Incremental Use of Multiple Transmitters for Low-Complexity Diversity Transmission in Wireless Systems

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Abstract—In this paper we develop and analyze low-complexity approaches called incremental multiple-input multiple-output (IMIMO) for exploiting multiple antennas for reliable wireless communications. The proposed schemes leverage consecutive uses of a single transmit antenna combined with automatic repeat request (ARQ) feedback. Unlike multiple-input multiple-output (MIMO) communications without feedback, the schemes we propose do not require a space-time encoder or decoder because they only use one transmit antenna at a time. We compare the performance of IMIMO schemes with the corresponding MIMO system without feedback using outage probability and average long-term throughput as metrics for comparison. These comparisons show that for relatively low rates IMIMO schemes have better performance than the corresponding MIMO system without feedback. For higher rates, MIMO with higher complexity performs better up to a certain signal-to-noise ratio, but beyond this threshold IMIMO schemes again have better performance.

Index Terms—multiple antennas systems, automatic repeat request, incremental redundancy, diversity, outage, throughput.

I. INTRODUCTION

Multiple-antenna wireless systems provide both reliable and high data rate communication. The corresponding multiple-input multiple-output (MIMO) system models have been well studied since [1], [2] predicted linear capacity increases in the number of antennas in rich scattering environments. The field of space time codes (STC) has developed to exploit the benefits of multiple antennas at the transmitter [3]–[7], but complexity remains a challenge. Because MIMO induces interference among the multiple transmitters, the optimal receiver for spatial multiplexing systems is a joint maximum likelihood (ML) decoder. However, the complexity of the ML decoder is exponentially increasing with the number of antennas and is often impractical.

Our desire is to develop low-complexity multiple transmit and receive antenna wireless systems that provide the increased diversity and rate due to multiple antennas. Our approach is to simplify the encoder, by exploiting automatic repeat request (ARQ) feedback, which in turn simplifies the decoder. The simplified encoder uses only one transmitter from the set of all possible transmitters, and at any given time the receiver can decode the message optimally with comparatively low complexity. To take advantage of the multiple transmitters, we incorporate ARQ into the system in such a way that different transmit antennas are used for diversity transmissions.

The proposed incremental multiple-input multiple-output (IMIMO) protocols combine consecutive emissions from individual transmit antennas with different types of ARQ feedback. This structure is illustrated in Fig. 1. The schemes encode and transmit a message through a randomly chosen transmit antenna, and wait for ARQ feedback; if the transmission is successful, they transmit the next message in the transmitter queue and, if not, they encode and transmit the same message through a different transmitting antenna to exploit space diversity. The idea of the incremental use of individual transmit antennas was inspired by [8], in which incremental use of a single relay is used for cooperative communications. However, in contrast to [8], in IMIMO there can be multiple retransmissions which complicates the analysis and design.

A. Related Work

To reduce the complexity of the decoder in multi-antenna systems, some suboptimal receivers such as non-linear successive interference cancelation (SIC), linear zero forcing (ZF), and minimum mean square error (MMSE) can be employed, but their performance is substantially inferior of that of optimal detectors [9]. Some other near-optimal receiver algorithms have been proposed, such as sphere decoding (SD) [10], [11] and QR decomposition with the M-algorithm (QRD-M) [12] for uncoded systems and soft SD [13], [14] for coded systems, but their complexity remains high. The vertical Bell Labs layered space time (V-BLAST) transmitter in conjunction with MMSE-SIC receiver and diagonal BLAST (D-BLAST) transceiver are the optimal structures for fast and slow fading environments, respectively, but they are also highly complex.
[9]. There has been some effort in designing near-optimal low-complexity schemes [15]–[18]; however, these still either exhibit complexity problems or their performance gap to the optimal schemes is significant.

Combinations of MIMO and ARQ have been studied in [19]–[22]. In [20], [21] several ARQ schemes have been proposed to improve the throughput of MIMO systems. In [22] a specific hybrid ARQ (H-ARQ) scheme is analyzed and shown to decrease the performance gap to the capacity of a slowly varying MIMO channel. The proposed schemes have high complexity since they use all possible transmit and receive antennas. To the best of our knowledge, no prior work has explored low-complexity schemes that incrementally make use of individual transmit antennas.

In [19] a diversity-multiplexing-delay-tradeoff (DMDT), similar to the diversity-multiplexing-tradeoff (DMT) [23], is introduced to evaluate the performance of MIMO-ARQ schemes. However, the DMT, like the DMT, is a metric for the high signal-to-noise ratio (SNR) regime and does not provide a complete picture for evaluating performance of ARQ schemes, since at high SNR ARQ is rarely used because the probability of obtaining a negative acknowledgement (NACK) is very small (proportional to $SNR^{-1}$). There has been a finite SNR DMT proposed in [24] for evaluating performance of MIMO schemes in the finite SNR regime, which can be generalized to the finite DMDT and used to evaluate the performance of MIMO-ARQ schemes. Both the DMT and DMDT are derived from outage probability, so they can in principle be deduced from our outage analysis in the sequel.

### B. Summary of Results

In this paper we develop and analyze three low-complexity feedback schemes for communicating using multiple transmit antennas and possibly multiple receive antennas, without requiring STC or joint decoding. We consider rich-scattering, low-mobility, and narrow-band environments modeled by a flat and slow Rayleigh fading channel. Since the Shannon capacity for such a flat fading model is zero, to evaluate the performance of the proposed schemes, we use the notion of outage events and probability of outage [25]. However, outage analyses are limited to a finite number of fading blocks. Thus, to evaluate the long-term performance of IMIMO schemes, we also use the long-term throughput (LTT) as a performance metric. Our modeling and analysis suggest that, in some wide SNR and rate regimes, the IMIMO schemes achieve performance superior to MIMO systems without feedback that have the same number of transmit and receive antennas, use all transmitters simultaneously, and perform outage-optimal processing at the transmitter and receiver.

The main contributions of this paper are as follows:

1) We develop three low-complexity and high-performance feedback schemes to communicate using multiple transmitters and multiple receivers based on three types of feedback and decoding, namely, ARQ, H-ARQ with chase combining (CC), and H-ARQ with incremental redundancy (IR).

2) We develop exact outage probability and tight bounds on outage probability and LTT of these schemes. Performance comparisons of these schemes with a corresponding MIMO system without feedback reveal the high performance of IMIMO for a wide range of rates and SNRs.

### C. Paper Structure

The remainder of the paper is organized as follows. In Section II we describe the baseline system model. We elaborate on IMIMO and compare it with MIMO in Section III, followed by outage and long-term throughput analysis of the protocols in Sections IV and VI, respectively. We conclude by providing numerical results and concluding remarks in Sections VII and VIII, respectively. Proofs can be found in the Appendix.

