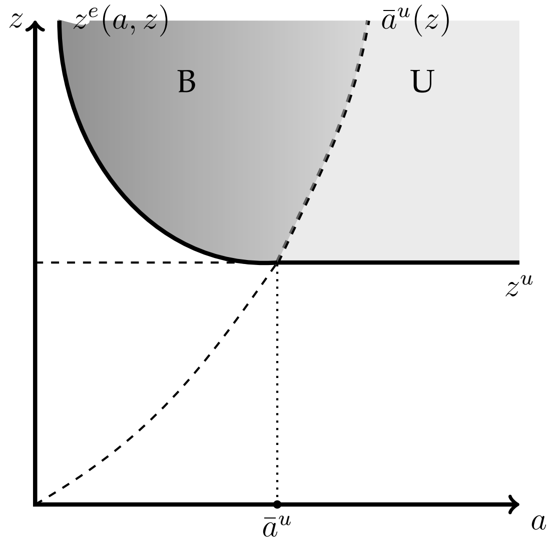


Figure B2: Cases 2 and 3: $k^u(z) < a$ and $0 \leq k - a < \frac{\phi(z(a+l)^\alpha n^\theta - wn)}{\tilde{r}}$. Light gray shaded area shows the measure of agents who are unconstrained entrepreneurs. Dark gray shaded area shows the measure of agents who are not constrained borrowers.

such that for all agents with $a < k^b(a,z) < \frac{\phi(z(d+l)^\alpha n^\theta - wn)}{\tilde{r}}$, and $z \geq z^b(a,z)$, then agents are entrepreneurs. Observe that $\frac{\partial z^b(a,z)}{\partial a} < 0$.

Cases 2 and 3 imply that for all $a \in [0, \bar{a}^u]$, there will be a productivity level $z^e(a,z) = \max\{z^u, \min(z^{nb}(a,z), z^b(a,z))\}$ such that $z^e(\tilde{r}, w, \bar{a}^u(r,w)) = z^u(r,w)$, $\frac{\partial z^e(\tilde{r},w,a)}{\partial a} < 0$, and $\lim_{a \to 0} z^{nb}(a,z) = \infty$. In addition, whenever $z \geq z^e(\bar{a}^u, z)$, then the agent is an entrepreneur. See Figure B3

Case 4: If $d = a > 0$ and $l = \frac{\phi(z(a+l)^\alpha n^\theta - wn)}{\tilde{r}}$, then $\mu > 0$ and $\lambda > 0$. Consequently:

$$\theta \frac{y}{n} = w, \ \alpha \frac{y}{k}\left(1 + \lambda \frac{\phi}{\tilde{r}}\right) = r + \mu, \ \text{ and } \ \alpha \frac{y}{k}\left(1 + \lambda \frac{\phi}{\tilde{r}}\right) = \tilde{r} + \lambda.$$

Given that the amount of capital is constrained, it must be the case that $\alpha \frac{y}{k} > \tilde{r}$. The labor first-order condition yields:

$$n(w; k^c, z) = \left(z\,(k^c)^\alpha \left(\frac{\theta}{w}\right)\right)^{\frac{1}{1-\theta}},$$

where k^c solves

$$k^c = a + \frac{\phi(z(k^c)^\alpha n(w; k^c, z)^\theta - wn(w; k^c, z))}{\tilde{r}}.$$