### II. System Model

In this section we describe the channel model for a wireless system with multiple transmitters and receivers. Then we review coding and decoding for systems equipped with ARQ feedback. Finally, we specialize the channel model and ARQ coding/decoding schemes to our proposed IMIMO structure.

#### A. Channel Model

Let $M$ and $N$ be the number of antennas at the transmitter and receiver, respectively. We consider slow Rayleigh fading that remains constant throughout a fixed number of transmissions, and changes independently thereafter, a typical model for rich scattering environments and limited mobility. The focus is on frequency non-selective (flat) fading with small delay spreads, appropriate for narrowband systems and indoor environments. To address wideband systems with mobility, one would incorporate frequency selectivity and time variation into the models, making them considerably less tractable analytically. In each transmission, $T$ consecutive uses of the channel are considered to transmit a message from the transmitter to the receiver. No channel state information (CSI) is utilized at the transmitter (no CSIT), but full CSI is obtained at the receiver (CSIR). We note that CSIT is always imperfect and delayed, and also for large number of antennas there can be substantial overhead in obtaining accurate CSIT; thus, lack of CSIT results in a simpler system.

The baseband-equivalent discrete-time channel model is

$$y_l[t] = H_l x_l[t] + z_l[t]$$

(1)

where $l = 1,2,...,L$ and $t = 1,2,...,T$ count the transmission and channel use, respectively. The effects of multipath fading are captured by the matrix $H_l = H$ for $l = 1,2,\ldots,L$ for slow fading, where

$$H = \begin{pmatrix} h_{1,1} & h_{1,2} & \cdots & h_{1,M} \\ h_{2,1} & h_{2,2} & \cdots & h_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N,1} & h_{N,2} & \cdots & h_{N,M} \end{pmatrix} \in \mathbb{C}^{N \times M}$$
and \( h_{n,m} \) is the fading coefficient between the \( m^{th} \)
transmit antenna and \( n^{th} \) receive antenna, is independent
of other coefficients, and is modeled by a zero-mean circularly symmetric complex Gaussian random variable with unit variance, i.e., \( \mathcal{CN}(0,1) \). The column vectors\(^1\) \( x_t[t] = [x_{1,t}[t], x_{2,t}[t], \ldots, x_{M,t}[t]]^T \) and \( y_t[t] = [y_{1,t}[t], y_{2,t}[t], \ldots, y_{N,t}[t]]^T \) correspond to the input and output of the channel, respectively, where \( x_{m,t}[t] \in \mathbb{C} \) is the symbol transmitted through antenna \( m \) and \( y_{n,t}[t] \in \mathbb{C} \) is the received signal from antenna \( n \), both at round \( l \) and channel use \( t \). The sequence \( z_t[t] = [z_{1,t}[t], z_{2,t}[t], \ldots, z_{N,t}[t]]^T \) with independent identically distributed (i.i.d.) elements captures the receiver thermal noise and possibly other interference and is assumed to be independent of the channel matrix and input of the channel. Here \( z_{m,t}[t] \in \mathbb{C} \) is assumed to be \( \mathcal{CN}(0, \sigma_z^2) \). Without loss of generality, we assume \( \sigma_z^2 = 1 \) throughout this paper.

In addition, each block must satisfy the average input power constraint given by
\[
\frac{1}{T} \|x_t\|^2_F \leq P \tag{2}
\]
for \( l = 1,2,\ldots,L \), where \( \| \cdot \|_F \) denotes the Frobenius norm [26]. The SNR per received antenna is denoted by \( \rho = P/\sigma_z^2 \).

### B. Coding/Decoding with ARQ

In a communication system with \( M \) transmit and \( N \) receive antennas equipped with ARQ, a message is sent using a maximum of \( L \) transmissions and up to \( L-1 \) ARQ feedback transmissions. Each message \( i \) is encoded using a codebook, \( B[i] \), which itself consists of up to \( L \) codebooks
\[
B[i] = \{ C^i_l \in \mathbb{C}^{M \times T} : l = 1,2,\ldots,L \}, \quad i = 1,2,\ldots \tag{3}
\]
where \( C^i_l \) is the codebook for message \( i \) at transmission \( l \). In other words, for each transmission, \( l \), the message \( i \) is encoded using the corresponding codebook \( C^i_l \).

Since errors should be detected in the ARQ protocol, the decoder needs to use an algorithm that is capable of detecting errors instead of the optimal ML decoder. There exists a suboptimal joint typicality decoder that can detect errors and is optimal in the limit of large retransmissions \( L \) [27].

Different constructions of the codebooks result in different ARQ system designs. Here we summarize three common designs of the codebooks and decoders.

1) If the decoder considers only the current received codeword in decoding message \( i \), then the encoder only needs to design a single codebook \( C^i_l \in \mathbb{C}^{M \times T} \), and it can use this codebook for all transmissions. We analyze the performance of random coding, in which the codebook is generated according to a given probability density function \( q(x) \), with i.i.d. components in time. The input distribution \( q(x) \) that maximizes the mutual information for a given SNR \( \rho \) is circularly symmetric complex Gaussian input with some covariance matrix \( \mathcal{CN}(0, Q_x) \), such that \( \text{Tr}(Q_x) \leq P \). Using this input and the mentioned joint typicality decoder, the mutual information between the input and output of the channel takes the form\(^2\)
\[
\log \det \left( I_M + \frac{1}{\sigma_z^2} H Q H^* \right),
\]
for all transmissions.

2) If the decoder considers all previously received codewords in decoding the current message \( i \), and if the codebooks are generated randomly but identically for each transmission \( l \), with i.i.d. components in time, according to a given probability density function \( q(x) \), it results in so-called H-ARQ with CC design. The input distribution \( q(x) \) that maximizes the mutual information for a given SNR \( \rho \) is again circularly symmetric complex Gaussian input with some covariance matrix \( Q_x \), \( \mathcal{CN}(0, Q_x) \), such that \( \text{Tr}(Q_x) \leq P \). Using this input the mutual information at transmission \( L \) takes the form of
\[
\log \det \left( I_M + \frac{1}{\sigma_z^2} H^L Q H^L \right)
\]
considering the same joint typicality decoder as in previous design. Here \( H^L \) is a \( LN \times M \) matrix resulting from vertically concatenating the channel matrices \( H_1,\ldots,H_L \), i.e.,
\[
H^L = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_L \end{bmatrix}.
\]

3) If the decoder considers all previously received codewords in decoding the current message \( i \), and if the codebooks are generated randomly and independently for each transmission, with i.i.d. component in time, according to a given probability density function \( q(x) \), it results in so-called H-ARQ with IR design. Like the previous two designs, the input distribution \( q(x) \) that maximizes the mutual information at transmission \( l \) is circularly symmetric complex Gaussian input with some covariance matrix \( Q_{x,l} \), \( \mathcal{CN}(0, Q_{x,l}) \), such that \( \text{Tr}(Q_{x,l}) \leq M \). Using this input the mutual information at transmission \( L \) in this case is accumulated over different transmissions using the joint typicality decoder [27]
\[
\log \det \left( I_M + \frac{1}{\sigma_z^2} H L Q H^L \right)
\]
for ARQ schemes with a maximum of \( L \) transmissions, an error occurs if a decoding error occurred at transmissions \( l < L \) and the decoder fails to detect it, or if a decoding error occurs during transmission \( l = L \).

We note that the increase in mutual information in H-ARQ schemes in comparison to ARQ is at the expense of adding memory in the receiver to retain the previously received data.

### C. Specializing to IMIMO

We define one round of IMIMO as up to \( L \) possible transmissions and up to \( L-1 \) error free, zero-delay ARQ feedback signals. Each transmission is performed using one transmit antenna and possibly multiple receiver antennas. Transmitting through one antenna in IMIMO is equivalent to

\(^1\) \( x^T \) represents the transpose of vector \( x \).

\(^2\) All logarithms are in base of 2 throughout the paper.
choosing the corresponding column and row of \( \mathbf{H}_l \) and \( \mathbf{x}_l \) in (1), respectively, and setting the other columns and rows to zero. Thus, the effective channel matrix for transmission \( l \) through transmitting antenna \( m \) within a round of IMIMO is \( \mathbf{h}_{m,l} = [h_{1,m}, h_{2,m}, \ldots, h_{N,m}]^\top \), independent of \( l \) for slow fading.

The fading matrix, \( \mathbf{H}_l \), is assumed to remain constant during \( L \) transmissions and changes independently thereafter. Also as mentioned before, the transmitter changes the transmit antenna for each transmission within an IMIMO round and since the columns of \( \mathbf{H}_l \) are independent, the effective channel matrix \( \mathbf{h}_{m,l} \) changes independently for each transmission within each round of IMIMO. Therefore, although the encoder could obtain some estimate of CSI through ACK/NACK, this partial CSIT cannot be used to select the best antenna at the transmitter. In other words, because of using only one transmit antenna for each transmission, the CSIT provides information only about the current antenna but not the other antennas for which the channels are different. However, the transmitter can do power adaption in each retransmission within each round of IMIMO based on the partial CSIT which is done in [28].

It is important to note that the messages are transmitted with an a priori fixed rate of \( R \) b/s/Hz in the first transmission within a round of IMIMO, i.e., there is no rate adaption in our schemes, which could be a future direction for extending this work. Furthermore, it makes sense that we let \( L \geq M \) to enable the full diversity benefit of \( M \times N \).

### III. Incremental MIMO

In this section, we describe the three schemes for IMIMO in more detail and compare with MIMO in terms of the complexity of their implementations.

The first scheme uses ARQ (i.e., no H-ARQ), the second scheme uses H-ARQ with CC, and the third scheme uses H-ARQ with IR for which analysis is more involved. For an initial transmission at rate \( R \) b/s/Hz, the transmitter creates a sequence of i.i.d. uniform messages \( w_1, w_2, \ldots, w_L, \ldots \) such that \( w_l \in [1, 2^{TR}] \). IMIMO can be described at a high level as follows:

- At the beginning of a round of transmission for message \( w_1 \), choose one of \( M \) transmitting antennas uniformly and initiate transmission of the first code of the first message \( C_1^l(w_1) \).
- If the transmission is successful, initiate a new round of transmissions for message \( w_2 \); if the transmission is not successful, choose another transmitting antenna uniformly from the remaining \( M - 1 \) antennas and transmit the corresponding codeword for the first message, \( C_1^l(w_1) \) for ARQ and H-ARQ with CC and \( C_2^l(w_1) \) for H-ARQ with IR.
- Additional transmissions are continued until the receiver decodes a message correctly or reaches the maximum of \( L \) transmissions.

If the system uses ARQ, decoding of a message is done based on the current received codeword for that message; and if it uses H-ARQ, decoding is based on all the signals received over the different transmissions within an IMIMO round.

For the case in which the transmitter gets an acknowledgement (ACK) from the receiver, it can transmit the next codeword through either the same antenna (the same channel matrix) or a different one (independent channel matrix). However, we leave this as an option for the transmitter to choose based on the channel conditions and the performance metric of interest.

It is known that H-ARQ with IR has better performance than H-ARQ with CC [29], and it is apparent that the performance of H-ARQ with CC is superior to ARQ. However, the complexity of ARQ is less than H-ARQ with CC, and H-ARQ with CC is in turn simpler than H-ARQ with IR. In fact, H-ARQ introduces memory at the receiver, however, this is the only complexity added in the physical layer to IMIMO using H-ARQ in comparison to SIMO.

Using a single antenna for each transmission allows for a low-complexity transmitter and receiver as it does not require space-time encoders and decoders like in MIMO, which uses all transmit antennas at the same time. IMIMO requires only a single radio frequency (RF) chain in the transmitter that can be shared among the transmit antennas, making the transmitter circuitry much simpler compared to the \( M \) RF chains required by MIMO.

### IV. Outage Analysis

In this section, we derive the outage probability of the three IMIMO schemes presented in Section III. For IMIMO using H-ARQ with IR we also derive an upper bound on its outage probability, because the exact outage probability expression is involved. We also derive a lower bound on the outage probability of a MIMO system without feedback for comparison. Such a comparison is reasonable since it compares either the exact or worst-case performance of IMIMO with the best-case performance of MIMO.

We define an outage event at transmission \( l \) within a round of IMIMO as the event in which the transmitter gets a NACK in all transmissions from 1 to \( l \). The outage probability at transmission \( l \) is the probability of the corresponding outage event.

#### A. ARQ

We begin with the outage analysis of IMIMO using ARQ. Specializing Section II-B, for \( M = 1 \) using circularly symmetric complex Gaussian scalar inputs and a joint typicality decoder, the mutual information between the input and output of the channel to decode a specific message takes the form of \( \log(1 + S_1) \), where the received SNR \( S_1 \) in transmission \( l \) is simply \( \rho \| \mathbf{h}_l \|^2 \). The following fact gives the outage probability of IMIMO using ARQ.

**Fact 1.** For an incremental MIMO system with \( M \) transmit antennas, \( N \) receive antennas, and using ARQ with a maximum number of \( L \) transmissions within each round, the exact outage probability for Rayleigh fading in the channel model...
of (1) is
\[ p_{\text{out,IMIMO}}^{\text{ARQ}}(\rho, L, R) := \prod_{l=1}^{L} \mathcal{P}\{ \log (1 + \rho \| h_l \|^2) < R \} \]
\[ = \left( \mathcal{P} \left( \frac{2R - 1}{\rho}, LN \right) \right)^L, \tag{5} \]
where \( \mathcal{P}(x, a) \) is the normalized incomplete Gamma function defined as \( \mathcal{P}(x, a) = \frac{1}{\Gamma(a)} \int_1^\infty t^{a-1} e^{-tx} dt, \Gamma(a) \) is the Gamma function, \( R \) is the transmission rate in b/s/Hz, and \( \rho \) is the SNR per received antenna.