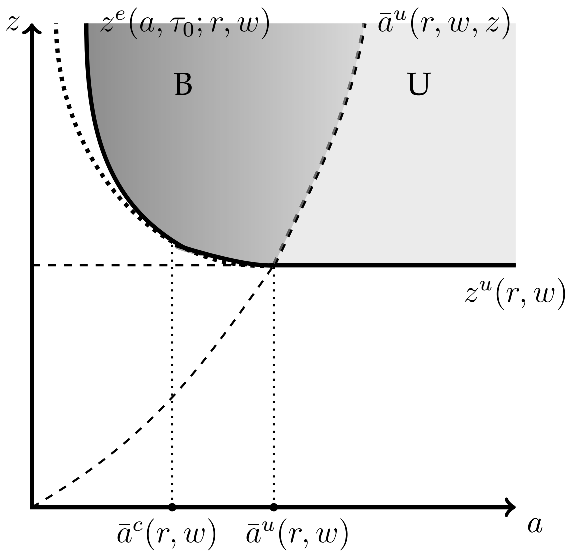


Figure B3: Cases 4: $k^u(r,w) < a$ and $k - a = \frac{\phi(z(a+l)^\alpha n^\theta - wn)}{\tilde{r}}$. Light gray shaded area shows the measure of agents who are unconstrained entrepreneurs. Dark gray shaded area shows the measure of agents who are not constrained borrowers.

This equation defines

$$k^c = k^c(\tilde{r}, w; z, a), \text{ with } \frac{\partial k^c}{\partial a} > 0, \ \frac{\partial k^c}{\partial z} > 0.$$

The derivatives can be checked using the Implicit Function Theorem.

We have that

$$y^c(\tilde{r}, w; z, a) = \left(z k^c(\tilde{r}, w; z, a)^\alpha \left(\frac{\theta}{w} \right)^\theta \right)^{\frac{1}{1-\theta}}$$

and

$$\pi^c(\tilde{r}, w; z, a) = (1-\theta)(1-\phi) y^c(\tilde{r}, w; z, a) - ra - \kappa.$$

Condition $\pi^c(\tilde{r}, w; z, a) \geq w$ defines a threshold ability level $\bar{z}^c(\tilde{r}, w; a)$, which is decreasing in a as long as $\lambda > 0$. We can show that $\lim_{a \to 0} \bar{z}^c(\tilde{r}, w; a) = \infty$. Observe that when $\lambda = 0$ and $l = \frac{\phi(z(a+l)^\alpha n^\theta - wn)}{\tilde{r}}$, then for agents who are indifferent to be entrepreneurs or workers, we have that $\bar{z}^c(\tilde{r}, w; a) = \bar{z}^b(\tilde{r}, w; a)$. This defines a value $\bar{a}^c(w, \tilde{r})$, such that whenever $a < \bar{a}^c(w, \tilde{r})$ and $\bar{z}^b(\tilde{r}, w; a) \leq z \leq \bar{z}^c(\tilde{r}, w; a)$, the leverage constraint is binding. For such agents, then $l = \frac{\phi(z(a+l)^\alpha n^\theta - wn)}{\tilde{r}}$ and $\lambda > 0$, and $\bar{z}^c(\tilde{r}, w; a) > \bar{z}^b(\tilde{r}, w; a)$, in order to compensate for the low capital used. Therefore for any $\bar{z}^b(\tilde{r}, w; a) \leq z \leq \bar{z}^c(\tilde{r}, w; a)$ and $a < \bar{a}^c(w, \tilde{r})$, the occupational choice is restricted by the leverage ratio. This is shown in the figure below.

C Quantitative Results for the Small Open Economy

Table C2: Small Open Economy: Impacts of Credit Frictions on Development

Value Relative to Perfect Credit:	Perfect Credit (1)	Benchmark (2)	No Spread Disp. (3)	Only Quant. Constr. (4)
Aggregate values relative to perfect credit world:				
GDP	1.00	0.33	0.34	0.41
TFP	1.00	0.72	0.76	0.84
Wage	1.00	0.36	0.41	0.43
Capital	1.00	0.09	0.09	0.12
Credit/GDP	1.00	**0.08**	**0.08**	**0.08**
Interest rate	**0.02**	0.02	**0.02**	**0.02**
Firm growth	0.08	0.03	-0.02	0.04
Exit rate	0.29	0.10	0.17	0.15
Avg. firm size	9	14	9	8

Notes: Column (2) parameter values are those calibrated in Table 2. The other columns keep the interest rate constant while changing parameters. Relative to benchmark values, Column (1): $\tau_i = \chi = 0$, Column (3): $\tau_a = \tau_z = \chi = 0$ and $\tau_0 = 0.30$, and Column (4): $\tau_i = \chi = 0$ and $\phi = 0.04$, calibrated to match credit/GDP in the benchmark (and data).