**Proof**: The proof is based on the exact outage probability for a SIMO system which is derived in [1] and we include it in Appendix A for completeness.

By using the following approximation [30]
\[ \mathcal{P}(x, N) \approx \frac{x^N}{N \Gamma(N)} \quad as \quad x \rightarrow 0 \tag{6} \]
we obtain a high-SNR approximation for the outage probability of IMIMO using ARQ as
\[ p_{\text{out,IMIMO}}^{\text{ARQ}}(\rho, L, R) \approx \left( \frac{\rho (N \Gamma(N))^{N-1}}{2R - 1} \right)^{-NL} \propto \rho^{-NL} \tag{7} \]
which shows the diversity order of IMIMO systems using ARQ is \( NL \) and its coding gain at high SNR is \( (N \Gamma(N))^{N-1} \).

It is clear that the outage probability of IMIMO using ARQ is strictly less than the outage probability of SIMO without feedback. However, as we will see in Section VII, IMIMO using ARQ has better performance than MIMO without feedback in a wider range of rates and SNRs than IMIMO using ARQ.

**B. H-ARQ with CC**

Aiming for better performance, next we consider IMIMO that exploits H-ARQ using CC. Again specializing Section II-B, for \( M = 1 \) using repetition codebooks across transmitters with circularly symmetric complex Gaussian scalar inputs and a joint typicality decoder, the mutual information between the input and output of channel to decode a specific message takes the form
\[ \log \left( 1 + \sum_{l=1}^{L} S_l \right). \]
The following fact gives the outage probability of IMIMO using H-ARQ with CC.

**Fact 2.** For an incremental MIMO system with \( M \) transmit antennas, \( N \) receive antennas, and using H-ARQ with CC and a maximum number of \( L \) transmissions within each round, the exact outage probability for Rayleigh fading in the channel model of (1) is
\[ p_{\text{out,IMIMO}}^{\text{H-ARQ, CC}}(\rho, L, R) := \mathcal{P}\{ \log (1 + \rho \| h_l \|^2) < R \} \]
\[ = \mathcal{P} \left( \frac{2R - 1}{\rho}, LN \right) \tag{8} \]
\[ \approx \left( \frac{\rho (L \Gamma(L))^L}{2R - 1} \right)^{-NL} \propto \rho^{-NL} \tag{9} \]

**Proof**: The proof is again based on the exact outage probability for a SIMO system which is derived in [1] and we include it in Appendix B for completeness.

Using the approximation (6), we obtain a high-SNR approximation for the outage probability of IMIMO using H-ARQ with CC as
\[ p_{\text{out,IMIMO}}^{\text{H-ARQ, CC}}(\rho, L, R) \approx \left( \frac{\rho (L \Gamma(L(N)))^{N-1}}{2R - 1} \right)^{-NL} \propto \rho^{-NL} \tag{10} \]
which shows the diversity order of IMIMO systems using H-ARQ with CC is \( NL \) and its coding gain at high SNR is \( (L \Gamma(L(N)))^{N-1} \).

As we will see in Section VII, IMIMO using H-ARQ with CC has better performance than MIMO without feedback in a wider range of rates and SNRs than IMIMO using ARQ.

**C. H-ARQ with IR**

To further boost the performance of IMIMO, the system can use H-ARQ with IR at the expense of adding a bit more complexity in the encoder due to multiple codebooks.

Specializing Section II-B, for \( M = 1 \) using independently generated codebooks across transmit antennas with circularly symmetric complex Gaussian scalar inputs and a joint typicality decoder, the mutual information between the input and output of channel to decode a specific message takes the form
\[ \log (1 + S_1). \]
As we will see, computing the exact outage probability is more involved in this case compared to the previous cases, so we start with an upper bound on the outage probability in the following theorem.

**Theorem 1.** For an incremental MIMO system with \( M \) transmit antennas, \( N \) receive antennas, and using H-ARQ with IR and a maximum number of \( L \) transmissions within each round, the outage probability for Rayleigh fading in the channel model of (1) is upper bounded by
\[ p_{\text{out,IMIMO}}^{\text{H-ARQ, IR}} := \mathcal{P}\{ \sum_{l=1}^{L} \log (1 + \rho \| h_l \|^2) < R \} \]
\[ \leq \left( \frac{1}{(N - 1)!} \right)^L G_{1,L+1}^{L,1} \left( \frac{2R - 1}{\rho} \right)^L \tag{11} \]
where \( G_{p,q}^{m,n} (b_1, ..., b_q | a_1, ..., a_p, z) \) is the Meijer-G function [30].

**Proof**: See Appendix C.

Furthermore, by using the expansion [30, 9.33.1] and letting \( \rho \rightarrow \infty \), we can approximate the above upper bound at high SNR as
\[ \left( \frac{\rho ((N - 1))^{N-1}}{G_{1,L+1}^{L,1} \left( \frac{2R - 1}{\rho} \right)^L} \right)^{-NL} \propto \rho^{-NL} \tag{13} \]
which shows that the maximum diversity order of IMIMO using H-ARQ with IR is \( NL \).

Comparing (11) and (8), it is clear by convexity that
\[ p_{\text{out,IMIMO}}^{\text{H-ARQ, IR}} \leq p_{\text{out,IMIMO}}^{\text{H-ARQ, CC}}. \]
Although the result is more involved, the exact outage probability of IMIMO using H-ARQ with IR is given in the following theorem.

**Theorem 2.** For an incremental MIMO system with $M$ transmit antennas, $N$ receive antennas, and using H-ARQ with IR and a maximum number of $L$ transmissions within each round, the exact outage probability for Rayleigh fading in the channel model of (1) is

$$p_{\text{out,IMIMO}}^{\text{H-ARQ, IR}}(\rho, L, R) := \mathbb{P}\left\{ \sum_{i=1}^{L} \log \left( 1 + \rho \left\| h_i \right\|^2 \right) < R \right\},$$

$$= 2^R g(-R \ln 2) - g(0),$$

where

$$g(t) = \mathcal{L}^{-1} \left\{ \frac{1}{1 - s} \left[ \frac{U(N, N + s, \frac{1}{\rho})}{\rho^N} \right]^L \right\}$$

and $U(a, b, c)$ is the Tricomi confluent hypergeometric function [30], $R$ is the rate in b/s/Hz, $\rho$ is the SNR per received antenna, and $\mathcal{L}^{-1}$ denotes the inverse Laplace transformation with respect to $s$.

**Proof:** See Appendix D.

We note that the exact outage probability expression in Theorem 2 involves an inverse Laplace transform of the distribution of a product of generalized Gamma distributions, for which there is no simpler closed form. This is in contrast to the distribution of the product of Gamma distributions, for which there is a simpler closed form [31]. In any case, we can more efficiently compute the result of Theorem 2 using the following representation for $g(t)$.

**Proposition 1.** The function $g(t)$ in Theorem 2 can be evaluated as the convolution chain

$$g(t) = g_1(t) * g_2(t) * \cdots * g_L(t)$$

where

$$g_1(t) = -e^{-t}(1 - \mathcal{P}(N, \frac{e^{-t} - 1}{\rho}))u(-t) - e^{-t}u(t - 1)$$

$$g_i(t) = \frac{(e^{-t} - 1)^{N-1} e^{\frac{1-e^{-t}}{\rho}}}{\rho^N \Gamma(N)} u(-t), \quad i = 2, \ldots, L,$$

$u(t)$ is the unit step function defined as

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases},$$

and $*$ represents the continuous-time convolution operation.

**Proof:** See Appendix E.

The convolution chain in (17) can be easily evaluated numerically, which is illustrated in Section VII.

V. **Performance Comparisons with MIMO**

Since we want to compare the performance of IMIMO to that of MIMO without feedback, we can compare an upper bound (worst-case performance) and a lower bound (best-case performance) on outage probability for IMIMO and MIMO, respectively. A lower bound on the outage probability of MIMO without feedback is given by the following theorem.

**Theorem 3.** For a MIMO system with $M$ transmit and $N$ receive antennas and without feedback, the outage probability for Rayleigh fading in the channel model of (1) is lower bounded by

$$p_{\text{out, MIMO}} \geq \mathbb{P}\left( \frac{MN(2^\frac{R}{L} - 1)}{\rho}, MN \right),$$

where $\rho$ is the SNR per received antenna and $R$ is the rate in b/s/Hz.

**Proof:** See Appendix F.

Because potential retransmissions for a specific message result in transmissions with a lower effective rate and more effective power in IMIMO than in MIMO without feedback, the expected power and rate need to normalized for a fair comparison of these schemes. If the transmitter initially uses rate $R$ b/s/Hz and power $\rho$ Joules per two-dimension (J/2D) (since the noise is assumed to have unit power, transmitted power is the same as SNR), then the expected rate and power of IMIMO (for all three IMIMO schemes) can be obtained as

$$\bar{R} = Q(L - 1)R + \sum_{l=1}^{L-1} S(l)R$$

$$\bar{\rho} = Q(L - 1)LR + \sum_{l=1}^{L-1} S(l)l\rho$$

where $S(l) = \mathbb{P}\{\bar{A}_1 \cap \ldots \cap \bar{A}_{l-1} \cap \bar{A}_l\}, \quad Q(l) = \mathbb{P}\{\bar{A}_1 \cap \ldots \cap \bar{A}_l\}, \quad Q(0) = 1$ by definition, and $\bar{A}_l$ are the events of receiving an ACK and NACK after transmission $l$ within each round of IMIMO, respectively. Note that (21) and (22) are generic equations that hold for any system using any type of ARQ. Because there is no retransmission, for MIMO, we have

$$\bar{R} = R$$

$$\bar{\rho} = \rho.$$
VI. LONG-TERM THROUGHPUT ANALYSIS

If the channel changes independently for each round of IMIMO, then stopping transmission after receiving an ACK or reaching the maximum number of ARQ transmissions could be modeled as a renewal process [19], [27]. However, in the underlying channel model, the fading coefficients (channel state) remain constant during $L$ transmissions within each round of IMIMO. Furthermore, the transmitter could pick the same antenna (probably the same channel state) through which it has received an ACK for the next transmission. This means there could be multiple messages for which transmission stops during a single channel state. One example of such a scenario can be seen in Fig. 6, where for example the first and second rounds of ARQ share the same channel state, $h_{1,2}$. Therefore, the interarrival times between two consecutive stopping events may not be independent, because these events may occur during a single channel state, and stopping transmission of the current message after receiving an ACK or reaching the maximum number of ARQ rounds is no longer a renewal process.

Analyzing the exact LTT for IMIMO is involved due to the potential dependence of stopping events discussed above. We instead develop a lower bound on LTT by assuming independent interarrival times in the underlying model. Interarrival times are not independent in general because the same transmit antenna is used after a successful transmission, and by the independence assumption, we assume that always a different antenna is picked for the next transmission even after a successful transmission. In this way, the transmitter is losing the benefit of a conditionally better antenna for transmission and this is why we get a lower bound on LTT.

By assuming independent interarrival times, we can lower bound the LTT of the IMIMO as follows. If $T$ is the random variable representing the interarrival times of the renewal process, by using the renewal-reward theorem [27], the long-term average throughput in terms of transmitted b/s/Hz is

$$\eta_{\text{MIMO}}(\rho) \geq \liminf_{\tau \to \infty} \frac{1}{\tau} \sum_{t=1}^{\tau} R'[t]$$

(29)

$$= \mathbb{E}\left[\frac{R'}{\mathbb{E}[T]}\right]$$

(30)

$$= \frac{R(1 - Q(L))}{\sum_{l=0}^{L-1} Q(l)}$$

(31)

where $R$ is the rate of the initial IMIMO transmission and $R'[t]$ is the effective rate of transmission, i.e., the actual rate of data that is received in slot $t$, which is $R$ if a renewal event occurs at time $t$ because of the successful transmission and 0 otherwise. Then (30) follows from using renewal-reward theorem and (31) follows from a simplification in [19]. The lower bound on LTT derived in (31) shows good agreement with Monte Carlo simulation results in Section VII. We note that this bound can serve as the exact LTT for MIMO systems without feedback by letting $L = 1$.

VII. NUMERICAL RESULTS

In this section we verify through Monte Carlo simulations the exact outage probability of Theorem 2 and the tightness of the upper and lower bounds of Theorems 1 and 3, respectively. We also compare the performance of IMIMO schemes and MIMO without feedback in terms of outage probability and LTT.

Fig. 3 compares the exact outage probability from Theorem 2, the upper bound on outage probability from Theorem 1, and Monte Carlo simulation results for a $2 \times 2$ IMIMO system using H-ARQ with IR and $L = 2$. We observe that the exact outage probability from Theorem 2 is visually indistinguishable from the Monte Carlo simulation results, and the upper bound becomes tighter as the transmission rate increases. Fig. 4 compares the lower bound on outage probability from Theorem 3 and Monte Carlo simulation results for a $2 \times 2$ MIMO system without feedback. As we can see, the lower bounds become tighter as the transmission rate decreases.