Table C3: Small Open Economy: Isolated Impacts of Spread-Causing Frictions

		Eliminating Frictions				Single Friction Calibrations			
	Benchmk	No Market Power	No Uniform Cost	No $a-$ depend. Cost	No $z-$ depend. Cost	All Market Power[†]	All Uniform Cost	All $a-$ depend. Cost	All $z-$ depend. Cost
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Aggregate values relative to perfect credit world:									
GDP	0.33	0.33	0.36	0.35	0.43	0.25	0.35	0.24	0.35
TFP	0.72	0.72	0.72	0.75	0.74	0.58	0.77	0.59	0.75
Wage	0.36	0.36	0.40	0.40	0.47	0.26	0.42	0.25	0.38
Capital	0.09	0.10	0.12	0.10	0.19	0.08	0.09	0.06	0.10
Credit/GDP	0.08	0.08	0.17	0.09	0.31	0.13	0.09	0.00	0.00
Firm credit spread moments:									
Interest rate	0.02	0.02	0.02	0.02	0.02	0.07	0.02	0.02	0.02
Avg. (weighted)	**0.28**	0.17	0.27	0.11	0.12	**0.03***	**0.28**	**0.17***	**0.28**
Avg. (unweighted)	0.69	0.67	0.55	0.40	0.33	0.02	0.28	0.17	0.51
Std. deviation	0.34	0.33	0.33	0.06	0.28	0.01	0.00	0.01	0.11
Frac. with credit	0.23	0.23	0.28	0.38	0.46	0.52	0.58	0.00	0.29
Firm growth	0.03	0.03	0.05	-0.00	0.12	0.04	-0.01	0.03	0.00
Exit rate	0.10	0.10	0.11	0.14	0.14	0.03	0.19	0.02	0.12
Avg. firm size	14	14	17	11	21	9	9	7	11

Notes: Column (1) parameter values are those calibrated in Table 2. The other columns keep the interest rate constant while changing parameters. Relative to these values, Column (2): $\chi = 0$, Column (3): $\tau_0 = 0$, Column (4): $\tau_a = 0$, Column (5): $\tau_z = 0$, Column (6): $\tau_i = 0$ and $\chi = 0.01$, Column (7): $\tau_a = \tau_z = \chi = 0$ and $\tau_0 = 0.28$, Column (8): $\tau_0 = \tau_z = \chi = 0$ and $\tau_a = 457$, and Column (9): $\tau_0 = \tau_a = \chi = 0$ and $\tau_z = 1.65$. For Columns (6) and (8), the calibrated value is chosen to match the weighted spread in the benchmark.

[†]: The single parameter alone cannot match the weighted spread of 0.28. Hence, Columns (6) and (8) yield spreads substantially below target but approaching their peaks.

D Additional Results for One Friction at a Time

Table D4: Benchmark: One Friction at a Time

	Benchmark (1)	Only Market Power (2)	Only Uniform Cost (3)	Only a-depend. Cost (4)	Only z-depend. Cost (5)
GDP	0.61	0.99	0.82	0.88	0.68
TFP	0.72	1.00	0.90	0.89	0.79
Wage	0.68	0.99	0.91	0.86	0.81
Capital	0.59	0.98	0.74	0.97	0.64
Credit/GDP	0.32	0.99	0.81	1.04	0.65
Interest rate	0.019	0.144	0.100	0.112	0.036
Avg. Unweighted	0.692	0.005	0.084	0.195	0.302
Avg. Weighted	0.282	0.007	0.084	0.020	0.169
Std. Deviation	0.343	0.003	0.000	0.246	0.063
Frac. with credit	0.232	0.962	0.909	0.761	0.602
Firm growth	0.034	0.060	0.055	0.228	0.033
Exit rate	0.098	0.297	0.292	0.287	0.262
Avg. firm size	14	11	11	30	16

Notes: Column (1) provides results for the benchmark, with the parameters from Table 2. The other columns shut off all spread-causing parameters but one: Column (2) has $\tau_i = 0$ and $\chi = 0.10$; Column (3) has $\chi = \tau_a = \tau_z = 0$ and $\tau_0 = 0.08$; Column (4) has $\chi = \tau_0 = \tau_z = 0$ and $\tau_a = 0.73$; Column (5) has $\chi = \tau_a = \tau_0 = 0$ and $\tau_z = 0.93$.