Fig. 5 compares the exact outage probabilities of a $2 \times 2$ IMIMO system with $L = 2$ using ARQ, H-ARQ with CC, and H-ARQ with IR to the corresponding Monte Carlo simulation results for a $2 \times 2$ MIMO system without feedback. For fairness, we normalize the expected transmit power and expected rate and compare the outage probabilities of these four schemes for $R = 1, 2, 6$ b/s/Hz. Specifically, we determine $\bar{R}$

3Here $t$ refers to the absolute index of rounds starting from the beginning of the transmission.

4Since the exact analytical outage probability for MIMO systems without feedback is less tractable, we use Monte Carlo simulation results instead.
and \( \bar{P} \) for IMIMO \(^5\), and then show \( P_{\text{out}, \text{MIMO}}(R, \bar{P}) \). We note that due to these normalizations the behavior of the outage probability for MIMO is not monotonic in SNR. We can see that for the low rate regime \( (R = 1 \text{ b/s/Hz} \text{ in this case}) \), all three IMIMO schemes are superior to MIMO without feedback for almost all examined SNR. However, for the higher rate regime \( (R = 6 \text{ b/s/Hz} \text{ in this case}) \), MIMO without feedback does better than IMIMO using ARQ and H-ARQ with CC for almost all examined SNR. It does better than IMIMO using H-ARQ with IR up to an SNR threshold \( (\rho \approx 16 \text{ dB} \text{ in the case of } R = 6 \text{ b/s/Hz}) \), but beyond this threshold, IMIMO using H-ARQ with IR is again superior in terms of outage probability. This SNR threshold becomes smaller in the moderate rate regimes \( (\rho \approx 3 \text{ dB} \text{ in the case of } R = 2 \text{ b/s/Hz}) \). The upper and lower bounds on outage probability of IMIMO using H-ARQ with IR and MIMO without feedback, respectively, have been depicted in Fig. 5(c), which shows that these bounds correctly mimic the behavior of the exact outage probability. Another observation is that IMIMO schemes have better performance in terms of outage probability than MIMO without feedback at sufficiently high SNR for all rates.

The ITU-R target for cell spectral efficiency in 4G systems is \( R = 2 \text{ b/s/Hz} \) [32]. From the above comparisons, we observe that 2 \times 2 IMIMO using H-ARQ with CC and IR have better performance than 2 \times 2 MIMO without feedback for large ranges of SNR (SNR \( \geq 3 \text{ dB} \) and SNR \( \geq 4 \text{ dB} \), respectively). Fig. 6 compares the lower bound on LTT of a 2 \times 2 IMIMO system using H-ARQ with CC and H-ARQ with IR and \( L = 2 \) from (31) to Monte Carlo simulations for \( R = 6 \text{ b/s/Hz} \). In this plot the exact LTT of a 2 \times 2 MIMO and a 1 \times 2 SIMO system without feedback also have been illustrated. This comparison reveals that the lower bound is close to the Monte Carlo simulation results in the low and high SNR regimes. Intuitively, this tightness at low and high SNRs can be explained as follows. In the high SNR regime, the transmitter receives an ACK with high probability (proportional to \( 1 - \frac{\sigma}{\rho} \)) from the first transmission of each codeword. Thus, it does not matter if the transmitter changes the transmitting antenna (independent inter-arrival times) or if it transmits through the same antenna for which the transmitter has received an ACK since all channels are good with high probability. In this case both the lower bound and the simulation approach \( \eta = R \) b/s/Hz. At low SNR, the transmitter receives \( L - 1 \) consecutive NACKs for transmitting \( L \) codewords for a specific message with high probability (proportional to \( \frac{1}{\rho} \)). Since the fading remains constant over \( L \) rounds of IMIMO, the block codes corresponding to different messages experience different and independent channels, i.e., multiple ACKs do not occur during a single channel state, and the independence assumption is

\(^5\)It is important to note that for \( L = 2 \), \( R \) and \( \bar{P} \) are the same for all three IMIMO schemes which makes it possible to compare their performance with MIMO without feedback on the same plot in Fig. 5.
valid for this case. In this case both the approximation and the simulation approach \( \eta = R/L \). Another useful observation from this plot is the fact that the IMIMO system (H-ARQ with IR) outperforms SIMO and MIMO systems in terms of LTT at moderate SNRs, but for high SNR MIMO systems do better than IMIMO. However, we should note that the complexity of IMIMO systems is much less than of a MIMO system without feedback, and this makes IMIMO systems appealing.

VIII. CONCLUSION

We developed three incremental MIMO schemes as low-complexity options for wireless communications with multiple antennas. IMIMO does not need complex space-time coding and decoding algorithms, and only requires one RF chain in the transmitter. The minor added complexities in comparison to SIMO without feedback are feedback in the link layer, i.e., one bit ARQ feedback per transmission, and memory in the physical layer on the receive side for H-ARQ schemes. Outage performance comparisons showed that, in the low rate regime, IMIMO schemes outperform MIMO without feedback for almost all examined SNR. For higher rates, MIMO without feedback does better up to an SNR threshold that increases with rate, but beyond this threshold IMIMO schemes again have better performance. This observation suggests that IMIMO suffers some finite SNR multiplexing loss relative to conventional MIMO, which is not surprising. Nevertheless, with significantly less complexity in the physical layer compared to MIMO without feedback, and with better performance in broad rate and SNR regimes, IMIMO appears to be an intriguing option for larger, multiple-antenna radios envisioned for next generation wireless systems.

APPENDIX

A. Proof of Fact 1

Recalling the channel model (1), the outage event of IMIMO using ARQ at round \( l \) can be written as

\[
\tilde{A}_l = \{ I(x_l; y_l) = \log (1 + \rho \| h_l \|^2) < R \}
\]

(32)

So, the outage probability of IMIMO using ARQ is

\[
P_{\text{out,IMIMO}}^{\text{ARQ}} = P \left( \bigcap_{l=1}^{L} \tilde{A}_l \right).
\]

(33)

From the independence of the \( h_i \)'s we can conclude the independence of events \( \tilde{A}_i \)'s. Therefore,

\[
P_{\text{out,IMIMO}}^{\text{ARQ}} = \prod_{l=1}^{L} P \left( \tilde{A}_l \right)
\]

(34)

\[
= \prod_{l=1}^{L} P \left\{ \log (1 + \rho \| h_l \|^2) < R \right\}
\]

(35)

\[
= \prod_{l=1}^{L} P \left\{ \| h_l \|^2 < \frac{2R - 1}{\rho} \right\}
\]

(36)

Since \( h_l \) is a zero-mean complex circularly symmetric Gaussian vector with variance 1, the \( h_i \)'s have normalized Chi-square distribution with \( 2N \) degrees of freedom, i.e., its PDF is

\[
f_h(h) = \frac{1}{(N-1)!} h^{N-1} e^{-h} dh
\]

(37)

Substituting into (36), the outage probability can be derived as follow

\[
P_{\text{out,IMIMO}}^{\text{ARQ}} = \prod_{l=1}^{L} \int_0^{\frac{2R-1}{\rho}} \frac{1}{(N-1)!} h^{N-1} e^{-h} dh
\]

(38)

\[
= \prod_{l=1}^{L} \left( P \left( \frac{2R-1}{\rho}, N \right) \right)^{L}
\]

(39)

where the last step is due to [30, 3.351.1].

B. Proof of Fact 2

The outage event of IMIMO using H-ARQ with CC at round \( l \) can be written as

\[
\tilde{A}_l = \left\{ I(x_l; y_l^1) = \log \left( 1 + \rho \sum_{i=1}^{L} \| h_i \|^2 \right) < R \right\}
\]

(40)

where \( y_l^1 = [y_1, y_2, ..., y_l]^T \) denotes the vector of all outputs from the first to the \( l \)th transmission. We can see from (40) that \( \tilde{A}_L \subseteq \tilde{A}_{L-1} \subseteq ... \subseteq \tilde{A}_1 \), so that the outage probability of IMIMO using H-ARQ with CC is

\[
P_{\text{out,IMIMO}}^{\text{H-ARQ, CC}} = P \left( \bigcap_{l=1}^{L} \tilde{A}_l \right) = P \{ A_L \}.
\]

(41)

Therefore,

\[
P_{\text{out,IMIMO}}^{\text{H-ARQ, CC}} = P \left\{ \log \left( 1 + \rho \sum_{i=1}^{L} \| h_i \|^2 \right) < R \right\}
\]

(42)

Since \( \| h_l \|^2 \) has the normalized Chi-square distribution with \( 2N \) degrees of freedom, \( \sum_{i=1}^{L} \| h_i \|^2 \) has normalized Chi-square distribution with \( 2LN \) degrees of freedom. The rest of the proof follows the proof of Fact 1.
C. Proof of Theorem 1

The outage event of IMIMO using H-ARQ with IR at round \( l \) can be written as
\[
\mathcal{A}_l = \left\{ \mathbf{I} (\mathbf{x}_l; \mathbf{y}_1) = \sum_{i=1}^{L} \log (1 + \rho \| \mathbf{h}_i \|^2) < R \right\}
\]
(43)

Since we again have \( \mathcal{A}_L \subseteq \mathcal{A}_{L-1} \subseteq \cdots \subseteq \mathcal{A}_1 \), following the first few steps of proof of Fact 2, the outage probability is given by
\[
P_{\text{out}}^{\text{H-ARQ, IR}} = \mathbb{P} \left\{ \mathcal{A}_L \right\} = \mathbb{P} \left\{ \sum_{i=1}^{L} \log (1 + \rho \| \mathbf{h}_i \|^2) < R \right\} = \mathbb{P} \left\{ \prod_{i=1}^{L} (1 + \rho \| \mathbf{h}_i \|^2) < 2^R \right\}.
\]
(44)

We can rewrite the expression \( \prod_{i=1}^{L} (1 + \rho \| \mathbf{h}_i \|^2) \) as
\[
\det \left( I_L + \rho \begin{pmatrix} \| \mathbf{h}_1 \|^2 & 0 & \cdots & 0 \\ 0 & \| \mathbf{h}_2 \|^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \| \mathbf{h}_L \|^2 \end{pmatrix} \right)
\]
(47)

where \( I_L \) is the \( L \times L \) identity matrix. By applying Minkowski’s inequality [26, Theorem 7.8.8] to (47) we get
\[
\det( I_L + B ) \geq (1 + \det(B)^{\frac{1}{L}})^L
\]
(49)

Where
\[
B = \rho \begin{pmatrix} \| \mathbf{h}_1 \|^2 & 0 & \cdots & 0 \\ 0 & \| \mathbf{h}_2 \|^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \| \mathbf{h}_L \|^2 \end{pmatrix}
\]
(50)

So, we can upper bound the outage probability as
\[
P_{\text{out,imimo}}^{\text{H-ARQ, IR}} \leq \mathbb{P} \left\{ \left( 1 + \left( \rho \prod_{i=1}^{L} \| \mathbf{h}_i \|^2 \right)^{\frac{1}{L}} \right)^L < 2^R \right\}
\]
(51)

where (55) follows from the integral [30, 7.811.2].

D. Proof of Theorem 2

Following the first few lines of proof of Theorem 1, we can get the outage probability as
\[
P_{\text{out}}^{\text{H-ARQ, IR}} = \mathbb{P} \left\{ \sum_{i=1}^{L} \log (1 + \rho \| \mathbf{h}_i \|^2) < R \right\} = \mathbb{P} \left\{ \prod_{i=1}^{L} (1 + \rho \| \mathbf{h}_i \|^2) < 2^R \right\}.
\]
(43)

Since \( \| \mathbf{h}_i \|^2 \) has the normalized Chi-square distribution with \( 2N \) degrees of freedom, \( \mathbf{y}_1 = (1 + \rho \| \mathbf{h}_i \|^2) \) has the generalized Gamma distribution with the power parameter 1, shape/power parameter \( N \), scale parameter \( \rho \) and location parameter \( 1 \) [33], i.e., its distribution is
\[
f_{\mathbf{y}_1}(y) = \frac{1}{\rho (N-1)!} y^{N-1} e^{-\frac{y}{\rho}} u(y)
\]
(58)

To derive the distribution of \( \mathbf{w} = \prod_{i=1}^{L} \mathbf{y}_i \) we first derive the Mellin transform [34] of \( \mathbf{y}_1 \) as follow
\[
\mathcal{M}(f_{\mathbf{y}_1}(y) \mid s) = \int_{0}^{\infty} f(y) y^{s-1} dy
\]
(59)

\[
= \int_{0}^{\infty} \frac{1}{(\rho)^N(N-1)!} y^{N-1} (y+1)^{s-1} e^{-\frac{y}{\rho}} dy
\]
(60)

\[
= \frac{U \left( N, N+s; \frac{1}{\rho} \right)}{(\rho)^N}
\]
(61)

where (61) follows from [35, 13.2.5]. Since the \( \mathbf{y}_i \)'s are independent, the Mellin transform of \( \mathbf{w} \) can be written as [34]
\[
\mathcal{M}(f_{\mathbf{w}}(w) \mid s) = \prod_{i=1}^{L} \frac{U \left( N, N+s; \frac{1}{\rho} \right)}{(\rho)^N}
\]
(62)

So, we can get the distribution of \( \mathbf{w} \) as an inverse Mellin transform as
\[
f_{\mathbf{w}}(w) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} w^{-s} \mathcal{M}(f(w) \mid s) ds
\]
(64)

\[
= \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} w^{-s} \left( \frac{U \left( N, N+s; \frac{1}{\rho} \right)}{(\rho)^N} \right)^L ds
\]
(65)