E Calibration for the Scenario with Moderate Spreads

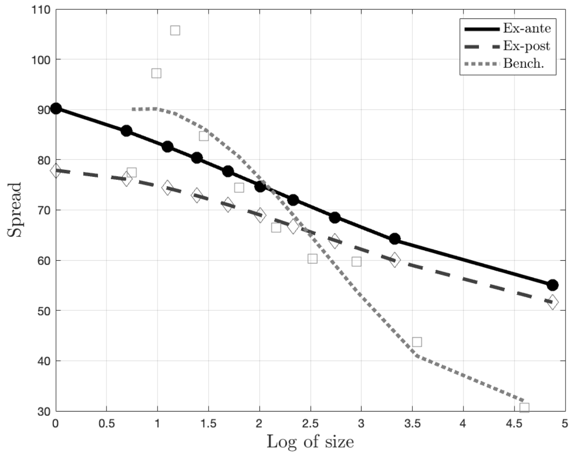
Table E5: Calibration and Model Fit, Moderate Spreads

Parameter Values		
Parameter	**Description**	**Value**
2 assigned parameters		
σ	Coefficient of relative risk aversion	1.50
δ	Depreciation rate	0.03
11 calibrated parameters		
ρ	Subjective discount rate	0.24
α	Elast. of y with respect to k	0.33
θ	Elast. of y with respect to n	0.39
κ	Fixed cost of production	0.66
η	Curvature of the Pareto distr.	3.41
γ	New productivity arrival rate	0.30
ϕ	Enforcement parameter	0.20
τ_0	Interm. costs - independent factor	0.05
τ_a	Interm. costs - elast. of assets	0.91
τ_z	Interm. costs - elast. of productivity	0.29
χ	Bank barg. power in a loan	0.52

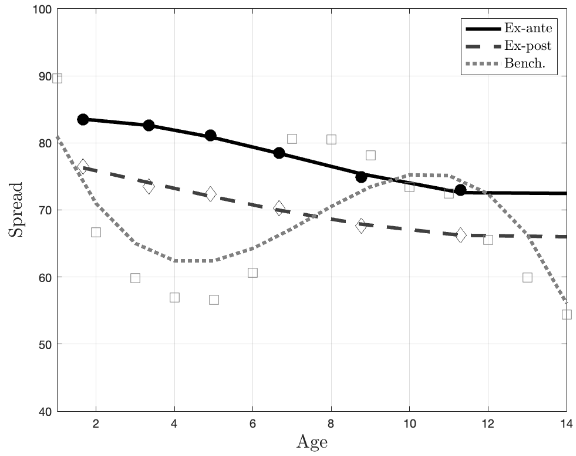
Model Fit		
12 Targeted Moments	**Data**	**Model**
Risk-free bond rate	0.020	0.019
Capital-output ratio	2.55	1.92
Average firm growth rate	0.034	0.035
Top 10% earners' income share	0.56	0.55
Average firm size	13	17
Top 10% firms' employment share	0.77	0.75
Firm exit rate	0.11	0.07
External finance to GDP ratio	0.49	0.57
Fraction firms with credit	0.26	0.30
Average spread (unweighted)	0.28	0.34
Average spread (credit-weighted)	0.14	0.14
Standard deviation of spread	0.23	0.19

F The Relationship between Spreads and Firm Characteristics: Model and Data

Figure F4: Ex Ante and Ex Post Spreads



(a) Spreads by firm size.



(b) Spreads by firm age.

Notes: Ex ante spreads are based on contracted interest rates. Ex post spreads are calculated by setting the interest rate to -100% for loans in default.