Equivalently, the probability of outage can be derived as
\[
P_{\text{out,imimo}}^{\text{H-ARQ, IR}} = \int_{1}^{2^R} f_{\mathbf{w}}(w) dw
\]
(66)

\[
= \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \left( \frac{U \left( N, N+s; \frac{1}{\rho} \right)}{(\rho)^N} \right)^L \int_{1}^{2^R} w^{-s} dw ds
\]
(67)

\[
= \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{2^{-Rs} - 1}{1 - s} \left( \frac{U \left( N, N+s; \frac{1}{\rho} \right)}{(\rho)^N} \right)^L ds
\]
(68)

\[
= 2^R g(-R \ln 2) - g(o)
\]
(69)
where $g(t)$ can be written as

$$g(t) = \mathcal{L}^{-1} \left( \frac{1}{1-s} \left( \frac{U \left( N, N + s, \frac{1}{\rho} \right)}{(p)^N} \right)^L \right)$$

(70)

and (69) follows from the definition of inverse Laplace transform.

E. Proof of Proposition 1

We define

$$G_i(s) = \frac{U \left( N, N + s, \frac{1}{\rho} \right)}{(1-s)^p}$$

(71)

$$G_i(s) = \frac{U \left( N, N + s, \frac{1}{\rho} \right)}{\rho^N}, \quad i = 2, \ldots, L$$

(72)

Then we have

$$g(t) = \mathcal{L}^{-1} \left( \prod_{i=1}^{L} G(s) \right) = g_1(t) \ast \cdots \ast g_L(t)$$

(73)

where

$$g_i(t) = \mathcal{L}^{-1} (G_i(s)), \quad i = 1, \ldots, L$$

(74)

Now we need to derive the $g_i(t)$. For $g_1(t)$ we have

$$g_1(t) = \mathcal{L}^{-1} \left( \frac{U \left( N, N + s, \frac{1}{\rho} \right)}{(1-s)^p} \right)$$

(75)

$$= \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{st} \int_{0}^{\infty} r^{N-1}(r+1)^{s-1} e^{-\frac{s}{p}dr} ds$$

(76)

$$= \int_{0}^{\infty} \left( \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{st} \frac{(r+1)^{s-1}}{1-s} dr \right) \frac{r^{N-1}e^{-\frac{s}{p}}}{\rho^N(N)}$$

(77)

where (76) follows from the definition of inverse Laplace transform and Tricomi confluent hypergeometric function [30]. By definition of Laplace inverse transformation we know that

$$\frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{st} (r+1)^{s-1} \frac{1}{1-s} ds = \mathcal{L}^{-1} \left( \frac{r+1)^{s-1}}{1-s} \right)$$

(78)

$$= \frac{1}{r+1} \mathcal{L}^{-1} \left( \frac{e^{\ln(r+1)}}{1-s} \right) = -e^t u(t + \ln(r+1))$$

(79)

Therefore, we have

$$g_1(t) = -\frac{e^t}{\rho^N(N)} \int_{0}^{\infty} r^{N-1}e^{-\frac{s}{p}}u(t + \ln(r+1))dr$$

(80)

$$= -\frac{e^t}{\rho^N(N)} \int_{-\infty}^{t} r^{N-1}e^{-\frac{s}{p}} dr$$

(81)

Using the integral formula of [30, 3.351.2], we can get

$$g_1(t) = -e^t \Gamma \left( N, \frac{e^{-t}}{\rho} \right) u(-t) - e^t u(t-1)$$

(82)

The derivation of $g_i(t)$ for $i = 2, \ldots, L$ follows exactly the same steps as in derivation of $g_1(t)$ and we omit the proof. \qed

F. Proof of Theorem 3

The outage probability of a MIMO system with $M$ transmit and $N$ received antennas, using all transmit antennas simultaneously with no CSIT and full CSIR can be written as [9]

$$p_{out,MIMO} = \mathbb{P} \left\{ \log \left( \det \left( I_N + \frac{\rho}{M} \mathbf{H} \mathbf{H}^H \right) \right) < R \right\}$$

(83)

which by singular value decomposition can be written as

$$p_{out,MIMO} = \mathbb{P} \left\{ \sum_{i=1}^{N} \log \left( 1 + \frac{\rho}{M} \lambda_i(\mathbf{H} \mathbf{H}^H) \right) < R \right\}$$

(84)

where $\lambda_i(\mathbf{H} \mathbf{H}^H)$’s are the eigenvalues of $\mathbf{H} \mathbf{H}^H$. We know that $\log(x)$ is a concave function in $x$ and so according to Jensen’s inequality we have [26, Appendix B]

$$g(\Sigma_i \gamma_i t_i) \geq \sum_i \gamma_i g(t_i)$$

(85)

By choosing $g(t) = \log(1 + \frac{\rho}{M} t)$ and $\gamma_i = \frac{1}{N}$ and $t_i = \lambda_i(\mathbf{H} \mathbf{H}^H)$ and using (85), the outage probability can be upper bounded by

$$p_{out,MIMO} \geq \mathbb{P} \left\{ N \log \left( 1 + \frac{\rho}{M} \sum_{i=1}^{N} \frac{1}{N} \lambda_i(\mathbf{H} \mathbf{H}^H) \right) < R \right\}$$

(86)

$$= \mathbb{P} \left\{ N \log \left( 1 + \frac{\rho}{MN} \text{Tr}(\mathbf{H} \mathbf{H}^H) \right) < R \right\}$$

(87)

$$= \mathbb{P} \left\{ \text{Tr}(\mathbf{H} \mathbf{H}^H) < \left( \frac{2^R - 1}{\rho} \right) M N \right\}$$

(88)

where $\text{Tr}(\mathbf{H} \mathbf{H}^H) = \sum_{i=1}^{N} \sum_{j=1}^{M} |h_{i,j}|^2$ is the sum of square of elements of $\mathbf{H}$ which has normalized Chi-square distribution with $2MN$ degrees of freedom, i.e. its PDF is

$$f_h(h) = \frac{1}{(MN-1)!} h^{MN-1} e^{-h} u(h)$$

(89)

Now we can lower bound the outage probability by integrating $f_h(h)$ over the interval $\left( 0, \left( \frac{2^R - 1}{\rho} \right) \right)$

$$p_{out,MIMO} \geq \int_{0}^{\left( \frac{2^R - 1}{\rho} \right) MN} f_h(h) dh$$

(90)

$$= \mathbb{P} \left\{ \left( \frac{2^R - 1}{\rho} \right) M N, M N \right\}$$

(91)

where (91) is due to [30, 3.381.1]. \qed

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